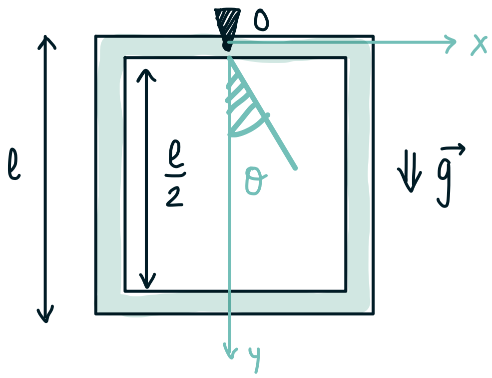
## Ejercicio - 1




* Suponemos que no hay disipación de energía

- $T = \frac{1}{2} I_0 \dot{\theta}^2$ ⊛
- $U = -\int_{y_0}^{y_F} Mg\,dy = -Mg(y_F - y_0)$

$$\begin{cases} y_0(\theta=0) = \ell/2 \\ y_F(\theta) = \ell/2 \cos(\theta) \end{cases}$$

Por tanto $U = Mg\,\ell/2\,(1-\cos(\theta))$

⊛ $$I_0 = \int_0^{\ell}\int_{-\ell/2}^{\ell/2} \rho_M (x^2+y^2)\,dx\,dy + \int_{-3\ell/4}^{3\ell/4}\int_{-\ell/4}^{\ell/4} (-\rho_M)(x^2+y^2)\,dx\,dy$$

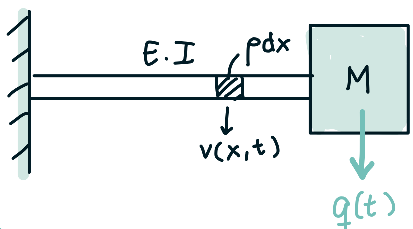
$$I_0 = \frac{5\rho_M \ell^4}{12} - \frac{\rho \ell^4 28}{6\cdot 64} = \rho \ell^4 \frac{11}{32}$$

- $\boxed{I_0\ddot{\theta} + Mg\,\ell/2 \sin(\theta) = 0}$ si $\theta << 0$ $\quad I_0\ddot{\theta} + Mg\frac{\ell}{2}\cdot\theta = 0$

$\boxed{\omega_0^2 = \frac{Mg\,\ell/2}{I_0}}$ → *frecuencia natural*

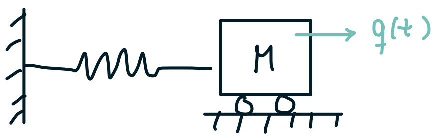
## Ejercicio - 2

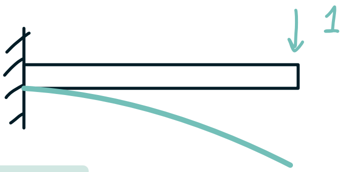
$M \equiv$ masa puntual sin momento de inercia




1) La masa de la viga es despreciable
2) Masa de la viga se tiene en cuenta de forma aproximada

1)




$R_{eq} = \frac{3EI}{L^3}$

Por tanto $\boxed{\omega_0^2 = \frac{3EI}{L^3 M}}$

- $T_v = \frac{1}{2}\int_0^L \rho\, v(x,t)^2\,dx$ con $v(x,t) \simeq \psi(x)\,\dot{q}(t)$
$\psi(L) = 1$

→ $\psi_1(x) = \frac{x}{L}$

→ $EI\frac{d^4\psi_2}{dx^4} = \delta(x-L)$ con $\psi_2(0) = 0 \quad \psi_2''(L) = 0$
$\psi_2'(0) = 0 \quad \psi_2'''(L) = 0$

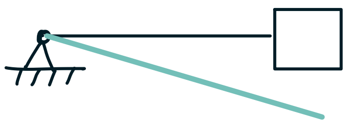
consecuencia de su propio peso

$$EI\frac{d^4\psi_3}{dx^4} = m(x)g$$

PARA $\psi_1$ $\Rightarrow$ $T = \frac{1}{2}\int_0^L \rho\left(\frac{x}{L}\right)^2 \dot{q}^2\,dx = \frac{1}{2}\rho L\dot{q}^2\int_0^1\left(\frac{x}{L}\right)^2 d\left(\frac{x}{L}\right) =$

$$= \frac{1}{2}\rho L\dot{q}^2\left(\frac{x}{L}\right)^3\frac{1}{3}\Big|_0^1 = \frac{1}{2}\left(\frac{1}{3}\rho L\right)\dot{q}^2 = \frac{1}{2}\left(\frac{1}{3}m_v\right)\dot{q}^2$$

