

First tests

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To start the resolution of the problem we compute with the Crank Nicolson method the time evolution for $g = 0$ and $V_{ext} = 0$ then the equation reads as:

$$i \frac{\partial \Psi(\tilde{z}, \tilde{t})}{\partial \tilde{t}} = - \frac{\partial^2 \Psi(\tilde{z}, \tilde{t})}{\partial \tilde{z}^2} \quad (1)$$

1 Gaussian

To start testing the method we compute the time evolution for a gaussian function centered at zero and with $\sigma = 1$. We expect it to evolve as it is not an eigenfunction of $H = -\frac{\partial^2}{\partial \tilde{z}^2}$.

We compute the method for a spacing of $\Delta \tilde{z} = 0.01$ with $\tilde{z} \in [-3, 3]$ and a time interval $\delta \tilde{t} = 0.001$.

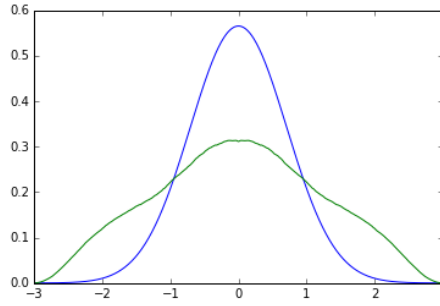


Figure 1: $|\Psi|^2$ at time $\tilde{t} = 0$ in blue and at time $\tilde{t} = 5$ in green

The maximum difference between the norm at $\tilde{t} = 0$ and the norm of the evolving function was: $1.16233793701 \times 10^{-7}$

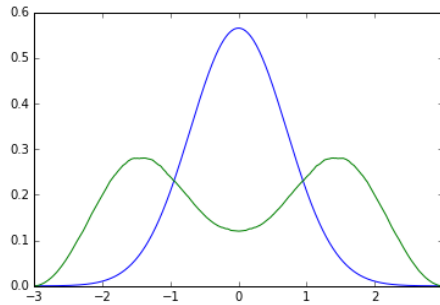


Figure 2: $|\Psi|^2$ at time $\tilde{t} = 0$ in blue and at time $\tilde{t} = 10$ in green

The maximum difference between the norm at $\tilde{t} = 0$ and the norm of the evolving function was: $1.16241938852 \times 10^{-7}$

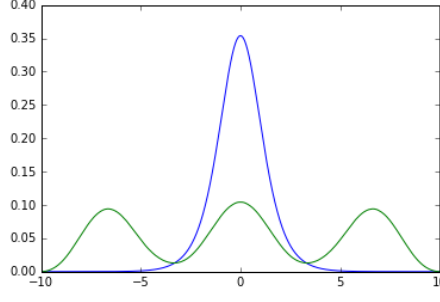


Figure 3: $|\Psi|^2$ at time $\tilde{t} = 0$ in blue and at time $\tilde{t} = 10$ in green

2 Bright soliton

2.1 null velocity

We compute the method for a spacing of $\Delta\tilde{z} = 0.01$ with $\tilde{z} \in [-10, 10]$ and a time interval $\delta\tilde{t} = 0.001$ for a bright soliton solution with null velocity and $n_0 = 1$

The maximum difference between the norm at $\tilde{t} = 0$ and the norm of the evolving function was: $1.459786764218 \times 10^{-9}$

2.2 0.5 velocity

We compute with $\tilde{v} = 0.5$ and two different time intervals: $\delta\tilde{t} = 0.001$ and $\delta\tilde{t} = 0.01$

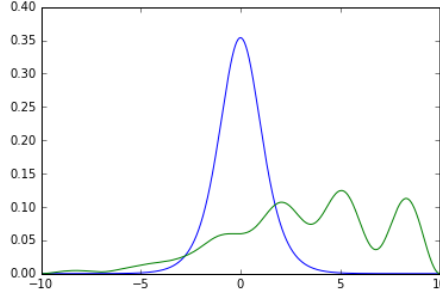


Figure 4: $|\Psi|^2$ at time $\tilde{t} = 0$ in blue and at time $\tilde{t} = 5$ in green with $\delta\tilde{t} = 0.01$

The maximum difference between the norm at $\tilde{t} = 0$ and the norm of the evolving function was: $3.06995195931 \times 10^{-9}$

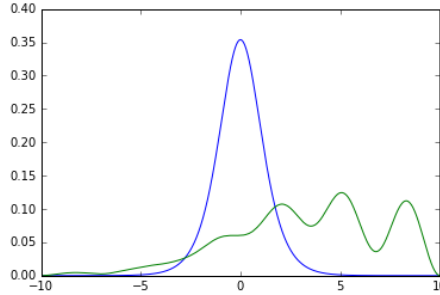


Figure 5: $|\Psi|^2$ at time $\tilde{t} = 0$ in blue and at time $\tilde{t} = 10$ in green with $\delta\tilde{t} = 0.001$

The maximum difference between the norm at $\tilde{t} = 0$ and the norm of the evolving function was: $3.45472594976 \times 10^{-9}$

Here we have also computed the time that the program's been running to obtain the result. For $\delta\tilde{t} = 0.01$ it was only 19.82s whereas the running time for $\delta\tilde{t} = 0.001$ it was 87.26s and yet there is no significant improvement on the result.

3 Grey soliton

We run the code for the grey solution for a spacing of $\Delta\tilde{z} = 0.01$ with $\tilde{z} \in [-10, 10]$ and a time interval $\delta\tilde{t} = 0.01$ and the results found are:

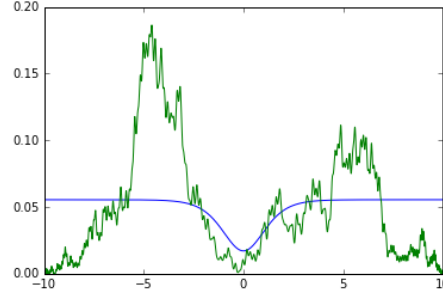


Figure 6: $|\Psi|^2$ at time $\tilde{t} = 0$ in blue and at time $\tilde{t} = 5$ in green with $\tilde{v} = 0.5$

The maximum difference between the norm at $\tilde{t} = 0$ and the norm of the evolving function was: $9.4415014253 \times 10^{-5}$

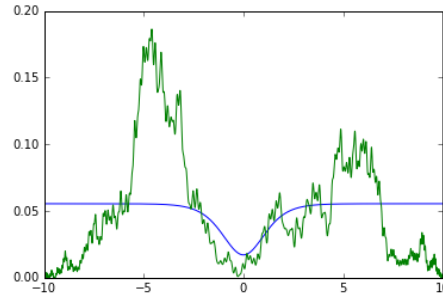


Figure 7: $|\Psi|^2$ at time $\tilde{t} = 0$ in blue and at time $\tilde{t} = 5$ in green with $\tilde{v} = 0$

The maximum difference between the norm at $\tilde{t} = 0$ and the norm of the evolving function was: $9.70332161562 \times 10^{-5}$