MATH3802 Time Series, Exercise Sheet 1

Ex 1.1: The `lynx` data set in R gives the annual number of lynx trapped at Mackenzie River between 1821 and 1934. Use R to both plot and describe the data.

The following R commands may be helpful for this question:

```
data(lynx)                    # connect to lynx data set
lynx                          # print lynx data on screen
plot(lynx, xlab="Year", ylab="Number", main="Title")
summary(lynx)                 # summary statistics
mean(lynx)
var(lynx)
sd(lynx)
```

Ex 1.2: The data below give the average fuel consumption of a 40-tonne rig in litres per 100 km and are given in file lorry.txt on the VLE, with `NA` denoting missing values.

Year	1965	1970	1975	1980	1985	1990	1995	1996	1997
Fuel consumption	52.5	50.0	48.0	43.5	41.5	39.0	38.0	36.5	35.0

Use the equations from the lectures to obtain a 95% confidence interval for the mean emission of lorries during the year 2000.

Repeat your analysis using R. The following commands may be helpful:

```
lorry = scan("lorry.txt") # assumes lorry.txt is saved in your file space
# missing values are denoted NA in the above file
# setup dates for all years between 1965 and 1997
dates = c(1965:1997)
# analyse linear model   lorry = A + B*dates
ll = lm(lorry~dates)
predict(ll, data.frame(dates=c(2000)), interval=c("confidence"),
        se.fit=TRUE, level=c(0.95))
```

Ex 1.3: The R data set `co2` (accessible in the variable `co2` in R) gives the monthly concentration of carbon dioxide in parts per million at Mauna Loa between January 1959 and December 1997. Use R to fit the following two models.

(a) CO_2 level is a linear trend.

(b) CO_2 level is a linear trend with a seasonal component.

Plot the residuals for each model.

Ex 1.4: Suppose n observations $X_1, X_2, \ldots, X_n$ have mean μ, variance σ^2, and $\operatorname{cor}(X_i, X_{i+1}) = \rho$ for all i. All other correlations equal zero.

(a) If

$$\bar{X} = \frac{1}{n}\sum_{t=1}^{n} X_t,$$

show that

$$\operatorname{var}(\bar{X}) = \frac{\sigma^2}{n}\Big(1 + \frac{2(n-1)\rho}{n}\Big).$$

(b) Deduce that if $\rho > 0$ the confidence interval for μ will be wider than in the case with $\{X_t\}$ independent. What happens if $\rho < 0$?

Hints:

(1) Recall that

$$\operatorname{var}(\bar{X}) = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\operatorname{cov}(X_i, X_j).$$

How many terms have $i = j$*? How many have* $i - j = \pm 1$*?*

(2) Recall that $\operatorname{cov}(X_i, X_j) = \operatorname{cor}(X_i, X_j)\sqrt{\operatorname{var}(X_i)\operatorname{var}(X_j)}$.