Optimization of German Highway Networks Using Multi-Colony Ant Colony Optimization

Maria Francis, *Center for Mind/Brain Sciences, University of Trento, maria.francis@stud.unitn.it*

***Abstract*—The optimization of highway networks is a critical challenge for reducing environmental impact and improving transportation efficiency. In Germany, where the transportation sector accounts for a significant portion of CO2 emissions, innovative solutions are needed to design cost-effective and efficient highway networks. This paper investigates the use of a simple multi-colony Ant Colony Optimization (ACO) algorithm to address the problem of minimizing highway construction costs while maximizing transportation efficiency. Colonies of ants are tasked with optimizing a single path between two cities, and influence solutions of other colonies by means of stigmergy through pheromones. A range of experimental parameters, including pheromone initialization, decay rates, and transition rule weights, were analyzed to evaluate their influence on network performance. While reasonable solutions are found, the simple algorithm does not effectively explore the trade-off space between the cost and efficiency optimization functions.**

***Index Terms*—Network Design, Ant Colony Optimization.**

I. Introduction

EFFICIENT transportation networks are a critical component of a nation's infrastructure, influencing economic activity, regional development, and citizens' quality of life. In Germany, optimizing the highway network is critical due to the increasing volume of traffic and the need to improve standards of sustainability. In 2022[1], road transportation was the largest source of CO2 emmisions in the EU, with Germany's transportation sector leading all EU countries in emmisions.[2] In 2023, Germany produced 2.9 billion tons of CO2 in road freight transport compared to 171.7 million tons from inland waterways freight and 358.8 million tons of CO2 from rail freight.[3] As such, effective highway network design holds great potential to reduce environmental impact by reducing travel distances, improving traffic flow, and reducing congestion.

Designing an efficient highway network is complex, largely due to the lack of a clear definition of 'efficiency'. An efficient network must satisfy multiple objectives: Simultaneously minimizing travel distance and construction costs while remaining resilient to disruptions like construction, natural disasters, or terrorist attacks. The large number of cities requiring access to the network creates a vast search space.

[1]https://www.statista.com/statistics/999398/carbon-dioxide-emissions-sources-european-union-eu/

[2]https://www.statista.com/statistics/1306876/transportation-emissions-european-union-bycountry/#:~:text=Germany%20was%20the%20biggest%20contributor,MtCO%E2%82%82%20and%2094.5%20MtCO%E2%82%82%2C%20respectively.

[3]https://www.destatis.de/EN/Themes/Economic-Sectors-Enterprises/Transport/Goods-Transport/_node.html

Ant Colony Optimization (ACO), inspired by ants' stigmergy, is a promising approach. ACO excels in large search spaces by exploring and refining multiple solutions simultaneously. While originally designed for single-objective problems, ACO can be adapted for multi-objective optimization, making it well-suited to address the complex challenges of efficient highway network design.

In this work, we investigate the use of a simple, multi-colony ACO algorithm to explore optimal solutions balancing highway construction cost and travel speed, applied to the design of a highway network for Germany.

II. Methods

We approximate the highway network as a an undirected graph, where nodes represent cities and edges represent highways connecting them. Edges are weighted by the air line distance between their respective cities. In this simplified representation, highways are only able to connect two cities, and cannot be split into multiple connections or merged. Topological features such as mountain ranges, rivers, and ground conditions are not included in the experiment.

For each pair of cities in the experiment, a separate colony of ants is tasked with finding the shortest path between them. As in traditional ACO, each ant's path is determined by a probabilistic transition rule that takes into account both the length of a given edge and its pheromone level. While each ant colony focuses on optimizing the path between two specific cities, the solutions they generate are used to update a global pheromone map. This encourages ants from different colonies to share more frequently traveled routes, even when the route in question is longer.

