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Question 1. Computing Transitions in a Representative Agent Economy. Consider the following closed optimal growth economy populated by a large number of identical infinitely lived households that maximize:

$$E_0\{\sum_{t=0}^{\infty} \beta^t u(c_t)\}\tag{1}$$

over consumption and leisure $u(c_t) = lnc_t$ subject to:

$$c_t + i_t = y_t \tag{2}$$

$$y_t = k_t^{(1-\theta)} (zh_t)^{\theta} \tag{3}$$

$$i_t = k_{t+1} - (1 - \delta)k_t \tag{4}$$

Set labor share to $\theta = 0.67$. Also, to start with, set $h_t = .31$ for all t. Population does not growth.

a) Compute the steady-state. Choose z to match an annual capital-output ratio of 4, and an investment-output ratio of .25.

First of all, before trying to solve the problem we need to observe how many equations and parameters are in this setting. One equation that is needed is the Euler Equation, to get this equation, we need to solve the Social Planner problem subject to the constraints. Plugging equation 3¹ and 4 in equation 2, the problem is simplified.

$$c_t + k_{t+1} - (1 - \delta)k_t = k_t^{1-\theta}(zh_t)^{\theta} \Rightarrow c_t + k_{t+1} = k_t^{1-\theta}(zh_t)^{\theta} + (1 - \delta)k_t$$

The maximization problem can be writen as:

$$\mathcal{L} = E_0 \{ \sum_{t=0}^{\infty} \beta^t u(c_t) \} + \lambda_t (k_t^{1-\theta} (zh_t)^{\theta} + (1-\delta)k_t - (c_t + k_{t+1})) \}$$

The FOC's of the problem are:

$$c_t: \beta^t * u_c(c_t) - \lambda_t = 0 \Rightarrow \lambda_t = \beta^t * u_c(c_t) \Rightarrow \lambda_t = \beta^t / c_t$$
(5)

$$c_{t+1} = \beta^{t+1}/c_{t+1} \tag{6}$$

$$k_{t+1} = -\lambda_t + \lambda_{t+1} \left[(1 - \theta) k_t^{-\theta} (zh_t)^{\theta} + (1 - \delta) \right] \Rightarrow \lambda_t = \lambda_{t+1} \left[(1 - \theta) k_t^{-\theta} (zh_t)^{\theta} + (1 - \delta) \right]$$
 (7)

Plugging equation 5 and 6 in 7, we obtain:

$$\frac{\beta^t}{c_t} = \frac{\beta^{t+1}}{c_{t+1}} \left[(1-\theta)k_t^{-\theta}(zh_t)^{\theta} + (1-\delta) \right]$$

¹The equations and graphs of this document have cross-references

$$c_{t+1} = c_t * \beta \left[(1 - \theta) k_t^{-\theta} (zh_t)^{\theta} + (1 - \delta) \right]$$

Since we are computing the steady state, we know that the variables grow in a constant rate in SS, therefore:

- $c_t = c_{t+1} = c$
- $k_t = k_{t+1} = k$
- $h_t = h_{t+1} = h$

$$\frac{c}{c} = \beta \left[(1 - \theta)k^{-\theta}(zh)^{\theta} + (1 - \delta) \right]$$

where $(1 - \theta)k^{-\theta}(zh)^{\theta} = F_k$ This is equivalent to:

$$1 = \beta(1 + F_k - \delta)$$

We have obtained the Euler Equation in the Steady State, the other equations are the production function 3, the budget constraint 2, the capital-output ratio (k/y)=4 and the investment-output ratio: $\frac{\delta *i}{Y}=0.25$, all of them are given in this exercise. The system is just-identified since we have 5 unknowns: β,δ , consumption, output and capital. From the previous ratios, we know that:

$$\delta = \frac{0.25}{4} = 0.0625$$

We can find different combinations of k^* and y^* for which the ratio is hold, let's normalize the output to 1, therefore the capital is equal to 4. Obviously, there are some combinations that produce the same result. Since δ is equal to 0.0625 and in steady state, $k_t = k_{t+1}$, the investment is equal to:

$$i = k - (1 - \delta)k = \delta * k = 0.0625 * 4 = 0.25$$

Therefore, the consumption, that is the difference between the income and the investment becomes:

$$c = y - i = 1 - 0.25 = 0.75$$

We have solved for consumption, output, capital and δ , therefore the only parameter that is left is β but before knowing the result, we need to calculate z, in the production function, 3:

$$\left(\frac{y}{k^{1-\theta}h^{\theta}}\right)^{\frac{1}{\theta}} = z^* \Rightarrow z^* = \left(\frac{1}{4^{0.33}h^{0.67}}\right)^{\frac{1}{0.67}} \Rightarrow z^* = 1.629 \approx 1.63$$
$$z^* = \frac{k^{\frac{\theta-1}{\theta}}}{h}$$

Plugging z in the euler equation and substituting the values for θ , k, h and δ , we obtain that $\beta = 0.9804 \approx 0.98$.

Let's calculate the capital in the steady state, to do so, we need two equations: the marginal productivity of capital and the Euler Equation.

$$f'(k) = (1 - \theta)k^{-\theta}(zh)^{\theta}$$

$$f'(k) = \frac{1}{\beta} - 1 + \delta$$

$$\frac{1}{\beta} - 1 + \delta = (1 - \theta)k^{-\theta}(zh)^{\theta} \Rightarrow 1 - \beta(1 - \delta) = \beta(1 - \theta)k^{-\theta}(zh)^{\theta} \Rightarrow k^{-\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)(zh)^{\theta}} \Rightarrow k^{\theta} = \frac{1 - \beta(1 - \delta)}{\beta$$

Now, let's do it computationally, first of all, we define the parameters:

```
#Set the parameters:
h=0.31
Theta=0.67
Delta=0.0625
#Normalize the output to 1 and the capital to 4
k=4
y=1
i= Delta*k
c=y-i
z= k**((Theta-1)/Theta) / h
```

Once we have done this, we need the production function to obtain the coefficient of productivity, z, and the Euler equation to get β . We obtain the same results that analitically:

```
The productivity parameter,z= 1.6297
Beta= 0.9804
Steady State for capital= 4.00
Steady State for output= 1.00
Steady State for consumption= 0.75
Delta 0.0625
```

b) Double permanently the productivity parameter z and solve for the new steady state.

The parameter θ , h, the annual capital-output ratio and an investment-output ratio are exactly the same. Therefore, we continue having the same δ and the same intertemporal discount rate. The introduction in this part is that z_t is the double compared with the part a, this means that it is equal to $z_t = 3.2594$. Once we have this, we can substitute in the steady state level of capital:

$$k^* = 3.26 * 0.31 \left(\frac{1 - 0.98(1 - 0.0625)}{0.98(1 - 0.67)} \right)^{\left(\frac{-1}{0.67}\right)} = 7.95 \approx 8$$
$$i^* = 0.0625 * 8 = 0.5$$
$$y^* = 8^{1 - 0.67} * (3.26 * 0.31)^{0.67} = 2$$
$$c^* = y^* - i^* = 2 - 0.5 = 1.5$$

We observe that when z is double, the capital is also doubled. Then, the investment is doubled and the production is doubled, and therefore, the consumption is also doubled. However, the h does not change, since it is exogeneous.

For doing this computationally, we need the Euler equation but we should substitute for the new capital. Once we have the capital, we can substitute for the other variables.

```
New productivity parameter,z= 3.2594
New Steady State for capital= 8.00
Beta= 0.9804
New steady State for output= 2.00
New steady State for consumption= 0.75
Delta 0.0625
```

	Parameters/Variables	Steady State a	Steady State b
0	z	1.629676	3.259352
1	С	0.750000	1.500000
2	i	1.000000	0.500000
3	у	1.000000	2.000000
4	k	4.000000	8.000000
5	h	0.310000	0.310000
6	Theta	0.670000	0.670000
7	Beta	0.980392	0.980392
8	Delta	0.062500	0.062500

c) Compute the transition from the first to the second steady state and report the timepath for savings, consumption, labor and output.

