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Model and simulation of four-wheeled robot based on Mecanum wheel

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Based on Mecanum wheels and "Y"-shaped planetary wheels, we combine these two kinds of wheels' respective motion principle with their advantages to design a new type of four-wheeled robot: install the Mecanum wheels at the end of "Y"-shaped planetary wheel group. The wheel designed based on Mecanum wheels and "Y"-shaped planetary wheel can adapt to the complex terrain such as stairs, steps, and at the same time it can achieve the rotation of the whole body in a limited space. This paper studies the adaptability of the four-wheeled robot to the stairs, analyzing and calculating the parameters of the four-wheeled robot and the stairs.

Keywords: Four-wheeled robot; Mecanum wheel; "Y"-shaped planetary wheel; obstacle avoidance.

1. Introduction

Wheeled robots have advantages of high speed and high efficiency. But the motion environment of wheeled robots cannot be too complicated. For example, it is difficult for wheeled robots to climb stairs. Wheeled robots also face the problem of low turning efficiency and large turning radius.[1] This paper puts forwards to a new type of wheeled robot combining Mecanum wheels which can achieve omni-directional movement with "Y"-shaped planetary wheels which can climb stairs more easily. Based on these two kind of wheels, this robot can achieve the functions of drifting on stairs to save space when turning around and climbing stairs.

The four-wheeled robot can be engaged in the elimination of explosives in public places where there are stairs and the situation is very dangerous for people to rescue.

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This paper makes static analysis of the states of the robot climbing and drifting on stairs. However, more sufficient conditions and states needs to be included and analyzed when the robot avoiding obstacle in different situation, this paper just put forwards a new way for wheeled robots to avoid obstacle on stairs.

2. Static Equilibrium Conditions for the Dumping Robot

The four-wheeled robot is mainly composed of the main body and wheel groups, and is simplified as a carriage in shape, as shown in Fig. 1. Since the wheel groups are symmetrical in structure, the position of the centroids of the wheel groups would not offset during movements. The position of the gravity center of the main body, which is marked as 'G' in Fig. 1, affects the most on the stability of the robot, specifically when the robot is climbing on slope or stairs, as illustrated in Fig. 1. Also, in Fig. 1, h_0 means the distance between the gravity center G to the under-frame of the main body; L means the distance between the front and rear supportive points on the wheel groups; θ means the pitching angle of the four-wheeled robot; φ means the angle between the wheel arm touching stair and the under-frame of the body and e means the small displacement of the gravity center in the process of climbing stairs. In other words, if the gravity center G in Fig. 1 is out of the area between two dotted lines passing the centers of the wheel groups, the robot will be overturned.The condition for the robot not to overturn is that the angle between the under-frame and the horizontal line is less than the incline degree of the stair which could be presented in formulation as follows[2]:

$$[h_0 + R \cdot \sin(\pi - \varphi)] \cdot \sin\theta + e\cos\theta < \left[\frac{L}{2} + R \cdot \cos(\pi - \phi)\right] \cdot \cos\theta. \tag{1}$$

By solving the above inequality, the following condition is required:

$$\tan\theta < \frac{\frac{L}{2} - R \cdot \cos\varphi - e}{h_0 + R \cdot \sin\varphi}, \tag{2}$$

when $\varphi = 0$, $\theta_{\max} = \arctan\frac{L-2R-e}{2h_0}$.

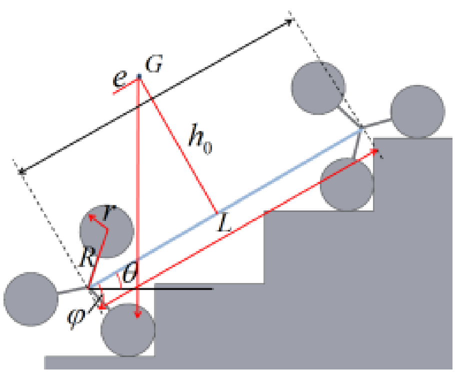

Fig. 1. Illustration of the robot when climbing stairs.

So, it can be seen from above formula that the decrease of h_0 and e is beneficial to improve the ability of the robot to climb stairs.[3]

As shown in the Fig. 2, geometrical subsumed condition is as follows: the robot has no other part to touch the stairs except the wheel in the process of climbing stairs, which also means the condition of the robot's geometric parameters and the geometric parameters of the stairs must be satisfied, and the equality is as follows:

$$\tan\theta = \frac{\frac{\sqrt{3}}{2}R + r}{(\frac{1}{2}R + r) - h}, \tag{3}$$

where R is the length of the "Y"-shaped planetary wheel's arm, r is the radius of one Mecanum wheel, h is the height of one stair.

