Harmonic oscillator

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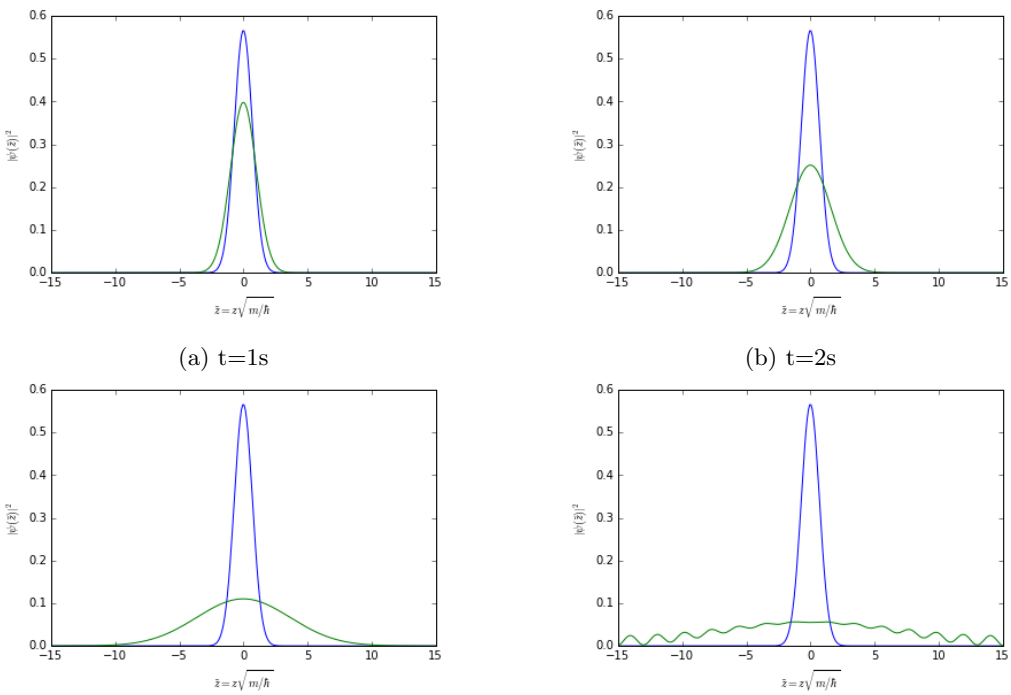
To make some checks to the Crank-Nicolson method developed we focus on a known case, the ground state of an 1-D harmonic oscillator. We know that the eigenfunction for the time independent Shrödinger equation is:

$$\Psi(\bar{z}) = AH_0(\bar{z})e^{-\frac{\bar{z}^2}{2}} = \frac{e^{-\frac{\bar{z}^2}{2}}}{\pi^{\frac{1}{4}}} \tag{1}$$

with $\bar{z} = \sqrt{\frac{m\omega}{\hbar}}z$.First of all we start by studiyng the evolution of this wavefunction in free space:

$$i\hbar\frac{\partial\Psi(z,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(z,t)}{\partial z^2} \Rightarrow i\frac{\partial\Psi(\tilde{z},t)}{\partial t} = -\frac{1}{2}\frac{\partial^2\Psi(\tilde{z},t)}{\partial \tilde{z}^2} \tag{2}$$

Wiht $\tilde{z} = \sqrt{\frac{m}{\hbar}}$. As the wave function is an eigenfunction for the harmonic potential we expect that when we compute the time evolution for free space it will mantaint the shape but it will widen with time. We run the method for a box of $\tilde{z} = [-15, 15]$ with intervals of $d\tilde{z} = 0.1$ and a time interval of $dt = 0.009$ which means that the parameter r for the Crank Nicolson method is $r = 0.499j$ we compute it for different values of t:



(a) t=1s (b) t=2s

(c) t=5s (d) t=10s

Figure 1: plots of the time evolution for different values of t in green and in blue $|\Psi|^2$ at $t = 0s$

As can be seen the method works properly for times no much bigger than t=5s.

	Norm difference	computing time (s)
t=1s	$2.664x10^{-15}$	0.329
t=2s	$5.33x10^{-15}$	2.234
t=5s	$9.202x10^{-8}$	4.895
t=10s	$8.979x10^{-6}$	10.650

Table 1: Computing time and norm diference for the plots in 1