The results for the time-path for savings, consumption, labor and output are represented in figure 1. In first place, the labor is constant along the transition, recall that it is given (h=0.31). In the long term, there is more capital, more production and therefore, more consumption. This is because capital is more productive, so it is worth accumulating more of it and use the proceeds to consume more in the long-run. In this case the wealth effect dominates the substitution effect, the savings decrease along this path.

With respect to the computational part, the important thing is that: when the transition is defined, we need to write the Euler Equation in term of capital. Therefore, we need to introduce this modification:

$$c_{t+1} = c_t * \beta \left[(1 - \theta) k_t^{-\theta} (zh_t)^{\theta} + (1 - \delta) \right]$$
$$y_{t+1} - i_{t+1} = \beta (y_t - i_t) * \left[(1 - \theta) k_t^{-\theta} (zh_t)^{\theta} + (1 - \delta) \right]$$

where:

$$(zh_t)^{\theta} = \frac{y}{k^{1-\theta}}$$
$$i_t = k_{t+1} - (1-\delta)k_t$$

Therefore:

$$(y_{t+1} - (k_{t+2} - (1 - \delta)k_{t+1})) = \beta(y_t - (k_{t+1} - (1 - \delta)k_t)) * \left[(1 - \theta)k_t^{-\theta} (\frac{y}{k^{1-\theta}}) + (1 - \delta) \right] \Rightarrow 0 = \beta(y_t - (k_{t+1} - (1 - \delta)k_t)) * \left[(1 - \theta)k_t^{-\theta} (\frac{y}{k^{1-\theta}}) + (1 - \delta) \right] - (y_{t+1} - (k_{t+2} - (1 - \delta)k_{t+1}))$$

In this problem we have three Euler equations: for the first steady state, for the transition and for the last steady state. We need to take into account the new value of the productivity shock. Once

the transition of capital has been solved, it's easy to compute the transition for production, savings (recall that it is equal to investment), consumption and labor.

d) Unexpected shocks. Let the agents believe productivity z_t doubles once and for all periods. However, after 10 periods, surprise the economy by cutting the productivity z_t back to its original value. Compute the transition for savings, consumption, labor and output.

The results of this unexpected shock can be seen in figure 1, the labor does not change since it is exogenous. Now the output and the consumption have a "hump-shaped", the first ten periods are explained using the same reasoning than before, but in this case there is an unexpected shock after period 10, that makes the economy return to the previous value of productivity. Since the productivity is lower, the production function is affected negatively and therefore, the consumption and capital decreases. The savings decrease for the first 10 periods and then increase until reaching the new level of steady state.

The computational part has been solved similar to before, we have consider the three Euler equation but in this case for the first value of productivity shock. Then, once transition for capital is solved, the transition for output, savings, consumption and labor is also solved. In this case for each variable, we need to consider the values of the first 10 periods (part c) and the new transition path, using the command "concatenate".

e)Bonus Question: Labor Choice Allow for elastic labor supply. That is, let preferences be:

$$u(c_t, 1 - h_t) = lnc_t - \kappa \frac{h_t^{1 + \frac{1}{\nu}}}{1 + \frac{1}{\nu}}$$
(8)

and recompute the transition as posed in Question 1.

In this case, the maximization problem is:

$$\mathcal{L} = E_0 \{ \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - ht) \} + \lambda_t (k_t^{1-\theta} (zh_t)^{\theta} + (1 - \delta)k_t - (c_t + k_{t+1})) =$$

$$= E_0 \{ \sum_{t=0}^{\infty} \beta^t (\ln c_t - \kappa \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}) \} + \lambda_t (k_t^{1-\theta} (zh_t)^{\theta} + (1-\delta)k_t - (c_t + k_{t+1})) \}$$

