

CptS 223 Homework #3 - Heaps, Hashing, Sorting

Due Date: Nov 20th 2020

Please complete the homework problems and upload a pdf of the solutions to blackboard assignment and upload the PDF to Git.

1. [6] Starting with an empty hash table with a fixed size of 11, insert the following keys in order into three distinct hash tables (one for each collision mechanism): {12, 9, 1, 0, 42, 98, 70, 3}. You are only required to show the final result of each hash table. In the very likely event that a collision resolution mechanism is unable to successfully resolve, simply record the state of the last successful insert and note that collision resolution failed. For each hashtable type, compute the hash as follows:

$$\text{hashkey(key)} = (\text{key} * \text{key} + 3) \% 11$$

Separate Chaining (buckets)

	3		0	12		9	70			
0	1	2	3	4	5	6	7	8	9	10

To probe on a collision, start at hashkey(key) and add the current $\text{probe}(i')$ offset. If that bucket is full, increment i until you find an empty bucket.

Linear Probing: $\text{probe}(i') = (i + 1) \% \text{TableSize}$

	3		0	12	1	98	9	70	42	
0	1	2	3	4	5	6	7	8	9	10

Quadratic Probing: $\text{probe}(i') = (i * i + 5) \% \text{TableSize}$

	42		0	12	3		9	70	1	98
0	1	2	3	4	5	6	7	8	9	10

2. [3] For implementing a hash table. Which of these would probably be the best initial table size to pick?

Table Sizes:

1

100

101

15

500

Why did you choose that one?

I don't know the total number of entries so I picked the maximum size to avoid a high load factor and rehashing.

3. [4] For our running hash table, you'll need to decide if you need to rehash. You just inserted a new item into the table, bringing your data count up to 53491 entries. The table's vector is currently sized at 106963 buckets.

- Calculate the load factor (λ):

$$53491 / 106963 \approx 0.5$$

- Given a linear probing collision function should we rehash? Why?

Yes, because the load factor is $> .5$

- Given a separate chaining collision function should we rehash? Why?

No, it's better to rehash when the load factor
 $> .75$

4. [4] What is the Big-O of these actions for a well designed and properly loaded hash table with N elements?

Function	Big-O complexity
Insert(x)	$O(1)$
Rehash()	$O(N)$
Remove(x)	$O(1)$
Contains(x)	$O(1)$

7. [3] I grabbed some code from the Internet for my linear probing based hash table at work because the Internet's always right (totally!). The hash table works, but once I put more than a few thousand entries, the whole thing starts to slow down. Searches, inserts, and contains calls start taking *much* longer than $O(1)$ time and my boss is pissed because it's slowing down the whole application services backend I'm in charge of. I think the bug is in my rehash code, but I'm not sure where. Any ideas why my hash table starts to suck as it grows bigger?

```
/**  
 * Rehashing for linear probing hash table.  
 */  
void rehash( )  
{  
    ArrayList<HashItem<T>> oldArray = array;  
  
    array = new ArrayList<HashItem<T>>( 2 * oldArray.size() );  
  
    for( int i = 0; i < array.size(); i++ )  
        array.get(i).info = EMPTY;  
    // Copy old table over to new larger array  
    for( int i = 0; i < oldArray.size(); i++ ) {  
        if( oldArray.get(i).info == FULL )  
        {  
            addElement(oldArray.get(i).getKey(),  
                      oldArray.get(i).getValue());  
        }  
    }  
}
```

The hash table needs to re-insert each individual value instead of re-inserting at the specified key location

8. [4] Time for some heaping fun! What's the time complexity for these functions in a Java Library priority queue (binary heap) of size N?

Function	Big-O complexity
push(x)	$O(\log n)$
top()	$O(1)$
pop()	$O(\log n)$
PriorityQueue(Collection<? extends E> c) // BuildHeap	$O(n)$

9. [4] What would a good application be for a priority queue (a binary heap)? Describe it in at least a paragraph of why it's a good choice for your example situation.

A shortest path algorithm would use a binary heap to efficiently get the minimum distance vertex from a set of vertices

10. [4] For an entry in our heap (root @ index 1) located at position i , where are it's parent and children?

Parent: $\frac{i}{2}$

Children:
Left child: $2i$
Right child: $2i + 1$

11. [6] Show the result of inserting 10, 12, 1, 14, 6, 5, 15, 3, and 11, one at a time, into an initially empty binary heap. Use a 1-based array like the book does. After insert(10):

10										
----	--	--	--	--	--	--	--	--	--	--

After insert (12):

12	10									
----	----	--	--	--	--	--	--	--	--	--

etc:

12	10	1								
----	----	---	--	--	--	--	--	--	--	--

14	12	1	10							
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14	12	1	10	6						
----	----	---	----	---	--	--	--	--	--	--

14	12	5	10	6	1					
----	----	---	----	---	---	--	--	--	--	--

15	12	14	10	6	1	5				
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15	12	14	11	6	1	5	3			
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15	12	14	11	6	1	5	3	10		
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12. [4] Show the same result (only the final result) of calling buildHeap() on the same vector of values: {10, 12, 1, 14, 6, 5, 15, 3, 11}

15	14	10	12	6	5	1	3	11		
----	----	----	----	---	---	---	---	----	--	--

13. [4] Now show the result of three successive deleteMin / pop operations from the prior heap:

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14. [4] What are the average complexities and the stability of these sorting algorithms:

Algorithm	Average complexity	Stable (yes/no)?
Bubble Sort		
Insertion Sort		
Heap sort		
Merge Sort		
Radix sort		
Quicksort		

15. [3] What are the key differences between Mergesort and Quicksort? How does this influence why languages choose one over the other?

16. [4] Draw out how Mergesort would sort this list:

24	16	9	10	8	7	20
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17. [4] Draw how Quicksort would sort this list:

24	16	9	10	8	7	20
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Let me know what your pivot picking algorithm is (if it's not obvious):