University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 1 EXAMINATIONS 2012/2013

STAT 30090 - STAT 40680

Models – Stochastic Models

Prof. Roger Payne

Dr. Patrick Murphy

Dr. Pierre Alquier*

Time Allowed: 2 hours

Instructions for Candidates

Attempt all questions. The number of marks for each question is given. The total number of marks is given.

Instructions for Invigilators

Candidates will not require
New Cambridge Statistical Tables.
Calculators are permitted.
Graph paper is not required.

1. Let X_n be a Markov chain with the following transition probability matrix:

$$P = \begin{bmatrix} 0.2 & 0.2 & 0.6 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0.1 & 0.4 \end{bmatrix}.$$

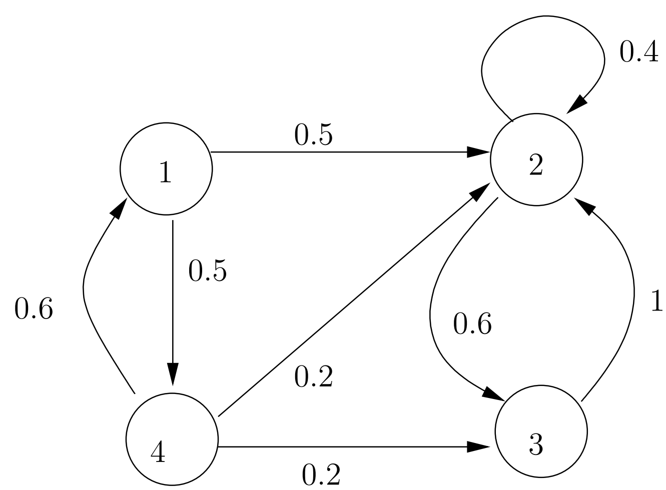
(i) The initial distribution of the chain is given by $\bar{p}_0 = (0, 0.4, 0.6)$. What is the distribution of X_2? [5]

(ii) Explain how the distribution of X_n is related to the initial distribution and the transition probability matrix. Hence or otherwise obtain the equation for the stationary distribution. [5]

(iii) Explain clearly why the stationary distribution of the Markov chain exists for P above. Find the stationary distribution. [8]

Total [18]

2. A finite Markov chain X_n with state space $\{1, 2, 3, 4\}$ has the following graph of transitions:



(i) Write down the corresponding probability transition matrix. [4]

(ii) Which states are transient? Which states are recurrent? [4]

(ii) Does this chain have a stationary distribution? Explain your reasons. [4]

(iv) Suppose the chain is initially in state 1. What states can the chain reach in the long term with non-zero probabilities? What are these probabilities? [8]

Total [20]

3. A continuous time Markov chain X_t with state space $\{1, 2, 3\}$ has the infinitesimal generator

$$A = \begin{bmatrix} -5 & 2 & ? \\ 2 & ? & 4 \\ ? & 3 & -5 \end{bmatrix}.$$

(i) Complete the matrix. [2]

(ii) The chain is currently in state 3. Find the probability that the next state will be 1. [3]

(iii) If the chain starts in state 2, what is the expected total time spent in state 2 before the chain first enters state 3. [10]

Total: [15]

4. The numbers of claims to an insurance company from smokers and non-smokers follow independent Poisson processes. On average 4 claims from non-smokers and 6 claims from smokers arrive every day independently of each other. The sizes of claims are independent, with an exponential distribution with mean 100€.

(i) Given that 8 claims arrived in a day, what is the probability that 5 of them were from smokers? [7]

(ii) What is the probability that over 3 days at most one claim will exceed 500€? [8]

(iii) What is the expected waiting time between claims in excess of 500€? [5]

Total [20]

5. A cell produces 0, 1 or 2 offspring with probabilities 0.2, 0.2 and 0.6 respectively. (You may assume that cells produce offspring independently of one another.)

(i) What is the extinction probability for a colony that initially consists of two cells? [6]

(ii) If a colony starts with a single cell, what is the probability that it is extinct in the third generation given that it did not die out in the second generation? [9]

Total [15]

6. Let W_t be the standard Brownian motion.

(i) Write down the defining properties of W_t. [3]

(ii) What is the variance of $5W_3 - 3W_2$? [4]

(iii) Calculate the stochastic differential dY_t for $Y_t = W_t^3$. [5]

Total [12]

oOo