The relationship between θ and h drawn by MATLAB is shown in Fig. 3.

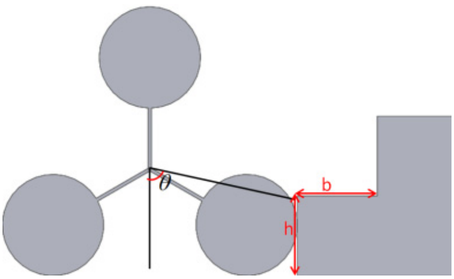

Fig. 2. Geometrical subsumed condition.

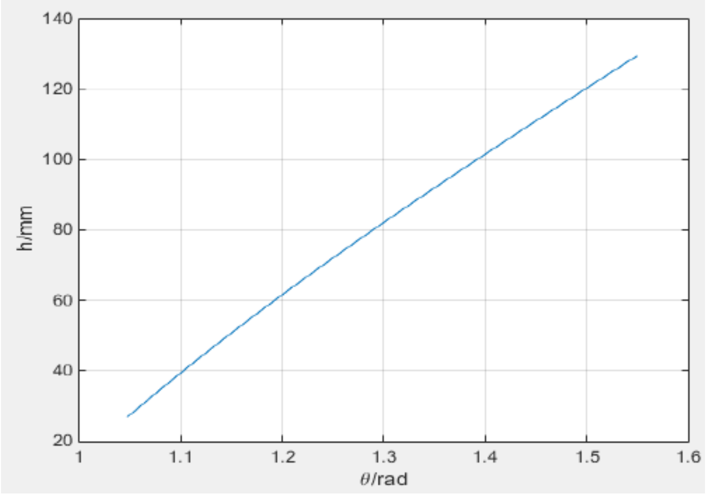

Fig. 3. The relationship between θ and h drawn by MATLAB.

By solving formula (3), we can get:

$$h = \frac{R}{2} + r - \frac{\frac{\sqrt{3}}{2}R + r}{\tan\theta}, \tag{4}$$

when $\theta = \theta_{\max} = \frac{\pi}{2}, h = h_{\max} = \frac{R}{2} + r$.

3. Static Analysis

In order to facilitate the analysis, the following assumptions should be proposed:

(1) The robot body, the driving wheel and the ground are treated as rigid bodies.
(2) The thickness of the wheel is ignored.
(3) Each Mecanum wheel is simplified into a round wheel.
(4) There only occurs pure rolling between the wheel and the ground, which means that the instantaneous speed of the Mecanum wheel when it contacts the ground is equal to zero.[4–6] Based on the above four assumptions, two conditions are considered, respectively, and are explained in the following sections.

3.1. *The initial stage of climbing*

In the initial stage of climbing, the force conditions are shown in Fig. 4 and presented as Eqs. (3) and (4):

$$f_1 = F_{N1} \cdot \mu_1 \geq G_0, \tag{5}$$

$$F_{N1} = 8f_0 = 8G_0 \cdot \mu_0, \tag{6}$$

$$\therefore\ \mu_1 \cdot \mu_0 \geq 0.125,$$

where f_1 is the friction force provided by the vertical plane of stairs, F_{N1} is the support force provided by the vertical plane of stairs, μ_0 is the friction coefficient

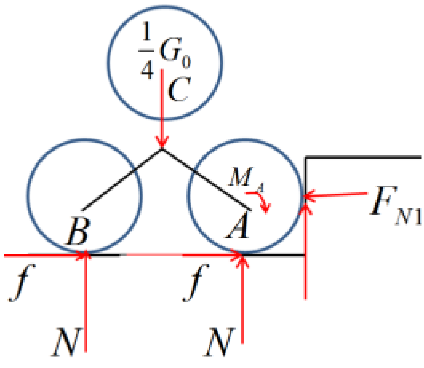

Fig. 4. The initial stage of the front wheel group.

between wheel and ground, μ_1 is the friction coefficient between wheel and stairs, G_0 is the gravity.