The FOC are:

$$c_t: \beta^t * u_c(c_t) - \lambda_t = 0 \Rightarrow \lambda_t = \beta^t * u_c(c_t) \Rightarrow \lambda_t = \beta^t / c_t$$
(9)

$$c_{t+1} = \beta^{t+1}/c_{t+1} \tag{10}$$

$$k_{t+1} = -\lambda_t + \lambda_{t+1} \left[(1 - \theta) k_t^{-\theta} (zh_t)^{\theta} + (1 - \delta) \right] \Rightarrow \lambda_t = \lambda_{t+1} \left[(1 - \theta) k_t^{-\theta} (zh_t)^{\theta} + (1 - \delta) \right]$$
(11)

$$h_t : -\beta^t \kappa (1 + \frac{1}{\nu}) \frac{h_t^{\frac{1}{\nu}}}{1 + \frac{1}{\nu}} + \lambda_t k_t^{(1-\theta)} (zh_t)^{\theta-1} * z = 0 \Rightarrow -\beta^t h_t^{\frac{1}{\nu}} + \lambda_t k_t^{(1-\theta)} (h_t)^{\theta-1} * z^{-\theta} \Rightarrow$$

$$\lambda_t = \frac{\beta^t h_t^{\frac{1}{v}}}{k_t^{(1-\theta)} (h_t)^{\theta-1} * z^{-\theta}}$$
 (12)

Combining the equations 9 and 10 in 11, we obtain the Euler Equation:

$$c_{t+1} = c_t \beta \left[(1 - \theta) k_t^{-\theta} (zh_t)^{\theta} + (1 - \delta) \right]$$

where $(1-\theta)k_t^{-\theta}(zh_t)^{\theta} = F_k$ This is equivalent to:

$$c_{t+1} = c_t \beta (1 + F_k - \delta)$$

In steady state:

$$1 = \beta(1 + F_k - \delta)$$

Combining the equations 9 and 12, we obtain the labbor supply:

$$\beta^{t}/c_{t} = \frac{\beta^{t} h_{t}^{\frac{1}{v}}}{k_{t}^{(1-\theta)} (h_{t})^{\theta-1} * z^{-\theta}} \Rightarrow c_{t} = \frac{k_{t}^{(1-\theta)} (h_{t})^{\theta-1} * z^{-\theta}}{h_{t}^{\frac{1}{v}}}$$

$$(13)$$

or
$$h_t = \left(\frac{k_t^{1-\theta} z^{-\theta}}{c_t}\right)^{\frac{v}{1-\theta v + v}}$$

If we compare with the previous exercise, we have one parameter more to estimate, the labor supply, but one more equation, the system is identified.

Our equations are:

1.
$$1 = \beta \left[(1 - \theta)k_t^{-\theta}(zh_t)^{\theta} + (1 - \delta) \right]$$

$$2. y_t = k_t^{(1-\theta)} (zh_t)^{\theta}$$

3.
$$i_t = k_{t+1} - (1 - \delta)k_t$$

$$4. (k/y) = 4$$

5.
$$\delta i/Y = 0.25$$

6.
$$h_t = \left(\frac{k_t^{1-\theta} z^{-\theta}}{c_t}\right)^{\frac{v}{1-\theta v + v}}$$

As before we can do a normalization of output equal to 1, therefore the capital is 4 in steady state (it is not affected by the transition), once we have this, for equation 5 δ =0.0625 and the investment is equal to 0.25 and the consumption 0.75. We need to suppose a value for v, taking a similar value to the literature, let's consider that it is equal to 2 and $\kappa = 1$. For getting the values for h and z we need the production function and the expression for the labor. If I solve the system I get that z=3.13 and h=0.161. The last step is solving for β that it is equal to 0.98.

$$z_t^* = \frac{k^{\frac{\theta - 1}{\theta}}}{h}$$

$$k_t^* = zh_t \left(\frac{1 - \beta(1 - \delta)}{\beta(1 - \theta)}\right)^{\left(\frac{-1}{\theta}\right)}$$

We observe how there is a variation in the parameters of productivity and labor with respect to part a. In this case, the productivity is higher and h is lower, you need less workers since they are more productive, all the other variables continue having the same values: capital, output, consumption and investment. All of them are not affected by endogeneous labor. The time path for labor cannot be a straight line in this case, since it is endogeneous, the transition for the variables will be a bit different with respect to the other parts. We can solve for part b, but in these case I have three equations and three unknowns, a bit tedious solving by hand...and I don't know how to solve the transition.