The size of each ant colony is determined by the importance of the cities it connects, thus increasing the pheromone level on routes linking influential cities. In a single iteration of the algorithm, each ant colony produces one solution per ant, and all solutions are compiled and used to update the global pheromone network. After completing all iterations, a variable heuristic using information from the resulting pheromone network is applied to select the optimal path for each city pair. These selected paths are then combined to form the final highway network. The final highway network is assessed using two evaluation metrics: one corresponding to the transportation performance of the network (referred to as 'efficiency') and the other corresponding to the total cost (henceforth 'cost').

In this study, we investigate how varying parameter values affects the algorithm's behavior and the quality of the resulting highway networks. Specifically, we explore the influence of the pheromone decay rate ρ, the transition rule threshold q_0, the weights α and β within the transition rule, and the method used to initialize the pheromone map. The following sections provide a detailed description of the experimental variables (Section II-C) and the key components of our algorithmic design.

A. City Selection and Map Design

To determine which cities to include in our experiment, we used information about the GDP and population size of German cities. Each city is assigned a 'city score', given by the sum of its GDP and population rank. For example, Frankfurt am Main has a city score of nine, as it ranks fourth in GDP and fifth in population. We include the 20 cities with the lowest city scores in the highway network. This results in the inclusion of the following cities: Berlin, Hamburg, Munich, Frankfurt am Main, Cologne, Stuttgart, Düsseldorf, Hanover, Bremen, Wolfsburg, Essen, Dortmund, Nuremberg, Leipzig, Ingolstadt, Dresden, Bonn, Regensburg, Halle (Saale), and Ludwigshafen am Rhein.

The latitude and longitude of each city are taken from Google Maps[4] and used as coordinates for their respective nodes. Initially, a distance matrix was created by calculating the Euclidean distance between each city pair. Because the Euclidean distance does not account for the Earth's curvature, the resulting distances were inaccurate and were later recalculated using the Haversine formula [1]. The cities' positions were plotted on a map of Germany using the Cartopy Python package[5].

B. Ant Distribution

In our algorithm, an ant colony of variable size is tasked with choosing the path that connects a specific city pair. Influential cities are assigned larger ant colonies, encouraging ants connecting less significant cities to share more popular routes. The colony size is proportional to the expected traffic between two given cities, which is determined using a simple gravity model [2]. Specifically, the number of ants connecting cities i and j is given by:

$$T_{ij} = \frac{P_i \cdot P_j}{D_{ij}^b * 1000000},$$

where P_i and P_j denote the GPD of i and j respectively, D_ij denotes the distance between i and j, and b is a distance decay parameter set to 1.2.

C. Pheromone Map and Transition Rules

We experiment with two distinct strategies of initializing the pheromone map, henceforth referred to as *uniform* and *superhighway*.

As a baseline, the edges are initialized with a uniform pheromone value:

$$\tau(0) = \frac{1}{(n \cdot \bar{L})}$$

where n is the number of nodes and $\bar{L}$ is the mean distance between city pairs.

In addition, we experiment with a setting in which the edges connecting the cities Berlin (B), Hamburg (H), Frankfurt am Main (F), and Munich (M) are initialized with an increased pheromone value:

$$\tau_{ij}(0) = \frac{1}{(n \cdot \bar{L})} + b \qquad \text{if } i \text{ and } j \in \{\text{B, H, M, F}\},$$

where we choose $b = 3$.

Ants are encouraged to travel along these edges due to the high population and GDP of the selected cities, as well as the even positioning of the cities throughout the country.

The ants build their solution paths incrementally using the traditional ACO transition rule: At each step t, each and k samples a random value q from $[0, 1]$. If $q < q_0$, where q_0 is a variable threshold, the ant chooses the edge with the highest pheromone value. Otherwise, it chooses the edge ij with the probability:

$$p_{ij}^k(t) = \frac{[\tau_{ij}(t)^\alpha] * [\eta_{ij}]^\beta}{\sum_{l \in \mathcal{N}_i^k} [\tau_{il}(t)]^\alpha \cdot [\eta_{il}]^\beta}$$

For all j not already on the solution path. α and β are variable parameters. Higher values for α increase the influence of the pheromone value of the edge, whereas higher values for β increase the influence of the length of the edge.