3.2. *The second stage of climbing*

Under the interactive force between Mecanum wheels and ground, the front wheel of the robot will turn over and the process is described as follows: Bottom wheel B leaves the ground first and then causes the center of the wheel group to move forward, subsequently leads the bottom wheel A to leave the ground. Finally, the top wheel C will contact horizontal plane of stairs due to inertia. The force analysis of this process is as follows:

$$\sum Y = 0, \quad -G_0 + 6N + 2f_s = 0, \tag{7}$$

$$\sum X = 0, \quad 6f - 2F_{N1} = 0, \tag{8}$$

$$f_s = F_{N1} \cdot \mu_1, \tag{9}$$

$$f = N \cdot \mu_0, \tag{10}$$

where f is the friction force provided by the ground, f_s is the friction force provided by the vertical plane of stairs.

Then take wheel A as the research object. The force analysis is shown in Fig. 5. And the moment of the center of wheel A needs to satisfy the following inequality:

$$\begin{gathered} M \geq f_s \cdot r + f_0 \cdot r, \\ \therefore\ M \geq \frac{G_0 \mu_0 r (3\mu_1 + 1)}{6(\mu_1 \mu_0 + 1)}. \end{gathered} \tag{11}$$

3.3. *Force analysis of the front wheel groups leaving the ground*

Similarly, take the four-wheeled robot as the research object in the first step. In this situation, only four Mecanum wheels in the two rear groups contact with the

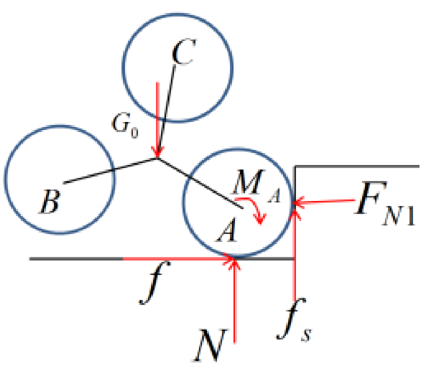

Fig. 5. The second stage of climbing.

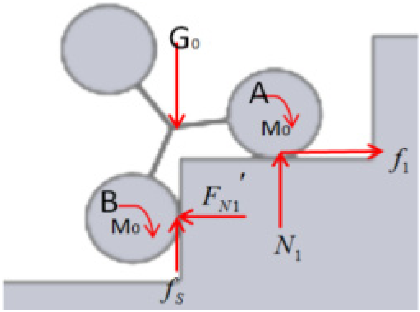

Fig. 6. Stress analysis diagram.

ground while the two wheels A in the two front groups are tangent with horizontal plane of the first stair. The stress analysis is shown in Fig. 6.

The robot's force should satisfy the following equations:

$$\sum X = 0, \quad 4f_0' - 2F_{N1}' = 0 \tag{12}$$

$$\sum Y = 0, \quad 4N_1 + 2f_s - G_0 = 0 \tag{13}$$

$$f_s = F_{N1}' \cdot \mu_1 \tag{14}$$

$$f_0' = N_1 \cdot \mu_0 \tag{15}$$

$$\therefore\ F_{N1}' = \frac{G_0\mu_0 7}{2(1+\mu_1\mu_0)}, \quad N_1 = \frac{G_0}{4(1+\mu_1\mu_0)},$$

where f_0' is the friction the two back wheel groups by the ground, F_{N1}' is the support force on the wheel B by the vertical plane, N_1 is the support force which is generated by the ground on the four back Mecanum wheels, f_s is the friction generated by vertical face of stairs on two front B wheels. M_0 is center moment of wheel A.

Then take the wheel A as the object: according to moment equilibrium, center moment of wheel A needs to satisfy the following equality:

$$\sum M = 0, \quad -M_0 + f_s \cdot r = 0, \tag{16}$$

$$\therefore\ M_0 = \frac{G_0\mu_1\mu_0 r}{2(1+\mu_1\mu_0)}.$$

3.4. *Force analysis of the state when the robot is on stairs*

In the process of climbing, there are three contact points at least contacting stairs.[7] Two possible critical cases should be analyzed:

(1) There are two wheels contacting stairs in each back wheel group while there is only one wheel in each front wheel group contacting the stairs.

(2) There is one wheel contacting ground in each front wheel group while there is only one wheel of the back wheel group contacting the stairs.

