Poker Skill, Hands, and Bet Limits Capstone Project

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[1]Bayesian Statistics: Techniques and Models - Capstone Project

Abstract

This project explores how poker skill, hand quality, and bet limits affect a player's final cash balance using a simulated dataset. I applied Bayesian statistics to analyze the main effects and interactions between these factors. The results from the initial visualization indicated that Skill level and Hand quality have impacts on the outcome. However, the model showed convergence issues, which calls into question the reliability of the parameter estimates. In this report, I discuss the convergence diagnostics and provide recommendations for improving convergence in the model.

1. Introduction

This report aim to investigate how three factors—poker skill, hand quality, and bet limits—affect the final cash balance of players in a poker game. By modeling these factors, I want to know the influence of each factor while accounting for uncertainty in the data. The results will allow us to draw conclusions about the relative importance of each factor on poker winnings.

2. Data

The dataset consists of 25 individuals in each of 12 different experimental conditions, resulting in a total of 300 observations. The factors being analyzed are:

- Skill: Two levels (1 = Expert, 2 = Average)
- Hand: Three levels (1 = Bad, 2 = Neutral, 3 = Good)
- Limit: Two levels (1 = Fixed, 2 = None)
- Final Cash Balance: Continuous variable representing the final cash balance in euros, which is the dependent variable.

The data was simulated to reflect realistic means and standard deviations for each condition, with a 3-Factor ANOVA framework.
To check for missing values in the "Final_Cash_Balance" column, I used the following code:

```
#Check for missing values
missing_values <- colSums(is.na(data_clean))
#Check for missing values in the Final Cash Balance column
sum(is.na(data_clean$Final_Cash_Balance))
```

The column contains 148 missing values. Since these missing values may affect the analysis, I removed any rows with missing "Final_Cash_Balance" values. As for visualizing the relationships between the factors and the final cash balance, I used a series of plots to answer key questions: How does skill level impact winnings? Does hand quality influence the cash balance, and how does the bet limit affect outcomes?. Scatter plots were

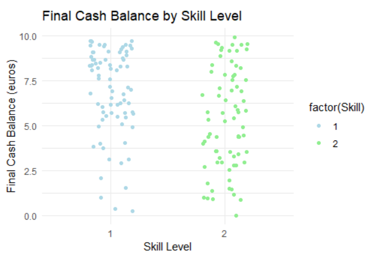

Figure 1. Final Cash Balance by Skill Level

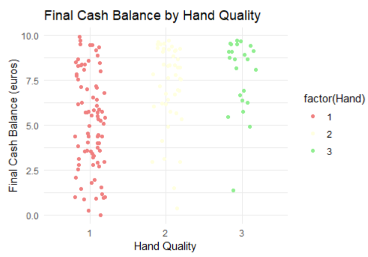

Figure 2. Final Cash Balance by Hand Quality

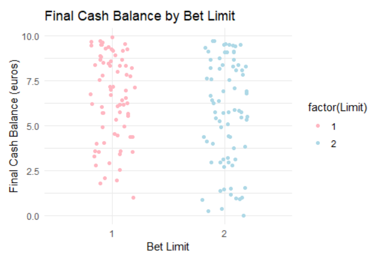

Figure 3. Final Cash Balance by Bet Limit

great for answering these questions and seeing if there were any unexpected patterns in the data.

Across all three factors (Bet Limit, Hand Quality, and Skill Level), there are clear distinctions in how these factors influence the final cash balance:

- Hand Quality and Skill Level appear to have a stronger impact on the final outcome compared to Bet Limit.
- The variability in Final Cash Balance is higher in Bet Limit and Hand Quality, whereas Skill Level seems to result in more predictable outcomes, especially for Expert players.

3. Model Development

The primary research question is to determine how skill level, hand quality, and bet limit impact a player's final cash balance. The hierarchical Bayesian model is well-suited to this task because:

- This model can handle both fixed effects (the main effects of skill, hand, and bet limit) and random effects (the variation in outcomes within each condition).
- Bayesian models allow for the integration of prior beliefs about the parameters, which can be adjusted.
- By using a hierarchical model, the model can estimate parameters for each group while sharing information across groups.

The hierarchical model is specified as:

$$Y_{ijk} \sim \mathcal{N}(\mu_{ijk}, \sigma^2)$$

Where:

$$\mu_{ijk} = \alpha + \beta_1 \cdot \text{Skill}_i + \beta_2 \cdot \text{Hand}_j + \beta_3 \cdot \text{Limit}_k + \epsilon_{ijk}$$

And where:

- Y_{ijk} is the final cash balance for individual k in condition (i, j, k).
- α is the intercept
- $\beta_1, \beta_2, \beta_3$ are the effects of skill level, hand quality, and bet limit.
- ϵ_{ijk} is the residual error for each observation.
- σ^2 is the variance of the residuals.

