STAT40380/STAT40390/STAT40850 Bayesian Analysis

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Catch-up

- We can now fit any model with any number of parameters using a combination of Gibbs sampling and Metropolis-Hastings
- Gibbs sampling allows us to update each parameter (or sets of parameters) in turn
- Metropolis-Hastings gives us a simple method for updating each parameter by sampling from a proposal distribution
- When we combine the two methods, this is known as *Metropolis within Gibbs*
- Today we will look at some of the more advanced models we can fit

Hierarchical models

- Most of the models we have looked at have been restricted to having one or two parameters
- Occasionally we want to look at models with many parameters, or where the parameters are *hierarchically related*
- A hierarchical model occurs when we give the parameters of our prior distribution their own probability distribution:

$$\begin{aligned} p(\boldsymbol{\theta}, \phi | \mathbf{x}) &\propto p(\mathbf{x} | \boldsymbol{\theta}, \phi) p(\boldsymbol{\theta}, \phi) \\ &= p(\mathbf{x} | \boldsymbol{\theta}) p(\boldsymbol{\theta} | \phi) p(\phi) \end{aligned}$$

- The parameters of a prior distribution on another parameter are known as *hyper-parameters* (here θ)
- We will go through 4 different examples and write out the steps required to produce samples from the posterior distribution

Steps in Bayesian Model fitting

- When fitting Bayesian models by hand (ie when not using WinBUGS) with a given likelihood and prior distribution(s):
  1. Write down the likelihood and the prior distributions
  2. Write out the posterior distribution up to the constant of proportionality
  3. Compute the complete conditionals for eack parameter (or parameter set) and see if they follow known probability distributions
  4. for parameters that do not follow known probability distributions, find a suitable proposal distribution
  5. Write out the steps to update each of these parameters in turn using Metropolis within Gibbs

Example 1: Linear regression

Example

A linear regression model is propsed for genetic data with p explanatory variables: $\mathbf{y} \sim N(\mathbf{X}\beta, \phi\mathbf{I})$. Prior distribution: $p(\beta, \phi) \propto 1/\phi$. Outline the Gibbs steps for creating samples from the posterior distributions of β and ϕ.

Example 2: Tumours in rats

Example

Some data are presented below containing observations on the number of rat tumours in 70 different experiments. An over-dispersed Binomial model is proposed, so that $y_i \sim Bin(n_i, \pi_i)$ with prior distributions $\pi_i \sim Be(\alpha, \beta)$ and $p(\alpha, \beta) \propto (\alpha + \beta)^{-5/2}$. Outline the Gibbs steps for creating posterior samples of $(\boldsymbol{\pi}, \alpha, \beta)$.

0/20	0/20	0/20	0/20	0/20	0/20	0/20	0/19	0/19	0/19
0/19	0/18	0/18	0/17	1/20	1/20	1/20	1/20	1/19	1/19
1/18	1/18	2/25	2/24	2/23	2/20	2/20	2/20	2/20	2/20
2/20	1/10	5/49	2/19	5/46	3/27	2/17	7/49	7/47	3/20
3/20	2/13	9/48	10/50	4/20	4/20	4/20	4/20	4/20	4/20
4/20	10/48	4/19	4/19	4/19	5/22	11/46	12/49	5/20	5/20
6/23	5/19	6/22	6/20	6/20	6/20	16/52	15/47	15/46	9/24

Example 3: Hepatitis C vaccination

Example

Some data on hepatitis vaccinations are available where y_{ij} is the log of the amount of surface antibody in the jth observation for the ith infant. An explanatory variable t_{ij} is the time the sample was taken. A linear mixed effects model is propsed: $y_{ij} \sim N(\alpha_i + \beta_i t_{ij}, \phi)$. The prior distributions are $p(\phi) \propto 1/\phi, \alpha_i \sim N(\alpha_0, \phi_\alpha), \beta_i \sim N(\beta_0, \phi_\beta)$ and $p(\alpha_0, \phi_\alpha, \beta_0, \phi_\beta) \propto 1/(\phi_\alpha \phi_\beta)$. Outline the Gibbs steps for creating a posterior sample of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \phi, \alpha_0, \beta_0, \phi_\alpha, \phi_\beta | \mathbf{y})$. (data omitted)

Example 4: Mining disasters

Example

An alternative model for the mining disasters data (see Lab 3) is a Poisson change-point model: $x_i \sim Po(\lambda)$ for $i = 1, \ldots, k$ and $x_i \sim Po(\gamma)$ for $i = k + 1, \ldots, n$. Prior distributions: $k \sim UD(1, n), \lambda \sim Exp(1), \gamma \sim Exp(1)$. Outline the Gibbs steps for creating posterior samples of (λ, γ, k).