3.4.1. *The first critical state when the robot contacting stairs*

As shown in Fig. 8, the static analysis for the front wheel group is as follows: there is only a single wheel contacting the stair in each front wheel group. Take the Mecanum wheels in the front wheel groups that contact stairs' horizontal plane as the research object. The force equation is as follows:

$$\sum X = 0, \tag{17}$$

$$\sum Y = 0, \quad -G_0 + 2N_1 = 0, \tag{18}$$

$$\sum M = 0, \quad -M_1 + R \cdot N_1 \cdot \sin\alpha = 0, \tag{19}$$

$$\therefore\ M_1 = \frac{1}{2} R \cdot G_0 \cdot \sin\alpha, \tag{20}$$

where N_1 means the support force on the front wheel groups by the stairs.

Then it comes the static analysis for the back wheel group: the stress analysis is shown in Fig. 7. The force equation is as follows:

$$\sum X = 0, \quad f_2 - F_{N2} = 0, \tag{21}$$

$$\sum Y = 0, \quad -\frac{1}{2} G_0 + 2N_2 + 2f_{22} = 0, \tag{22}$$

$$\begin{aligned}\sum M = 0, \quad & -M_2 + N_2 \cdot R \cdot \cos\beta + f_2(r - R\sin\beta) \\ & - F_{N2} \cdot R \cdot \cos\beta + f_{22}(r - R\sin\beta) = 0,\end{aligned} \tag{23}$$

$$f_2 = N_2 \cdot \mu_1, \tag{24}$$

$$f_{22} = F_{N2} \cdot \mu_1, \tag{25}$$

$$\begin{aligned}\therefore\ M_2 = & \frac{G_0 R \cdot \cos\beta}{4(\mu_1^2 + 1)} + \frac{G_0 \mu_1 (r - R\sin\beta)}{4(\mu_1^2 + 1)} \\ & - \frac{G_0 R \mu_1 \cos\beta}{4(\mu_1^2 + 1)} + \frac{G_0 \mu_1^2 (r - R\sin\beta)}{4(\mu_1^2 + 1)},\end{aligned} \tag{26}$$

where F_{N2} is the support force on the back wheel groups by the vertical plane of the stairs, f_{22} is the friction on the back wheel groups by the vertical plane of the stairs, N_2 is the support force on the back wheel groups by horizontal plane of the stairs, f_2 is the friction on the back wheel groups by the horizontal plane of the stairs.

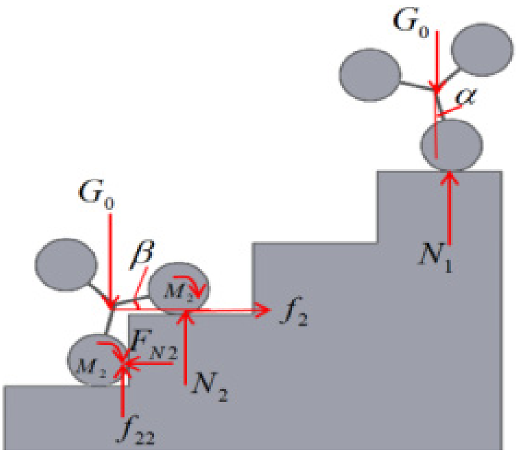

Fig. 7. The force analysis of first critical state.

3.4.2. *The second critical state when the robot contacting stairs*

As shown in Fig. 8, the static analysis for the front wheel group: there are two Mecanum wheels contacting the stairs in each front wheel group.The force equation is as follows:

$$\sum X = 0, \quad f_1 - F_{N1} = 0, \tag{27}$$

$$\sum Y = 0, \quad 2N_1 + 2f_{11} - \frac{1}{2}G_0 = 0. \tag{28}$$

$M_1' = f_1 \cdot r, M_1'$ means the moment of the wheel which is on the ground.
$M_1'' = f_{11} \cdot r, M_1'$ means the moment of the wheel which is hang on the stair.

$$f_1 = N_1 \cdot \mu_{01}, \tag{29}$$

$$f_{11} = F_{N1} \cdot \mu_1, \tag{30}$$

$$\therefore\ M_1' = \frac{G_0(1+2\mu_1)\mu_0 r}{4(1+\mu_0\mu_1)}, \quad M_1'' = \frac{G_0(1+2\mu_1)\mu_1\mu_0 r}{4(1+\mu_0\mu_1)},$$

$$\therefore\ M_A' = \max\{M_1', M_1''\}.$$

Then take the Mecanum wheels in the back wheel groups that contact the stairs as the research object. The force equation is as follows:

$$\sum X = 0, \tag{31}$$

$$\sum Y = 0, \quad -G_0 + N_2 = 0, \tag{32}$$

$$\therefore\ N_2 = G_0.$$

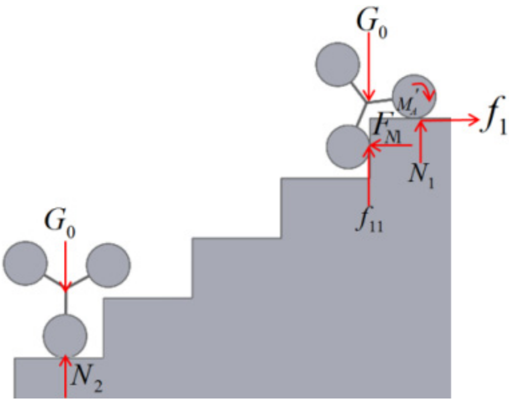

Fig. 8. The force analysis of second critical state.

3.5. *A dilemma each wheel group may face when climbing*

In the process of climbing stairs, the robot will face a critical situation that the length of stairs is shorter than the wheel group, in which the wheel A shown in Fig. 9 has to produce a relatively greater moment to climb stairs. Take the A wheel as the research object. The stair's edge provides the friction f_s and the support force N_{11} of wheel A. The stress analysis is shown in Fig. 9. The force equation is as follows:

According to statics analysis, Eqs. (33) and (34) could be obtained as follows:

$$f_{11} = \frac{G_0\mu^2}{4(\mu^2\cos\phi - 2\mu\sin\phi + \cos\phi)}, \tag{33}$$

$$f_{11} = \frac{G_0\mu}{4(\mu^2\cos\phi - 2\mu\sin\phi + \cos\phi)}, \tag{34}$$

$$\sin\varphi = \frac{\Delta h}{r}, \tag{35}$$

$$r_f = r\cos(\phi - \varphi), \tag{36}$$

$$r_{N11} = r\sin(\phi - \varphi), \tag{37}$$

$$\begin{aligned} \therefore M_A &= f_s r_f + N_{11} r_{N11}, \\ &= \cos\left[\phi - \arcsin\left(\frac{\Delta h}{r}\right)\right] \frac{G_0\mu^2 r}{4(\mu^2\cos\phi - 2\mu\sin\phi + \cos\phi)} \\ &\quad + \sin\left[\phi - \arcsin\left(\frac{\Delta h}{r}\right)\right] \frac{rG_0\mu}{4(\mu^2\cos\phi - 2\mu\sin\phi + \cos\phi)}, \end{aligned} \tag{38}$$

where Δh is the vertical distance between one wheel's gravity center and the horizontal plane of stairs, r_{N11} is the effective radius of N_{11}, r_f is the effective radius of f_s.

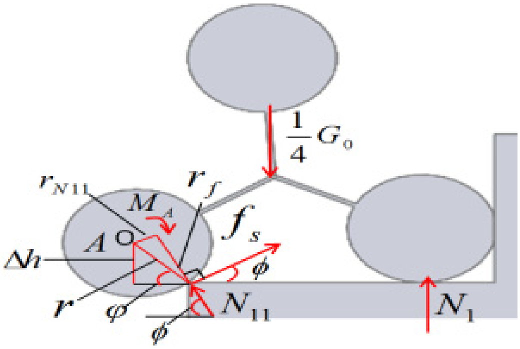

Fig. 9. The force analysis of second critical state.

3.6. *Force analysis of the robot when the robot is climbing the shorter stairs*

Since not all obstacles are standard stairs, this paper will analyze another situation where the height of a step is shorter than the radius of a mecanum wheel. As shown in Fig. 10, we take the front wheel of the robot as the research object. It is forced by the gravity of the whole robot body, the support force by both the step and the ground and the friction generated by the step. All these forces are combined with each other to keep the robot body balanced. Moreover, because height of step is less than the radius of one Mecanum wheel, A wheels in the two front wheel group will leave the ground first. The equations of statics analysis are as follows:

$$\sum X = 0, \quad 8f + 2f_s \cos\alpha - 2F_{N1}\sin\alpha = 0, \tag{39}$$

$$\sum Y = 0, \quad -G_0 + N + 2f_s \sin\alpha + 2F_{N1}\cos\alpha = 0, \tag{40}$$

$$\sum M = 0, \quad -M_s + f_s r + f_0 r \geq 0. \tag{41}$$

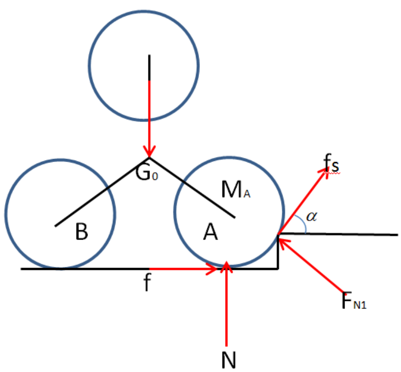

Fig. 10. The stress analysis diagram of the robot when the robot is climbing the shorter stairs.

