STAT40380/STAT40390/STAT40850 Bayesian Analysis

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Catch-up

- We can now fit any model with any number of parameters using a combination of Gibbs sampling and Metropolis-Hastings
- Gibbs sampling allows us to update each parameter (or sets of parameters) in turn
- Metropolis-Hastings gives us a simple method for updating each parameter by sampling from a proposal distribution
- When we combine the two methods, this is known as Metropolis within Gibbs
- Today we will look at some of the more advanced models we can fit





Hierarchical models

- Most of the models we have looked at have been restricted to having one or two parameters
- Occasionally we want to look at models with many parameters, or where the parameters are hierarchically related
- A hierarchical model occurs when we give the parameters of our prior distribution their own probability distribution:

$$p(\theta, \phi | \mathbf{x}) \propto p(\mathbf{x} | \theta, \phi) p(\theta, \phi)$$

= $p(\mathbf{x} | \theta) p(\theta | \phi) p(\phi)$

- The parameters of a prior distribution on another parameter are known as *hyper-parameters* (here θ)
- We will go through 4 different examples and write out the steps required to produce samples from the posterior distribution





Steps in Bayesian Model fitting

- When fitting Bayesian models by hand (ie when not using WinBUGS) with a given likelihood and prior distribution(s):
 - Write down the likelihood and the prior distributions
 - Write out the posterior distribution up to the constant of proportionality
 - Ompute the complete conditionals for eack parameter (or parameter set) and see if they follow known probability distributions
 - for parameters that do not follow known probability distributions, find a suitable proposal distribution
 - Write out the steps to update each of these parameters in turn using Metropolis within Gibbs





Example 1: Linear regression

Example

A linear regression model is propsed for genetic data with p explanatory variables: $\mathbf{y} \sim N(\mathbf{X}\boldsymbol{\beta}, \phi\mathbf{I})$. Prior distribution: $p(\boldsymbol{\beta}, \phi) \propto 1/\phi$. Outline the Gibbs steps for creating samples from the posterior distributions of $\boldsymbol{\beta}$ and ϕ .





Example 2: Tumours in rats

Example

Some data are presented below containing observations on the number of rat tumours in 70 different experiments. An over-dispersed Binomial model is proposed, so that $y_i \sim Bin(n_i, \pi_i)$ with prior distributions $\pi_i \sim Be(\alpha, \beta)$ and $p(\alpha, \beta) \propto (\alpha + \beta)^{-5/2}$. Outline the Gibbs steps for creating posterior samples of (π, α, β) .

0/20	0/20	0/20	0/20	0/20	0/20	0/20	0/19	0/19	0/19
0/19	0/18	0/18	0/17	1/20	1/20	1/20	1/20	1/19	1/19
1/18	1/18	2/25	2/24	2/23	2/20	2/20	2/20	2/20	2/20
2/20	1/10	5/49	2/19	5/46	3/27	2/17	7/49	7/47	3/20
3/20	2/13	9/48	10/50	4/20	4/20	4/20	4/20	4/20	4/20
4/20	10/48	4/19	4/19	4/19	5/22	11/46	12/49	5/20	5/20
6/23	5/19	6/22	6/20	6/20	6/20	16/52	15/47	15/46	9/24

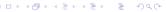


Example 3: Hepatitis C vaccination

Example

Some data on hepatitis vaccinations are available where y_{ij} is the log of the amount of surface antibody in the jth observation for the ith infant. An explanatory variable t_{ij} is the time the sample was taken. A linear mixed effects model is propsed: $y_{ij} \sim N(\alpha_i + \beta_i t_{ij}, \phi)$. The prior distributions are $p(\phi) \propto 1/\phi, \alpha_i \sim N(\alpha_0, \phi_\alpha), \beta_i \sim N(\beta_0, \phi_\beta)$ and $p(\alpha_0, \phi_\alpha, \beta_0, \phi_\beta) \propto 1/(\phi_\alpha \phi_\beta)$. Outline the Gibbs steps for creating a posterior sample of $(\alpha, \beta, \phi, \alpha_0, \beta_0, \phi_\alpha, \phi_\beta | \mathbf{y})$. (data omitted)





Example 4: Mining disasters

Example

An alternative model for the mining disasters data (see Lab 3) is a Poisson change-point model: $x_i \sim Po(\lambda)$ for $i = 1, \ldots, k$ and $x_i \sim Po(\gamma)$ for $i = k+1, \ldots, n$. Prior distributions: $k \sim UD(1, n), \lambda \sim Exp(1), \gamma \sim Exp(1)$. Outline the Gibbs steps for creating posterior samples of (λ, γ, k) .