↳ solamente el 33% de la masa de la viga participa en el modelo de 1gdl en la inercia de la viga si $\frac{1/3\, m_v}{M} \ll 1$ ✓



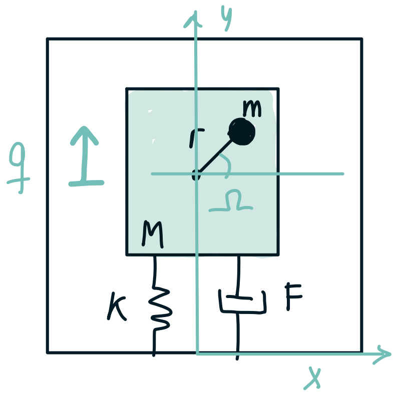
↳ en el caso real debe ser menor (empotramiento en vez de articulación)

PARA $\psi_2$ $\Rightarrow$ $\psi_2(x) = \frac{3}{L^3}\left(-\frac{x^3}{6} + \frac{Lx^2}{2}\right)$

- $T = \frac{1}{2}\int_0^L \rho\,\psi_2(x)^2\dot{q}(t)^2\,dx = \frac{1}{2}(0.2358\,\rho L)\,\dot{q}^2$ ¿ $\frac{0.236\rho L}{M} \ll 1$ ?

$$\boxed{\omega_0^2 = \frac{3EI/L^3}{(M + 0.236\,m_v)}}$$

## EJERCICIO-3




$M = 40$ kg
$mr = 0.2$ kgm
$\gamma = 0.2$
$\Omega = 1200$ rpm
$f_0 = 5$ Hz

1) amplitud mov. del tambor
2) fuerza transmitida
3) " " si $K \to \infty$ (atornillado rígidamente)

$q \equiv$ desplazamiento absoluto tambor

1)

$$T = \frac{1}{2}(M-m)\dot{q}^2 + \frac{1}{2}m\left(\dot{x}_m^2 + \dot{y}_m^2\right)$$

$$\begin{cases} x_m = r\cos(\Omega t) & \dot{x}_m = -r\Omega\sin(\Omega t) \\ y_m = r\sin(\Omega t) + q & \dot{y}_m = \dot{q} + r\Omega\cos(\Omega t) \end{cases}$$

POR TANTO $T = \frac{1}{2}(M-m)\dot{q}^2 + \frac{1}{2}m\left(r^2\Omega^2 + \dot{q}^2 + 2\dot{q}r\Omega\cos(\Omega t)\right)$

$$T = \frac{1}{2}M\dot{q}^2 + \frac{1}{2}m\left(r^2\Omega^2 + 2\dot{q}r\Omega\cos(\Omega t)\right)$$

$$V = \frac{1}{2}Kq^2$$

$$D = \frac{1}{2}F\dot{q}^2$$

$$\frac{\partial T}{\partial \dot{q}} = M\dot{q} + mr\Omega\cos(\Omega t) \longrightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) = M\ddot{q} - mr\Omega^2\sin(\Omega t)$$

POR TANTO $M\ddot{q} + F\dot{q} + kq = mr\Omega^2\sin(\Omega t)$

$$|q| = \frac{mr\Omega^2}{\left[(k-\Omega^2 M)^2 + (\Omega F)^2\right]^{1/2}} = \frac{mr\Omega^2}{K\left[\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\gamma\Omega}{\omega_0}\right)^2\right]^{1/2}}$$

con $\Omega = 40\pi$ rad/s, $\omega_0 = 5\cdot 2\pi$ rad/s, $K = \omega_0^2\cdot M = (10\pi)^2 40\ \frac{N}{m}$

$|q| = 0.0053$ m

2) $$F_{TR} = kq + F\dot{q} = \frac{\sqrt{1+\left(\frac{2\gamma\Omega}{\omega_0}\right)^2}\cdot mr\Omega^2}{\left[\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\gamma\Omega}{\omega_0}\right)^2\right]^{1/2}} = 395\ N$$

