5.13 Project: A Plant–Herbivore Model

This project is taken from Edelstein-Keshet (1988).
Assume that a population of herbivores of density y causes changesin the vegetation on which it preys. An internal variable x reflects some physical or chemical property of the plant, which undergo changes in response to herbivory. We refer to this attribute as the *plant quality* of the vegetation and assume that it may in turn affect the fitness or survivorship of the herbivores. If this happens in a graded, con- tinuous interaction, plant quality may be modeled by a system

$x' = f(x,y0, y' = yg(x,y).$

1. In one case, the function $f(x, y)$ is assumed to be

   $$f(x,y) = x(1-x)[\alpha(1-y)+x], 0 \leq x \leq 1.$$

   Sketch this as a function of x and reason that plant quality x always remains in the interval (0,1) if $x(0)$ is in this interval. Show that plant quality may either increase or decrease, depending on the initial value of x and the population of herbivores. For a given herbivore popultion density $y\hat{}$, what is the "break-even" point (the level of x for which the rate of change of x is zero?

2. It is assumed that the herbivore population undergoes logistic growth with re-productive rate β and a carrying capacity that is directly proportional to current plant quality. What is the function g?

3. Draw the nullclines of the system you have obtained. (There is more than one possibilty, depending on the parameter values.

4. Find the direction of motion along all nullclines obtained in part (c).

5. Show that there is a particular configuration with a set in the (x,y) plane that "traps" trajectories.

6. Define $\gamma = \alpha / (\alpha K - 1)$. Interpret the meaning of this parameter. Show that $(\gamma, K\gamma)$ is a steady state of your equations, and locate it on your phase plot. Find the other steady state.

7. Show that $(\gamma, K\gamma)$ is a saddle point if $\gamma > 1$ and a focusfocus if $\gamma < 1$.

8. Now show that as β decreases from large to small values, the steady state for $\gamma < 1$ undergoes a transition from a stable focus to an unstable focus.

9. Use the results you have obtained to comment on the existence of periodic solutions. Interpret your answer in biological terms.

[Reference: Edelstein-Keshet (1988)]