In each iteration of the algorithm, the solutions of each ant in each colony are compiled and the pheromones are updated simultaneously using the traditional ACO pheromone update rule:

$$\tau_{ij}(t+1) = (1-\rho) \cdot \tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^k(t) \qquad \forall (i,j)$$

where

$$\Delta\tau_{ij}^k(t) = \begin{cases} 1/L^k(t) & \text{if arc } (i,j) \text{ is used by ant } k \\ 0 & \text{otherwise} \end{cases}$$

D. Evaluation Metrics

The final network is evaluated along two metrics, *cost* and *efficiency*. The *cost* is calculated by summing the lengths of all edges in the network. It is then normalized by the sum of the lengths of the network's minimum spanning tree. *Efficiency* is calculated as the sum of the shortest distances between each pair of cities, obtained using Dijkstra's algorithm [3], and normalized by the sum of the shortest distances in the network's minimum spanning tree.

[4]maps.google.com

[5]https://scitools.org.uk/cartopy/docs/latest/

Fig. 1. Minimum spanning tree, calculated using SciPy

The minimum spanning distance, shown in Fig. II-D, was calculated using Python packages by SciPy[6] and NetworkX[7]. The sum of edge lengths of the minimum spanning tree is 1649, and the sum of all minimum distances of the tree is 97027. We find that the resulting network doesn't intuitively seem to be the true minimum spanning tree, but we were unable to calculate better solutions despite extensive troubleshooting. We therefore proceeded by normalizing the metrics using the values given by this tree.

III. Results

Table I shows the results of our experiments. We run the algorithm once for each parameter combination over 30 generations. To assess the impact of individual parameters, we varied one parameter at a time while keeping the others constant, except for α and β, which were varied simultaneously to examine their interaction within the same equation. Per experiment for each parameter, best values are given in bold and runners-up are underlined. For each parameter configuration, the best values are highlighted in bold, while the runners-up are underlined.

In our experiments with the threshold value q_0 (indices 1-4), which determines the likelihood of applying the pheromone-based transition rule instead of relying solely on pheromone information, we observe that higher values of q_0 consistently lead to improved performance in terms of both cost and efficiency. Larger values of q_0 increase the probability that ants will bypass the transition rule and instead choose the path with the highest pheromone level. Indeed, higher reliance on using pheromones for path selection yields better outcomes. This can also be observed when experimenting with the parameters α and β. Exceptional results are obtained when $\beta = 0$, which forces the transition rule to also choose solely based on pheromone information. $\beta = 0$ is conceptually equivalent to $_0 = 1$; the difference in results can be attributed to the algorithm's initialization, where pheromone values are uniformly distributed. When all pheromone values from a node are identical, path selection defaults to a random choice.

Index	q_0	α	β	ρ	Initialization	Cost	Efficiency
1	0.1	1	1	0.1	Uniform	10.971	1.382
2	0.5	1	1	0.1	Uniform	8.454	1.220
3	0.9	1	1	0.1	Uniform	5.366	0.878
4	1.0	-	-	0.1	Uniform	**4.511**	**0.798**
5	0.5	0	1	0.1	Uniform	8.706	1.061
6	0.5	1	0	0.1	Uniform	**3.829**	**0.771**
7	0.5	1	1	0.1	Uniform	10.440	1.142
8	0.5	1	2	0.1	Uniform	7.978	1.651
9	0.5	2	1	0.1	Uniform	8.867	1.145
10	0.5	2	2	0.1	Uniform	7.586	1.902
1	0.1	1	1	0.1	Uniform	10.971	1.382
11	0.5	1	1	0.3	Uniform	8.877	**1.183**
12	0.5	1	1	0.6	Uniform	8.424	1.287
13	0.5	1	1	0.9	Uniform	**7.572**	1.214
14	0.9	1	1	0.1	Superhighway	4.618↓	1.042↑
15	1.0	-	-	0.1	Superhighway	4.241↓	0.957↑
16	0.5	0	1	0.1	Superhighway	8.539↓	1.325↑
17	0.5	1	0	0.1	Superhighway	4.224↓	0.769↑
18	0.5	2	2	0.1	Superhighway	7.902↓	1.282↑
19	0.5	1	1	0.3	Superhighway	7.460↓	1.239↑
20	0.5	1	1	0.6	Superhighway	8.715↑	1.159↓
21	0.5	1	1	0.9	Superhighway	9.090↓	1.105↑