Question 2. Solve the optimal COVID-19 lockdown model posed in the slides.

a) Show your results for a continuum of β combinations of the [0,1] parameter (vertical axis) and the c(TW) [0; 1] parameter (hztal axis). That is, plot for each pair of β and c(TW) the optimal allocations of H, H_f , H_{nf} , $H_f = H$, output, welfare, amount of infections and deaths. Note that if H = N there is no lockdown, so pay attention to the potential non-binding constraint H < N. Discuss your results. You may want to use the following parameters: $A_f = A_{nf} = 1$; $\rho = 1.1$, $\kappa_f = \kappa_{nf} = 0.2$, $\omega = 20$, $\gamma = 0.9$, $i_o = 0.2$ and N = 1.

We consider an economy with only one sector associated with a pair HC (human contact) and TW (telework). The equations of the model are:

Production technology

$$Y = \left(A_f H_f^{\frac{\rho-1}{\rho}} + c(TW) A_f H_{nf}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

$$\tag{14}$$

The production in this economy can be done at the workplace or teleworking at come, the c(TW) represents the productivity associated with teleworking.

Contagion process

• Infection rate

$$i = \beta(HC)m(H_f)$$
 with $m(H_f) = \frac{i_o H_f}{N}$ (15)

Only the individuals that work at the workplace face contagion risk, the meeting probability is equal to the initial share of infections at work times the aggregate ours of telework (recall N=1).

• Number of infections

$$I = iH_f \tag{16}$$

Deaths

$$D = (1 - \gamma)I\tag{17}$$

Once we have obtained this, we can formulate the planners problem. The planner chooses the combination of consumption and hours worked (c,h) that maximizes the aggregate welfare of individuals, taking into account the deaths $(\omega, how much planner cares about death)$.

$$\max \sum_{i} (c_i - \kappa_f h_f - k_{nf} h_{nf}) - \omega D \tag{18}$$

Taking into account the resource constraint and the labor market clearing condition:

$$\max \sum_{i} (Y(H_f, H_{nf}) - \kappa_f H_f - k_{nf} H_{nf}) - \omega D \tag{19}$$

$$s.t$$
 (20)

$$H = H_f + H_{nf} \le N \tag{21}$$

Substituting the production function:

$$\max \sum_{i} \left(\left(A_f H_f^{\frac{\rho-1}{\rho}} + c(TW) A_{nf} H_{nf}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} - \kappa_f H_f - k_{nf} H_{nf} \right) - \omega D \tag{22}$$

$$s.t$$
 (23)

$$H = H_f + H_{nf} \le N \tag{24}$$

Substitute the deaths: first, the number of infections inside the deaths, then, the infection rate inside the number of infections, and the last step consists of substituting the meeting probability that meets with contagious individual, and we get:

$$D = (1 - \gamma)I = (1 - \gamma)iH_f = (1 - \gamma)\beta(HC)m(H_f)H_f = (1 - \gamma)\beta(HC)\frac{i_o H_f}{N}H_f$$
 (25)