NO HAY amplificación Dinamica

3) si $k \longrightarrow \infty$, $\omega_0 \longrightarrow \infty$ $\qquad F_{TR} = \frac{1}{1}\cdot mr\Omega^2 = 3158\ N$ (amplificacion con respecto a la estática)

# problemas de excitaciones aleatorias

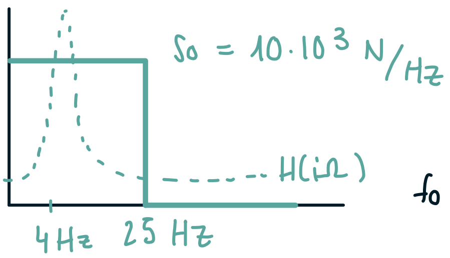
$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\,f/f_0}{\left(1-\left(\frac{f}{f_0}\right)^2\right)^2+\left(\frac{2\gamma f}{f_0}\right)^2}\simeq\frac{1}{4\gamma}$$

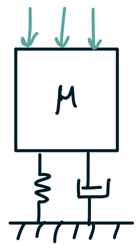
$$\frac{1}{2\pi}\int_{f_1}^{f_2}\frac{d\,f/f_0}{\left(1-\left(\frac{f}{f_0}\right)^2\right)^2+\left(\frac{2\gamma f}{f_0}\right)^2}=\frac{1}{4\gamma}\left[L\left(\frac{f_2}{f_0},\gamma\right)-L\left(\frac{f_1}{f_0},\gamma\right)\right]$$

$$L\left(\frac{f}{f_0},\gamma\right)=\frac{1}{\pi}\,tg^{-1}\left[\frac{2\gamma f/f_0}{1-\left(\frac{f}{f_0}\right)^2}\right]+\frac{\gamma}{2\pi\sqrt{1-\gamma^2}}\ln\left[\frac{1+\left(\frac{f}{f_0}\right)^2+2\sqrt{1-\gamma^2}\,\frac{f}{f_0}}{1+\left(\frac{f}{f_0}\right)^2-2\sqrt{1-\gamma^2}\,\frac{f}{f_0}}\right]$$

## ejercicio-1

$m = 60$ kg




DETERMINAR LA MEDIA CUADRÁTICA DE LA RESPUESTA

$$|H(i\Omega)|=\frac{1}{k\left[\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right)^2+\left(\frac{2\gamma\Omega}{\omega_0}\right)^2\right]^{\frac{1}{2}}}$$

$f_0 = 4$ Hz , $\gamma = 0.05$

$$\bar{y}^2=\frac{1}{2\pi}\int_{-\infty}^{\infty}|H(i\Omega)|^2 S_{xx}(\Omega)\,d\Omega=\frac{1}{2\pi}\int_{-\infty}\frac{f_0\cdot S_{xx}\cdot 2\pi}{K^2\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2+\left(\frac{2\gamma f}{f_0}\right)^2\right]}d\left(\frac{f}{f_0}\right)$$

$$=\frac{f_0}{K^2}\int_0^{25}\frac{S_{0xx}\,d(f/f_0)}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2+\left(\frac{2\gamma f}{f_0}\right)^2\right]}\quad \text{con}\quad f_0=4\text{ Hz}\quad (*)$$

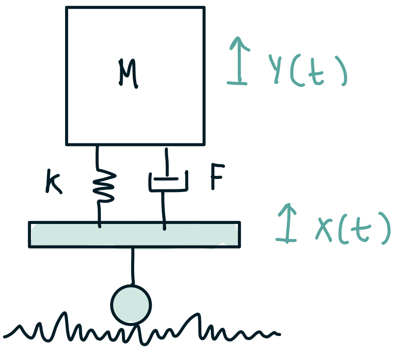
- como la frecuencia natural cae en el rango de frecuencias de excitación y el coeficiente de amortiguamiento es pequeño (~5%) el sistema ve la excitación como ruido blanco

$$(*)\simeq\frac{f_0}{k^2}S_{0xx}\int_{-\infty}^{\infty}\frac{d\,f/f_0}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2+\left(\frac{2\gamma f}{f_0}\right)^2\right]}=S_{0xx}\frac{f_0}{k^2}\cdot\frac{2\pi}{4\gamma}=\frac{S_{0xx}\,\omega_0}{k^2\,4\gamma}=$$

$$=\frac{S_{0xx}\,\omega_0}{\omega_0^4 m^2 4\gamma}=\boxed{\frac{S_{0xx}}{\omega_0^3 m^2 4\gamma}=\bar{y}^2}$$