TABLE I

Parameter settings and their effects on cost and efficiency. Best values are in bold, runners-up are underlined

Our findings indicate that the simple choice of pheromone initialization has a systematic impact on the performance of the generated maps in terms of cost and efficiency. Specifically, initializing pheromones with an additional weight of 3 on edges connecting influential cities (referred to as the Superhighway initialization) leads to an decrease in cost but an increase in efficiency, meaning that while the network becomes more cost-effective to construct, it results in longer average travel times between cities. This trend was observed in all but one of the cases tested. For this evaluation, we selected parameter configurations that produced either the best or runner-up results from each experimental category to test under the alternative initialization setting.

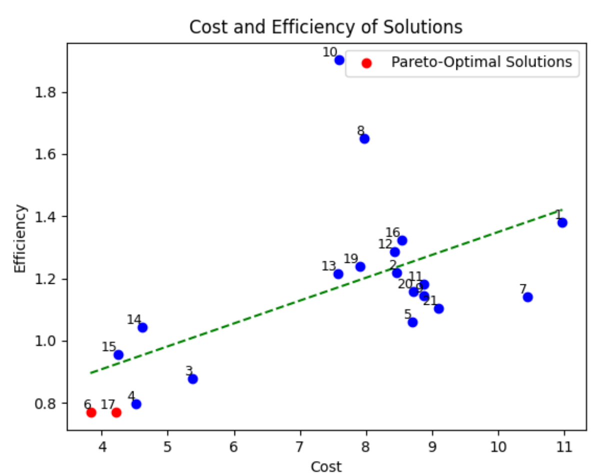

Fig. 2. Solutions plotted by cost and efficiency.

[6]https://docs.scipy.org/doc/scipy/reference/generated/scipy.sparse.csgraph.minimum_spanning_tree.html

[7]https://networkx.org/documentation/stable/auto_examples/graph/plot_mst.html

Within our set of solutions, we investigate whether we can observe a trade-off between cost and efficiency. Figure III presents each solution, labeled by index, plotted according to its corresponding cost and efficiency. A simple linear regression analysis revealed a significant positive relationship between cost and efficiency ($b = 0.074, SE = 0.025, p = 0.008, R^2 = 0.33$), indicating the absence of a trade-off between the two metrics. The lack of solutions that optimize either cost or efficiency, but not both, shows there is room for improvement in both dimensions. Further analysis of the Pareto-optimal solutions in the set shows that the Pareto front contains only two solutions, which are also the best overall solutions for both metrics. This limited Pareto set suggests that we were unable to sufficiently explore a diverse range of solutions along the Pareto front.

Fig. III shows Solutions 6 and 17, the two solutions comprising the Pareto set. Both solutions share identical parameter settings but differ in the initialization strategies for their respective pheromone maps. Solution 6 effectively connects geographically proximate cities - for example, the cities in the Ruhrpott region (Dortmund, Essen, Düsseldorf, Cologne, and Bonn) are well connected, as are Nuremberg, Regensburg, Ingolstadt, and Stuttgart. However, efficient connections between major, influential cities are lacking. Berlin, for instance, is directly connected to only three cities and lacks efficient links to key points such as Frankfurt am Main and Hamburg. In Solution 17, the use of the Superhighway initialization strategy partially addresses these deficiencies. However, even here, some critical connections are missing - for instance, neither Berlin nor Munich is directly connected to Hamburg. Furthermore, Bremen and Hamburg remain disconnected in both solutions. Solution 17 does leverage centrally positioned cities such as Wolfsburg, Hanover, and Halle as intermediate nodes to connect cities separated by greater distances. However, the potential of these intermediate cities to act as efficient ”bridges,” thereby improving overall network efficiency while keeping costs low, remains underutilized.

