STAT40380/STAT40390/STAT40850 Bayesian Analysis

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More on Bayesian hypothesis testing

- The Bayes factor is defined as:

$$BF = \frac{p(\mathbf{x}|\mathcal{M}_1)}{p(\mathbf{x}|\mathcal{M}_0)}$$

  where $\mathcal{M}_0$ and $\mathcal{M}_1$ are different models (possibly corresponding to hypotheses H_0 and H_1)
- The Bayes factor is the odds in favour of $\mathcal{M}_1$ against $\mathcal{M}_0$
- The numerator and denominator can be calculated from:

$$p(\mathbf{x}|\mathcal{M}_i) = \int p(\mathbf{x}|\boldsymbol{\theta}, \mathcal{M}_i)p(\boldsymbol{\theta}|\mathcal{M}_i)d\boldsymbol{\theta}$$

- Today we will go through a set of examples where we can calculate the Bayes Factor

Example 1: simple (and slightly unrealistic)

Example

Let x be the number of heads obtained after tossing a possibly biased coin 20 times, so that $x \sim Bin(20, \theta)$. Two hypotheses are proposed:

$$H_0 : \theta = 0.5 \text{ vs } H_1 : \theta = 0.75$$

Find the Bayes factor and determine whether H_0 or H_1 is most favoured by the data when $x = 14$

Example 2: comparing different priors

Example

Let $\mathbf{x}$ be the number of accidents occuring in a year for 30 drivers, so that $x_i \sim Po(\lambda)$ for $i = 1, \ldots, 30$ where λ is the rate at which accidents occur.
Two different prior distributions are proposed:

$$H_0 : \lambda \sim Ga(3.8, 8.1) \text{ vs } H_1 : \lambda \sim Ga(4, 4)$$

After a year, we observe $\sum x_i = 9$. Calculate the Bayes factor in favour of H_1 against H_0

Example 2 picture

Example 3: beware Lindley's paradox

Example

Let $\mathbf{x_i} \sim N(\theta, \phi)$ with ϕ known. Two hypotheses are proposed:

$$H_0 : \theta = \theta_0 \text{ vs } H_1 : \theta \neq \theta_0$$

Find the Bayes factor for the two hypotheses.

Example 4: Comparing different likelihoods

Example

Let $\mathbf{x_i}$ be the number of children born to couple i. Two scientists are arguing over how couples choose to conceive. Scientist A says that parents stop after they have a child of each gender. Scientist B says that parents choose how many children to have before they start. Assuming Scientist A's views match to a Geometric distribution, and his prior distribution is $Be(4, 4)$, and that Scientist B's views match to a Poisson distribution with prior $Ga(3, 1)$, find the Bayes factor for the two models when $\mathbf{x} = \{4, 3, 1, 5, 2, 2, 3, 1, 2, 2, 3\}$.

Some things to remember

- Specifying hypotheses is the same as specifying a model...
- if we can specify a model then we can specify parameters...
- thus, the model can be seen as another parameter. We can use the Bayes factor to obtain the ratio of posterior distributions of various models given the data
- Remember that the Bayes factor is a *relative* measure of model fit. There are other *absolute* measures (eg predictive distributions) that we should use in conjunction with the Bayes factor
- It is much harder to calculate Bayes factors for more complex models. We will use other techniques to measure model fit in these circumstances.