$\boxed{RMS_y = 0.0935 \text{ m}}$

# ejercicio - 2




$$\rightarrow \; M\ddot{y} + F(\dot{y} - \dot{x}) + K(y - x) = 0$$

- x posee media nula
- y definido respecto posición equilibrio de la media

$$M\ddot{y} + F\dot{y} + Ky = F\dot{x} + Kx$$

$$\ddot{y} + \frac{F}{M}\dot{y} + \frac{K}{M}y = \frac{F}{M}\dot{x} + \frac{K}{M}x$$

con $\frac{F}{M} = \frac{F}{2k}\frac{2k}{M} = \gamma\frac{2k}{\sqrt{kM}} = 2\gamma\omega_0$

por tanto $\ddot{y} + 2\gamma\omega_0\dot{y} + \omega_0^2 y = 2\gamma\omega_0\dot{x} + \omega_0^2 x = \omega_0^2 f(t)$

con $f(t) = x + \frac{2\gamma}{\omega_0}\dot{x}$

- suponemos que la PSD dato corresponde a $f(t)$
- la función de transferencia del sistema

$$H_{yf}(i\Omega) = \frac{\omega_0^2}{\left[(\omega_0^2 - \Omega^2)^2 + (2\gamma\omega_0\Omega)^2\right]^{1/2}} = \frac{1}{\left[\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\gamma\Omega}{\omega_0}\right)^2\right]^{1/2}}$$

$$\overline{y^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{S_0 f_0 \, d(f/f_0)}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2\gamma f}{f_0}\right)^2\right]} = S_0 f_0 \frac{1}{2\pi}\int_{-\infty}^{\infty \, (*)}\frac{df/f_0}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2\gamma f}{f_0}\right)^2\right]} =$$

$$= \frac{S_0 f_0}{4\gamma} \longrightarrow \boxed{RMS\ \overline{y^2} = \sqrt{\frac{S_0 f_0}{4\gamma}}}$$

(*) tendríamos que aportar e. infinita (idealización matemática)

$$\overline{y^2} = \frac{1}{2\pi}\int_{f_1}^{f_2}\frac{S_0 f_0}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2\gamma f}{f_0}\right)^2\right]} d\left(\frac{f}{f_0}\right) = \frac{S_0 f_0}{2\pi}\int_{f_1}^{f_2}\frac{df/f_0}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2\gamma f}{f_0}\right)^2\right]} =$$

¿ $f_0 \in [f_1, f_2]$ ó $\gamma << 1$ ? no lo sabemos

$$= \frac{S_0\omega_0}{4\gamma}\left[L\left(\frac{f_2}{f_1}, \gamma\right) - L\left(\frac{f_1}{f_0}, \gamma\right)\right]$$ y por tanto

$$\boxed{RMS\ \overline{y_2} = \sqrt{\frac{S_0\omega_0}{4\gamma}\left[L\left(\frac{f_2}{f_1}, \gamma\right) - L\left(\frac{f_1}{f_0}, \gamma\right)\right]}}$$

# Ejercicio - 3

* octava → Rango de frecuencias entre una frecuencia de referencia y su doble ($\Omega_{ref}$, $2\Omega_{ref}$, $4\Omega_{ref}$, $8\Omega_{ref}$ ...)
  Dadas dos frecuencias $\Omega_y$ y $\Omega_{REF}$, el nº de octavas se define

$$\frac{\Omega_y}{\Omega_{ref}} = 2^y \quad \text{por tanto} \quad \boxed{y = \frac{1}{\log 2} \log\left(\frac{\Omega_y}{\Omega_{REF}}\right)}$$

* decibelio → $10 \cdot \log\left[\frac{X}{X_{REF}}\right]$

Una PSD se dice que tiene "r" decibelios por octava si se cumple :