Therefore the problem can be writen as:

$$\max \sum_{i} \left(\left(A_f H_f^{\frac{\rho-1}{\rho}} + c(TW) A_{nf} H_{nf}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} - \kappa_f H_f - k_{nf} H_{nf} \right) - \omega (1 - \gamma) \beta (HC) \frac{i_o H_f}{N} H_f \qquad (26)$$

$$s.t$$
 (27)

$$H = H_f + H_{nf} \le N \qquad (28)$$

The FOC'S of the problem are:

$$\frac{\partial}{\partial H_f} = \frac{\rho}{\rho - 1} \frac{\rho - 1}{\rho} A_f H_f^{\frac{-1}{\rho}} \left(\left(A_f H_f^{\frac{\rho - 1}{\rho}} + c(TW) A_{nf} H_{nf} \right)^{\frac{1}{\rho - 1} \frac{\rho}{\rho}} - \kappa_f - \omega (1 - \gamma) \beta (HC) \frac{i_o H_f}{N} - \lambda = 0 (29) \right)$$

$$A_f H_f^{\frac{-1}{\rho}} * Y^{\frac{1}{\rho}} - \kappa_f - \omega (1 - \gamma) \beta (HC) \frac{i_o H_f}{N} - \lambda = 0$$

$$\frac{\partial}{\partial H_{nf}} = \frac{\rho}{\rho - 1} \frac{\rho - 1}{\rho} A_{nf} c(TW) H_{nf}^{\frac{-1}{\rho}} Y^{\frac{1}{\rho}} - \kappa_{nf} - \lambda = 0$$
(30)

Doing this computationally, the following graphs are obtained: 3, 4, 5, 6, 7, 8, 9 and 10. Focusing on the interpretation, each point in the graph is a pair of telework and human contact score. This is a world that people born and died in this period, the planner decides if individual stay at home or go to work. The only way that the social planner can reduce infection is working at home, but if they work from home, they are less productive, so there is a <u>trade-off</u>.

Another important fact is that the planner cares equally about all individuals, he aggregates consumption. The figure 3 have the same color for every possible point since it is equal to 1 for every possible combination.

Recall that c(TW) is a factor that captures a productivity associated with teleworking while $\beta(HC)$ is the (conditional) infection rate which depends on the extent of human contact. Then, figure 5 shows that if the productivity of telework is high, the planner is going to choose that individuals telework for that reason the right hand side of the graph has slightly less colour.

Looking at figure 4, if the productivity of telework is high, the planner chooses that individuals work at home, however if it is small, the value of aggregate hours at workplace increases. As we can see in the previous reasoning H_f and H_{nf} have opposite directions. The explanation for 6 is exactly the same as 4.

The figure 8 and 7 have the same tendency, if the productivity of telework is high, the probability of infection is smaller since they have less human contact. However, if the productivity of telework is small, the planner chooses that these individuals work at place, so the probability that they get infected increases. This has a positive relationship with the number of deaths, if the proportion of the individuals that get infected increases significantly, there will be more deaths. The last function to analyze is the 9, if the productive of telework is high, this means that the majority of people work at home, therefore they meet with less people and there is a smaller probability to get infected, so there are less deaths in the society. This is the perfect scenario for the social planner, since the utility is the highest possible. The opposite case consists of low productivity of telework, so people go to work in person so there is more probability of getting infected and therefore more deaths, since the social planner cares about deaths, the total welfare is smaller and the worst possible. The output is smaller when the productivity of the workers that telework is lower.

b) What happens to your results when you increase (decrease) ρ or ω .

We have done two modifications changing the values of the parameters in the previous code:

- 1. The first one consists of increasing ω up to 150, the graphs are 11, 12,13, 14, 15, 16,17 and 18. The agregate hours are very low if the productivity of telework is lower as well, since the social planner cares more than in part a about the deaths of the society, he prefers less production (23) but less deaths. The aggregate hours at the workplace are lower if the productivity of individuals are higher (right-hand side of the figure) since they telework, however if the productivity is very low, they spend more hours at the workplace, but for values of 0.1 it depends on the value of the probability of getting infected. The opposite case happens for the aggregate hours of teleworking. The figure 14 displays the same results that the aggregate hours of the economy. With respect to the infections of the economy, these are higher for values of 0.2-0.5 of productivity of telework and higher values of β , the same for the deaths since both variables experiment the same movements. If we compare with the previous values, this probability is around 0.04, before it was around 0.100, so increasing the value of ω (how much SP cares about deaths) the deaths of the economy are reduced. Regarding the welfare is exactly the same as before.
- 2. The second one consists of increasing ρ until 10 and we get the following graphs:19, 20, 21, 22, 23, 24, 25 and 26. The parameter ρ measures the elasticity of substitution, the SP substitutes easily aggregates hours at workplace and telework, so we can see the shape of perfect substitutes in the graphs. The aggregate hours at the workplace increases when the productivity of telework is small, but do it progressively the same for the aggregate hours of teleworking. As the rate of substitution is high, there are more infections in the economy when the productivity of telework is smaller than 0.5, the same for deaths. We observe that the rate of infection is so large if we compare which the case where SP cares about deaths and it is a bit higher with respect to part a. The welfare now and the output are smaller if we compare both with the previous results.