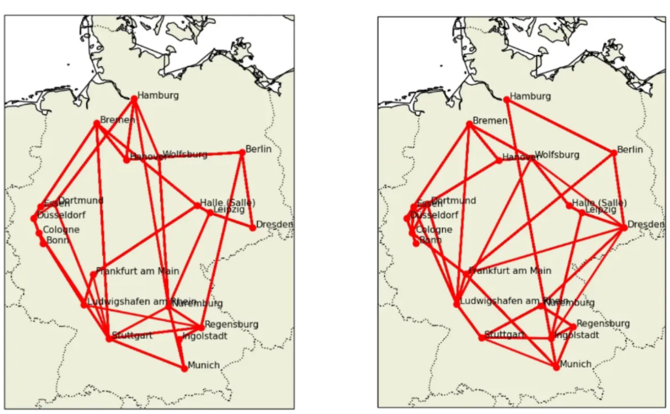

Fig. 3. Pareto set: Solutions 6 (left) and 17 (right).

IV. A Note on Lessons Learned

After implementing the core algorithm and running my first experiments, I found that simple algorithms can get you surprisingly far - The simplest version of my algorithm, which only accounted for pheromone trails and completely ignored edge lengths, outperformed every other variation I tried. Given the complexity of the problem, seeing an algorithm built from such basic building blocks produce decent results was impressive to me.

With that said, I quickly found that my simple solution only took me so far, and I attribute this to the fact that my algorithm does not explicitly improve based on fitness values - it relied on a heuristic only loosely tied to the objective functions. In hindsight, I regret not using the cost and efficiency metrics more directly to refine solutions in a more goal-driven way. If I could redo the project (albeit with a better programmer than me in my team), I would lean more into the problem's multi-objective nature in two ways. Firstly, I would try updating the global pheromone map based on solutions that directly improve a fitness function, which I would design by scalarizing the cost and efficiency metrics. Secondly, I would try creating two distinct pheromone maps - one optimizing cost, the other optimizing efficiency - and then combine partial solutions from both. I believe that these solutions would allow me to explore the extremes within the cost-efficiency trade-off in a far better way, which I am disappointed did not come out strongly in my project.

Another key takeaway was the importance of domain knowledge towards creating solutions that are indeed useful in real-life scenarios. I came to appreciate how much goes into designing a real highway network. Key connection points cannot naively be limited to cities. Topographical features influence not only the cost of building highways, but also set constraints as to what areas are accessible to the network. Estimating traffic flow between cities depends on a vast set of factors I cannot begin to imagine.

V. Conclusion

This project explored the potential of a multi-colony Ant Colony Optimization (ACO) algorithm for designing an efficient highway network for Germany. The algorithm consists of multiple individual ant colonies, each focusing on optimizing a smaller sub-goal while interacting with one another to influence their solutions. Our findings indicate that solutions relying exclusively on pheromone information tend to outperform those that also incorporate edge length data. While the algorithm generates reasonable results, the absence of a goal-driven fitness function limits its ability to fully optimize, leaving much of the Pareto front unexplored.

References

[1] de Mendoza, José. Memoria sobre algunos métodos nuevos de calcular la longitud por las distancias lunares: y aplicación de su teórica a la solución de otros problemas de navegación. En la Imprenta real, 1795.

[2] Erlander, Sven, and Neil F. Stewart. The gravity model in transportation analysis: theory and extensions. Vol. 3. Vsp, 1990.

[3] Dijkstra, Edsger W. ”A note on two problems in connexion with graphs.” Edsger Wybe Dijkstra: his life, work, and legacy. 1959. 287-290.