So, we can conclude that

$$M_s = G_0\mu_0 r + \frac{4G_0\mu_0 r(\mu_1 - 2\mu_0\mu_1 \sin\alpha - 2\mu_0 \cos\alpha)}{\mu_1(8\mu_0 \sin\alpha - \cos\alpha) + 8\mu_0 \cos\alpha + \sin\alpha}, \tag{42}$$

where f means friction on the A wheels, N means support force to the four-wheeled robot by the ground, F_{N1} means support force on front wheels by steps' edge, f_s means the step edge friction on the front wheels, α means the angle between the horizontal direction and the f_s.

4. Electric Power Calculation

The structure of the wheel groups is shown in Fig. 11. The power calculation of the four-wheeled robot is as follows:

We can assume that:

(1) The efficiency of the coupling is η_1.
(2) the bearing efficiency of the wheel group is $\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24}$.
(3) The efficiency of gear transmission is η_3.
(4) The efficiency of belt pulley transmission is η_0.
(5) The motor efficiency is η_{11}.
(6) The efficiency of coupling II is η_4.
(7) ε means the sliding rate.

$\therefore T_3 = 9550\frac{P_3}{n_3} = 9550\frac{P_0 \cdot \eta_1 \cdot \eta_{21} \cdot \eta_3 \cdot \eta_4 \cdot \eta_{22} Z_2 \cdot D_1(1-\varepsilon)}{n_1 \cdot Z_1 \cdot D_2} \times \frac{1}{27}$,

T_3 means the the moment of the axis of Mecanum wheel.

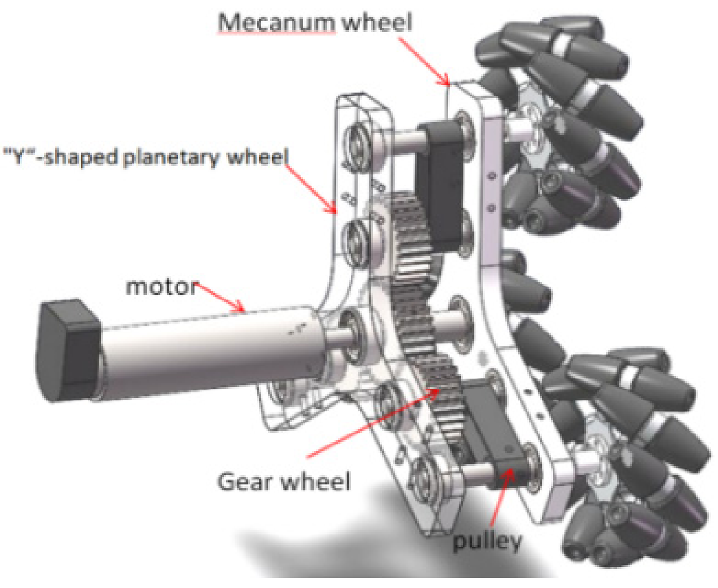

Fig. 11. Diagram of the transmission structure of the four-wheeled robot.

5. Experiment

5.1. *Calculation of moment*

Moreover, according to Eqs. (20) and (26), the following curves in Figs. 12 and 13 are drawn using MATLAB according to the analysis above.

As shown in Fig. 12, in the first critical state when the robot is contacting stairs, the changing process of moment can be divided into two parts: (1) α is less than 1.3 rad and (2) α is more than 1.3 rad. In the first process where α is less than 1.3 rad, the direction of M_1 is contrary to the assumed moment direction, and the maximum moment is 8 Nm. In the second process where α is more than 1.3 rad, the maximum moment is 13 Nm. Besides, when $\alpha = 1.3$ rad, M_1 reaches its minimum 0 Nm. As a result, no matter how the motor is turning or reversing, the maximum moment the motor can provide is 13 Nm.