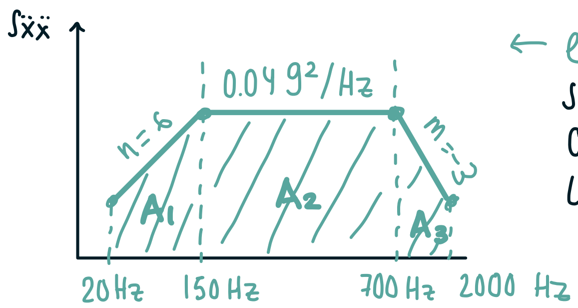
$$10 \cdot \log\left[\frac{S_{xx}(\Omega_y)}{S_{xx}(ref)}\right] = r\underbrace{y}_{\text{nº octavas}}$$

$$\log\left[\frac{S_{xx}(\Omega_y)}{S_{xx}(REF)}\right] = \frac{ry}{10} \; ; \quad \frac{S_{xx}(\Omega_y)}{S_{xx}(REF)} = 10^{\frac{ry}{10}} = 10^{\frac{r}{10\log 2}\log\left(\frac{\Omega_y}{\Omega_{ref}}\right)}$$

con $10\log 2 \approx 3$

$$\frac{S_{xx}(\Omega_y)}{S_{xx}(REF)} = 10^{\log\left(\frac{\Omega_y}{\Omega_{ref}}\right)^{r/3}} = \left(\frac{\Omega_y}{\Omega_{ref}}\right)^{\frac{r}{3}}$$

$$\boxed{\frac{S_{xx}(\Omega_y)}{S_{xx}(REF)} = \left(\frac{\Omega_y}{\Omega_{REF}}\right)^{\frac{r}{3}}}$$




← espectro del ariane en su lanzamiento
Se pide valor RMS del nivel de aceleración alcanzado durante su lanzamiento

$$\bar{\ddot{x}}^2 = \int_{-\infty}^{\infty} S_{\ddot{x}\ddot{x}}(\Omega)\, d\Omega = \int_{-\infty}^{\infty} S_{\ddot{x}\ddot{x}}(f)\, df$$

$$\frac{S_{\ddot{x}\ddot{x}}(20\text{ Hz})}{S_{\ddot{x}\ddot{x}}(150\text{ Hz})} = \left(\frac{20}{150}\right)^2 \longrightarrow \text{obtenemos } S_{\ddot{x}\ddot{x}}(20\text{ Hz}) = 0.0007\ \frac{g^2}{Hz}$$

$$\frac{S_{\ddot{x}\ddot{x}}(2000\text{ Hz})}{S_{\ddot{x}\ddot{x}}(700\text{ Hz})} = \left(\frac{2000}{700}\right)^{-1} \longrightarrow S_{\ddot{x}\ddot{x}}(2000\text{ Hz}) = 0.0140\ g^2/Hz$$

Integrando las áreas :

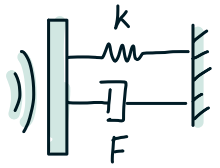
$$A_1 = \frac{S\ddot{x}\ddot{x}(150) + S\ddot{x}\ddot{x}(20)}{2}(150-20) = 2.6455\ g^2$$

$$A_2 = 0.04\ g^2/Hz\ (700-150)\,Hz = 22\ g^2$$

$$A_3 = \frac{S\ddot{x}\ddot{x}(2000) + S\ddot{x}\ddot{x}(700)}{2}(2000-700) = 35.1\ g^2$$

POR TANTO $\boxed{RMS_{\ddot{x}} = \sqrt{A_1 + A_2 + A_3} = 7.7295\ g}$

## ejercicio - 4




RUIDO BLANCO $S_0 = cte$

$$M\ddot{y} + F\dot{y} + Ky = P(t)$$

con F. transferencia $|H(i\Omega)| = \dfrac{1}{K\left[\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\gamma\Omega}{\omega_0}\right)^2\right]^{1/2}}$

$$\overline{y^2} = \frac{1}{\cancel{2\pi}}\int_{-\infty}^{\infty}\frac{S_0 \cdot f_0 \cdot d(f/f_0)}{K\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2\gamma f}{f_0}\right)^2\right]} = \frac{S_0 f_0}{K^2}\int_{-\infty}^{\infty}\frac{d(f/f_0)}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2\gamma f}{f_0}\right)^2\right]} =$$

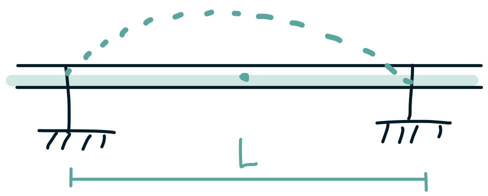
$$= \frac{S_0 f_0}{K^2}\cdot\frac{2\pi}{4\gamma} = \frac{S_0\omega_0}{K^2 4\gamma} = \frac{S_0\cdot\sqrt{\frac{\cancel{K}}{M}}}{K^2\cdot 4F/2\sqrt{KM}} = \frac{S_0 K}{K^2\ 2F} = \frac{S_0}{2KF}$$

$$\boxed{RMS\ \overline{y^2} = \sqrt{\frac{S_0}{2KF}}} = \sigma_y \quad si \quad \bar{x} = 0$$