1 APPENDIX

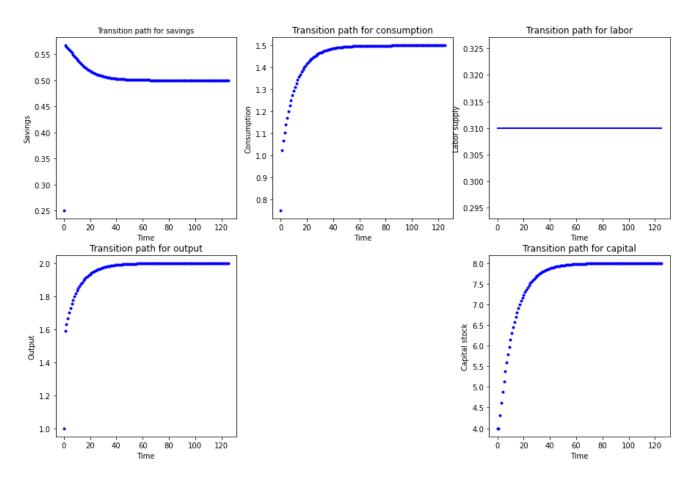


Figure 1: Time-path for the different variables, permanent productivity shock

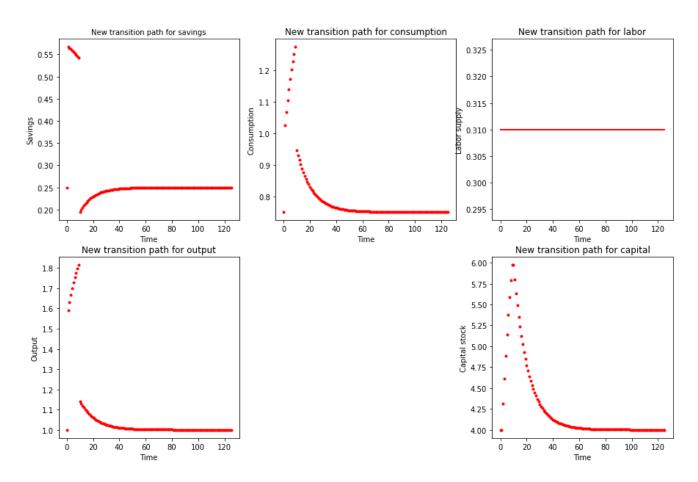


Figure 2: Time-path for the different variables, unexpected productivity shock

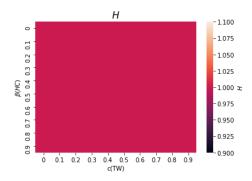


Figure 3: Aggregate hours

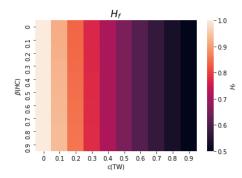


Figure 4: Aggregate hours at the workplace

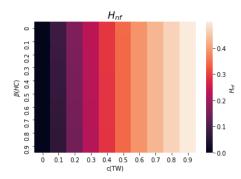


Figure 5: Aggregate hours of teleworking

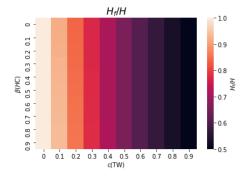


Figure 6: H_f/H

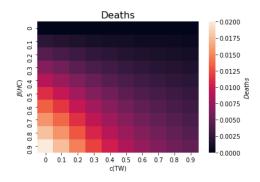


Figure 7: Deaths

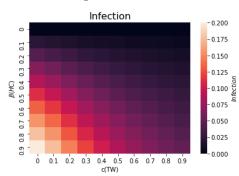


Figure 8: Infection

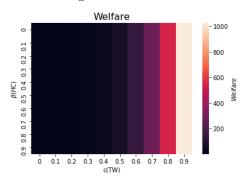


Figure 9: Welfare

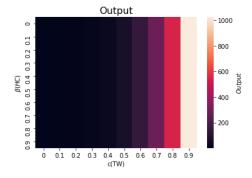


Figure 10: Output

MODIFICATION B.1

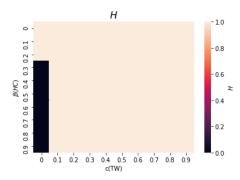


Figure 11: Aggregate hours

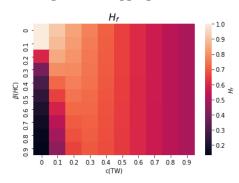


Figure 12: Aggregate hours at the workplace

