Homework 4: Quantitative Macro

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Contents

1 A simple wealth model

Consider the sequential problem of a household that maximizes over streams of future consumption,

$$\max_{c_t{}_t^T} E_0 \sum_{t=0}^{T} \beta^t u(c_t) \tag{1}$$

where $\beta = \frac{1}{1+\rho} \in (0,1)$. **Labor supply is inelastic and normalized to one. The budget constraint for this household at period t is:**

$$c_t + a_{t+1} = w_t y_t + (1 + r_t)a_t \tag{2}$$

where r_t is exogenously given. The individual faces a stochastic endowment process of efficiency units of labor $\{y_t\}_t^T$ with $y_t \in Y = \{y_1, y_2, ..., y_N\}$.. This endowment process is Markov with $\pi(y'|y)$ denoting the probability that tomorrow's endowment is y' if today's endowment is y, i.e. $\sum_{y'} \pi(y'|y) = 1$.

Preferences. Consider 2 utility functions:

1. ***Quadratic utility***

   $$u(c_t) - \frac{1}{2}(c_t - \bar{c})^2 \tag{3}$$

   where we can set $\bar{c}$ enough, say 100 times the maximal income, to avoid saturation of any consumer.

2. ***CRRA utility***

   $$u(c_t) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \tag{4}$$

   with $\sigma > 0$.

Make your program code flexible for a applying both of these utility functions under different parameter values for $\bar{c}$ and σ (what value of σ yields the log utility?).

Factors prices and subjective discount rate. **Assume $r = 4\% < \rho = 6\%$ (hence, in the presence of certainty equivalence, will agents like increasing or declining consumption profiles?). Normalize $w = 1$. Make your program code flexible in order to do potential sensitivity on these parameters.**

Income process. Assume a 2-state process (make your program code exible for a larger state space with cardinality N). Let

$$Y = \{1 - \sigma_y, 1 + \sigma_y\} \tag{5}$$

and the transition income matrix

$$\begin{pmatrix} \frac{1+\gamma}{2} & \frac{1-\gamma}{2} \\ \frac{1-\gamma}{2} & \frac{1+\gamma}{2} \end{pmatrix} \tag{6}$$

This way, the variance of the income process and its persistence are respectively:

$$Var(y) = \sigma_y^2 \tag{7}$$

and

$$Corr(y', y) = \frac{Cov(y', y)}{\sqrt{Var(y')}\sqrt{Var(y)}} = \gamma \tag{8}$$

Borrowing constraints. Consider two cases:

1. **The natural borrowing constraint $a_{t+1} \geq -\bar{A} = -\frac{1+r}{r} y_{min}$, the time horizon is infinite, use the constraint that agents cannot die in debt, that is, $a_{T+1} \geq 0$. This implies borrowing constraints for all ages of the form:**

$$\boldsymbol{a_{t+1} \geq -y_{min} \sum_{s=0}^{T-(t+1)} (1+r)^{-s}} \tag{9}$$

2. **$a_{t+1} \geq 0$, preventing borrowing altogether.**

In this part, we have to answer: what value of σ yields the log utility? The value of σ that yields the log utility is 1:

$$\lim_{\sigma \to 1} u(c) = lim_{\sigma \to 1} \frac{c^{1-\sigma} - 1}{1-\sigma} = \frac{0}{0}$$

I get an indetermination of type 0 divided by 0, so all I need to do consists in differentiating the numerator and the denominator, then, I take the limit. In other words, I am applying l´Hopital's Rule. Recall that $[c^{1-\sigma}]\prime = ln(c) * c^{1-\sigma} * -1$, then:

$$lim_{\sigma \to 1} u(c) = lim_{\sigma \to 1} \frac{c^{1-\sigma} - 1}{1-\sigma} = \frac{-ln(c) * c^{1-\sigma} *}{-1} = lim_{\sigma \to 1} ln(c) * c^{1-\sigma} = ln(c)$$

In turn, more curvature (σ) reflects a higher incentive to smooth consumption across time. The reason is that a large curvature means that the marginal utility will drop sharply if consumption rises and will increase sharply if consumption falls. I would like to emphasize some things before setting up the problem:

1. There is uncertainty and it is at the individual level - this comes from the endowment which is random and given by a Markov process of order one such that the probability of today of being in a state s is only given by the one of the previous period (this is what allows us to have only s as a state variable and not the whole sequence).