To fit the model, I used **JAGS** to perform a Bayesian analysis on the data. The model was specified as a hierarchical model, where the final cash balance was predicted by the factors Skill, Hand Quality, and Bet Limit. The outcome variable was modeled as normally distributed, with separate intercepts and slopes for the different categories of each factor. After my first attempt of fitting the first model, the PSRF value is 7.44, which is highly concerning, indicating that the chains have not converged well. To address the convergence issues, I included an interaction term between Skill and Hand Quality. This modification allows the model to account for the possibility that the effect of hand quality on the final cash balance depends on the player's skill level.

```
# Define the new model (with interaction between Skill and Hand Quality)
model_string_new <- "
model {
  for(i in 1:N) {
    Final_Cash_Balance[i] ~ dnorm(mu[i], tau)
    mu[i] <- alpha + beta_skill[Skill[i]] + beta_hand[Hand[i]] + beta_limit[Limit[i]] + beta_interaction[Skill[i], Hand[i]]
  }

  # Priors
  alpha ~ dnorm(0, 0.0001)  # Prior for intercept
  for(s in 1:2) {
    beta_skill[s] ~ dnorm(0, 0.01) # Priors for Skill effects (tighter)
  }
  for(h in 1:3) {
    beta_hand[h] ~ dnorm(0, 0.01) # Priors for Hand Quality effects (tighter)
  }
  for(l in 1:2) {
    beta_limit[l] ~ dnorm(0, 0.01) # Priors for Bet Limit effects (tighter)
  }
  for(s in 1:2) {
    for(h in 1:3) {
      beta_interaction[s, h] ~ dnorm(0, 0.01) # Interaction term between Skill and Hand Quality
    }
  }

  tau ~ dgamma(0.1, 0.1) # Prior for precision (inverse of variance)
}
"

# Prepare the data for JAGS
data_jags_new <- list(
  N = nrow(data_clean),
```

```
  Final_Cash_Balance = data_clean$Final_Cash_Balance,
  Skill = as.numeric(data_clean$Skill),
  Hand = as.numeric(data_clean$Hand),
  Limit = as.numeric(data_clean$Limit)
)

# Initial values for MCMC chains
inits_new <- function() {
  list(alpha = 0, beta_skill = c(0, 0), beta_hand = c(0, 0, 0),
     beta_limit = c(0, 0), beta_interaction = matrix(0, 2, 3),
     tau = 1)
}

# Set up the JAGS model
model_new <- jags.model(textConnection(model_string_new), data =
    data_jags_new, inits = inits_new, n.chains = 3)

# Burn-in and sampling
update(model_new, 2000)  # Increased burn-in period
samples_new <- coda.samples(model_new, variable.names = c("alpha
   ", "beta_skill", "beta_hand", "beta_limit", "beta_interaction
   ", "tau"), n.iter = 10000)

# Summary of the results
summary(samples_new)
```

This model was run with 3 MCMC chains, each with 10,000 iterations, after a burn-in period of 2,000 iterations.

After fitting the new model, we assessed convergence using the Gelman-Rubin diagnostic. The PSRF for the parameters were calculated, and the results are as follows:

- alpha: PSRF = 2.42, with an upper CI of 4.46. This suggests that the intercept term has not fully converged, and further iterations may be required.
- beta_hand (effects of Hand Quality): The PSRF values for beta_hand[1], beta_hand[2], and beta_hand[3] range from 1.11 to 1.46, showing an improvement in convergence, but further tuning is still needed.
- beta_interaction: The interaction terms between Skill and Hand Quality (‘beta_interaction‘) show relatively better convergence, with PSRFs ranging from 1.02 to 1.86.
- beta_limit: The PSRF values for ‘beta_limit[1]‘ and ‘beta_limit[2]‘ range from 1.19 to 1.57, which still indicates mild convergence issues.
- beta_skill: The PSRF for beta_skill[1] and beta_skill[2] range from 1.67 to 2.01, indicating that further improvement in convergence is needed.
- tau: The PSRF for the precision parameter tau is 1.00, indicating that this parameter has converged well.

The multivariate PSRF was calculated to be 2.11, which suggests that the overall model convergence is still not ideal. An alternative model could involve removing or changing the interaction terms or adjusting the priors further. It may also be beneficial to increase the number of chains and iterations to improve mixing.

4. Interpretation of Results

In the current analysis, I observed that the Gelman-Rubin diagnostics indicated that the model had not yet fully converged for several parameters, with PSRF values greater than 1. This suggests that the MCMC chains have not mixed properly, and the estimates may not be reliable.

Until proper convergence is achieved, interpretation of the model results is unrealiable. For now, the focus should be on improving convergence.

5. Limitations and Acknowledgments

While the model provides valuable insights, it has limitations, including issues with convergence for certain parameters, which may affect the reliability of the results. Additionally, the model assumes normality of residuals and does not account for unobserved factors such as player behavior or external influences. This analysis was based on a simulated dataset, and real-world data may present further challenges.

References

G. Meyer, M. von Meduna, T. Brosowski, T. Hayer (2012). "Is poker a Game of Skill or Chance? A Quasi-Experimental Study," Journal of Gambling Studies, Online First DOI 10.1007/s10899-012-9327-8