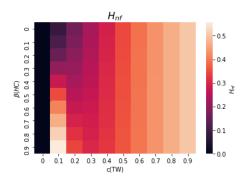


Figure 13: Aggregate hours of teleworking

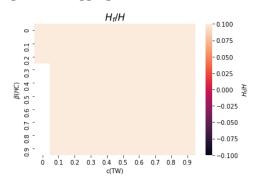


Figure 14: H_f/H

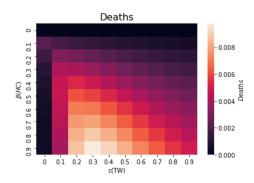


Figure 15: Deaths

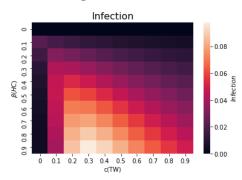


Figure 16: Infection

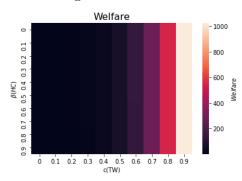


Figure 17: Welfare

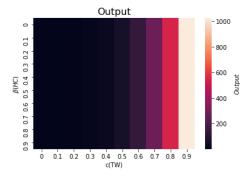


Figure 18: Output

MODIFICATION B.2

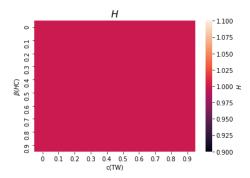


Figure 19: Aggregate hours

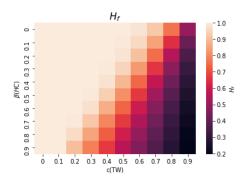


Figure 20: Aggregate hours at the workplace

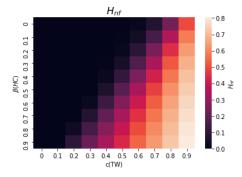


Figure 21: Aggregate hours of teleworking

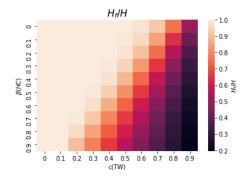


Figure 22: H_f/H

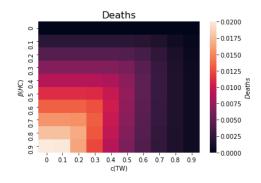


Figure 23: Deaths

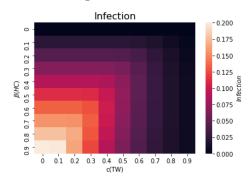


Figure 24: Infection

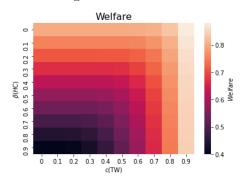


Figure 25: Welfare

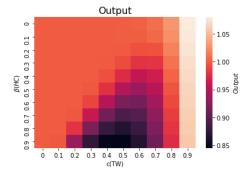


Figure 26: Output