In addition, as shown in Fig. 13, in the first critical state when the robot is contacting stairs, for back wheel groups, the changing process of moment can also be divided into two parts: (1) β is more than 2.3 rad along the clockwise direction and (2) β is less than 2.3 rad along the clockwise direction. In the first process where β is more than 2.3 rad, the direction of M_2 is contrary to the assumed moment direction, and the maximum moment is 8 Nm. In the second process where β is less than 2.3 rad, the maximum moment is 16 Nm, where $\beta = -0.5$ rad. Besides,

Table 1. The parameters of the four-wheeled robot.

G_0	r	R	μ_1	μ_0	L	b	H
500 Nm	63.5 mm	160 mm	0.3	0.5	1000 mm	280 mm	150 mm

Note: The width of the stairs is $b = 280$ mm, and the height is $H = 150$ mm.

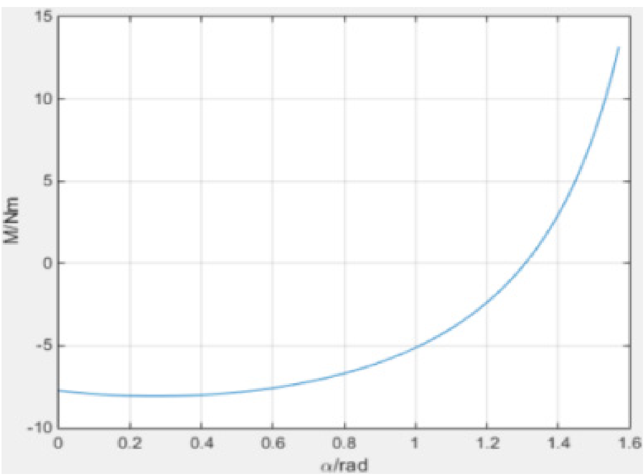

Fig. 12. The relationship between β and M_2.

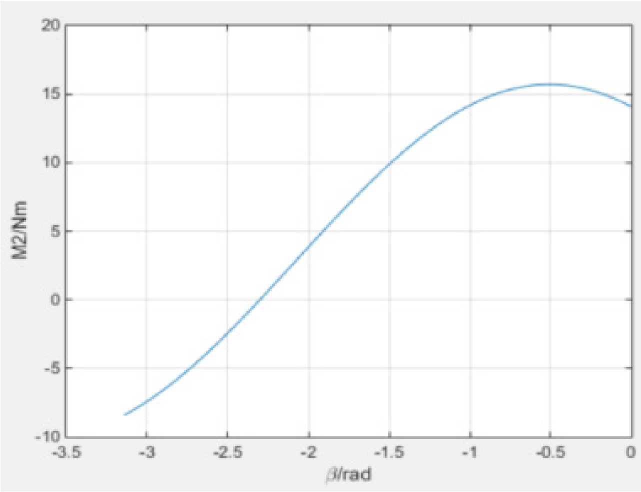

Fig. 13. The relation between α and M_S.

when $\beta = -2.3$ rad, M_2 reach its minimum 0 Nm. As a result, no matter how the motor is turning or reversing, the maximum moment the motor can provide is 16 Nm. Moreover, when $|\beta| > 0.5$ rad, M_2 changes with an upward trend, while when $|\beta| < 0.5$ rad, M_2 has a downward trend.

According to Eq. (38), the relation among M_A, ϕ and Δh provided by MATLAB is shown in Fig. 14.

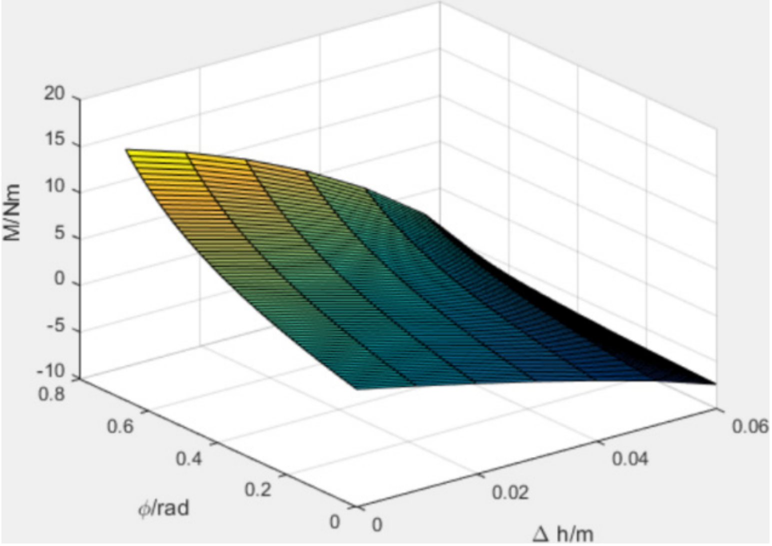

Fig. 14. The relation among M_A, Δh and ϕ.