NO DEPENDE DE LA MASA QUE SE LE PONGA AL CRISTAL

PROBABILIDAD MAYOR DE $\pm 2\sigma_y = 4.6\%$

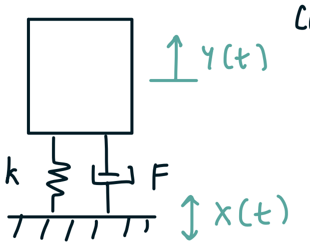
## ejercicio - 5




- APOYOS EXCITACIÓN ALEATORIA DE PSD, $S\ddot{x}\ddot{x} = 2\ g^2/Hz$ EN EL INTERVALO DE 10 A 1000 Hz

1) DESPLAZAMIENTO MEDIO CUADRÁTICO EN EL CENTRO DE LA TUBERÍA
2) ESFUERZO MEDIO CUADRÁTICO

CONSIDERAR $E = 200\cdot 10^9\ N/m^2$, $L = 1/2\ m$, $\varnothing ext = 1.25\ cm$, $\varnothing int = 1\ cm$, MASA POR U. LONGITUD $m = 0.1943\ kg/m$ CONSIDERANDO $\gamma = 0.02$




$y(t) \equiv$ MOV. RELATIVO CON RESPECTO A LOS APOYOS

$$T = \frac{1}{2}M(\dot{y}+\dot{x})^2$$

$$D = \frac{1}{2} F \dot{y}^2 \quad \longrightarrow \quad M(\ddot{y} + \ddot{x}) + F\dot{y} + Ky = 0$$

$$M\ddot{y} + F\dot{y} + Ky = -M\ddot{x}$$

$$U = \frac{1}{2} K y^2$$

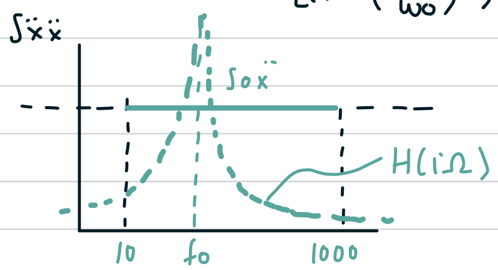
$$\hookrightarrow f_0 = \frac{\omega_0}{2\pi} = 165{,}4 \text{ Hz}$$

con $K|_{\text{viga biapoyada}} = \frac{48EI}{L^3}$ , $\omega_0 = \sqrt{\frac{48EI/L^3}{1/2\, mL}}$

$$m_{eq} = \frac{1}{2} mL = M \quad , \quad I = \frac{\pi}{64}\left(D_{ext}^4 - D_{in}^4\right) = 0.07075 \text{ cm}^4$$

$$|H(i\Omega)_{y\ddot{x}}| = \frac{-M}{\left[(K - \Omega^2 M)^2 + (\Omega F)^2\right]^{1/2}} = \frac{-M}{K\left[\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\gamma\Omega}{\Omega_0}\right)^2\right]^{1/2}} =$$

$$= \frac{-1/\omega_0^2}{\left[\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\gamma\Omega}{\Omega_0}\right)^2\right]^{1/2}}$$




se aproxima como ruido blanco:

$$\int_{10}^{1000} \frac{d(f/f_0)}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2\gamma f}{f_0}\right)^2\right]} \simeq \int_{-\infty}^{\infty} \frac{d(f/f_0)}{\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2\gamma f}{f_0}\right)^2\right]}$$

con $f_0 \in [10 - 1000]$

$$\bar{y}^2 = \frac{S_{0\ddot{x}\ddot{x}}}{\omega_0^3 4\gamma} \qquad \sigma_Y \quad (\text{con } \bar{x} = 0)$$

RMS $\bar{y}_2$ = 0.001464 m y por tanto el esfuerzo:

$$\sigma_{ESTR} = \boxed{\frac{\pi^2 D_{ext} E}{2L^2} \sigma_{\bar{y}}} = 72.25 \cdot 10^6 \text{ N/m}^2$$