As shown in Fig. 14, M_A increases with ϕ while decreases with the increase of Δh. When $\Delta h = 0$ and $\phi = 0.67\,\text{rad}$, the maximum of M_A is 16.93 Nm. Meanwhile, when $\Delta h > 0.03\,\text{m}$ and $\phi < 0.2\,\text{rad}$, M_A is contrary to the assumed moment direction, and the maximum moment is 7.4 Nm.

5.2. *Kinetic analysis of drifting on the stairs*

Assume the robot drifts towards right on stairs, its force diagram can be simplified as shown in Fig. 15. T_1, T_4, T_5, T_7 are the friction provided by the horizontal plane and T_2, T_3, T_6, T_8 are provided by the vertical plane of stairs. Meanwhile, the angle between T_1–T_8 and horizontal line is $\alpha = 45$. Based on statics analysis, the robot is balanced in vertical direction, and it is forced by horizontal component of T_1–T_8 in the horizontal direction. So the robot would move to the right on stairs. According to the principle of robot kinematics, Eq. (43) should be satisfied[8]

$$\begin{bmatrix}\varpi_1\\ \varpi_4\\ \varpi_6\\ \varpi_8\end{bmatrix} = \begin{bmatrix}\dfrac{1}{r\tan\alpha}\\ \dfrac{-1}{r\tan\alpha}\\ \dfrac{-1}{r\tan\alpha}\\ \dfrac{1}{r\tan\alpha}\end{bmatrix} v_x, \quad \begin{bmatrix}\varpi_2\\ \varpi_3\\ \varpi_5\\ \varpi_7\end{bmatrix} = \begin{bmatrix}\dfrac{1}{r\tan\alpha}\\ \dfrac{-1}{r\tan\alpha}\\ \dfrac{-1}{r\tan\alpha}\\ \dfrac{1}{r\tan\alpha}\end{bmatrix} v_x. \tag{43}$$

The velocity of center point O is vx and the angular velocity of one Mecanum wheel is ϖ_i, $\alpha = 45°\text{r} = 63.5\,\text{mm}$. Thus through Eq. (43), each Mecanum wheel's angular velocity can be gotten.

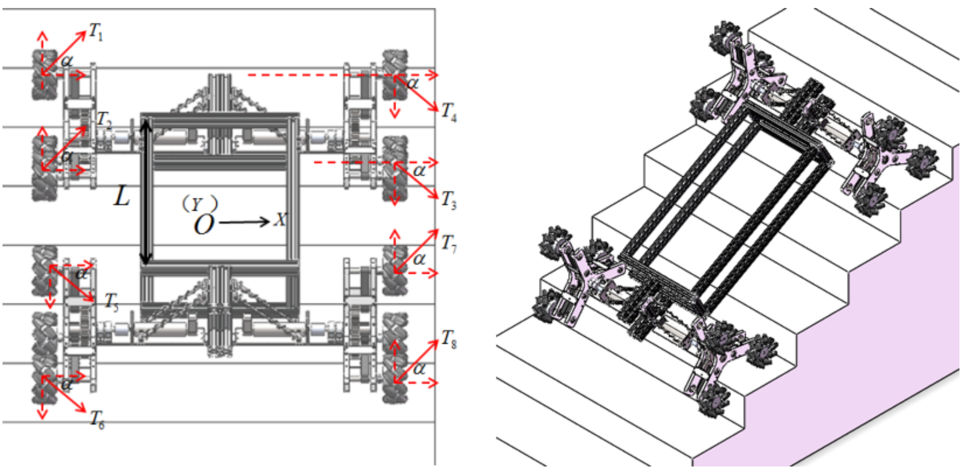

Fig. 15. Top view map and isometric map of drifting on the stairs drawn by solid works.

6. Conclusion

This paper designs a new type of obstacle avoidance four-wheeled robot based on Mecanum wheel and "Y"-type planetary wheel. The static stability of the robot in the process of stair climbing is analyzed in detail. The concluded torque value provides great security margin for the whole structure, and MATLAB is used to draw those torque curves. Finally, this paper analyzes the power demands for this robot to avoid obstacle.

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