2. What do incomplete markets mean? In the case of uncertainty, complete markets are only present when the individuals can insure against all risks (contingent claims mean that when a state s realizes, you can a payoff conditional on that particular state). Here, there is only one way to ensure against future risks - a riskless asset a which is however subject to a borrowing limit A so in fact they can't insure against potentially very bad risks.

3. Here $\bar{A}$ is the largest amount of debt that the agent can repay with probability one (by not consuming at all). The individuals must be able to return his/her debt, this prevents households to run in Ponzi Schemes (they cannot augment their consumption without bound).

4. If the exogenously imposed limit in question A is larger than the natural limit we derived, it won't be binding and therefore the borrowing limit can be specified as:

$$\bar{A} = min\{A, -\frac{1+r}{r} y_{min}\}$$

2 Solving the ABHI MODEL

1 The recursive formulation

Formulate the problem of the agent recursively, i.e. write down Bellmans equation and derive the stochastic Euler equation.

In this exercise we know that the price r is exogenous and constant, therefore we don't necessarily need to keep this as a state variable, since r'=r.

The recursive problem of the agents in recursive form is given by:

$$
\begin{aligned}
V(a,y) = max\ \ u(c) + \beta EV(a',y') \quad (10)\\
s.t \\
c_t + a_{t+1} = w_t y_t + (1+r_t)a_t \quad (11)\\
c \geq 0 \quad (12)\\
a' \geq -\bar{A} \quad (13)
\end{aligned}
$$

Note that in the recursive formulation, we only care about today and tomorrow's periods, and we denote tomorrow's period with a'. The equation 11 is the budget constraint, equation 12 the non-negativity condition for consumption, and the last one, equation 13, is the natural borrowing limit, which together with the assumption on the Markov process of order one for endowments, translates to:

$$V(a,y) = max\ \ u(c) + \beta \sum_{y'} \pi_{y'|y} V(a',y') \tag{14}$$

obviously subject to the constraints explained previously, since the expectation is taken over the distribution of the endowments y:

$$EV(a',y') = \sum_{y'} \pi_{y'|y} V(a',y')$$

First of all, we substitute the consumption from the budget constraint:

$$V(a,y) = max\ \ u(wy + (1+r)a - a') + \beta \sum_{y'} \pi_{y'|y} V(a',y') - \lambda_a(a' + A)$$

Then, we can take the first order condition with respect to a'.

FOC:

$$\frac{\partial V(a,y)}{\partial a'} = -u_c(wy + (1+r)a - a') + \beta \sum_{y'} \pi_{y'|y} \frac{\partial V(a',y')}{\partial a'} - \lambda_a = 0 \tag{15}$$

We need to add two extra conditions:

- Complementary Slackness:

  $\lambda_a(a' + A) = 0$

- Dual Feasibility:

  $\lambda_a \geq 0$

The last step consists in using the Envelope Theorem (the derivative of the value function with respect to a state variable is the derivative of the utility with respect to that state variable) to get the Euler

Equation. We need to derive $\frac{\partial V(a,y)}{\partial a}$, but taking into account the policy function g^a since it depends on a. Therefore:

$$V(a,y) = max\ \ u(wy + (1+r)a - g^a(a,y)) + \beta \sum_{y'} \pi_{y'|y} V(g^a(a,y), y') - \lambda_a(g^a(a,y) + A)$$

Taking the FOC:

$$\begin{aligned}\frac{\partial V(a,y)}{\partial a} =& u_c(wy + (1+r)a - g^a(a,y))(1+r) - u_c(wy + a - g^a(a,y))\frac{\partial g^a(a,y)}{\partial a} + \\ &+ \beta \sum_{y'} \pi_{y'|y} \frac{\partial V(g^a(a,y), y')}{\partial g^a(a,y)} \frac{\partial g^a(a,y)}{a} - \lambda_a \frac{\partial g^a(a,y)}{\partial a} = 0\end{aligned}$$

Since the policy function is at the maximum, all derivatives $\frac{\partial g^a(a,y)}{\partial a} = 0$. For this reason, we get that:

$$\frac{\partial V(a,y)}{\partial a} = u_c(c)(1+r) \longrightarrow \frac{\partial V(a',y')}{\partial a'} = u_c(c')(1+r)$$

Then, we can substitute this in expression 16:

$$\frac{\partial V(a,y)}{\partial a'} = u_c(wy + (1+r)a - a')(1+r) + \beta \sum_{y'} \pi_{y'|y} u_c(c') - \lambda_a = 0$$

Rearranging:

$$u_c(wy + (1+r)a - a')(1+r) + \lambda_a = \beta \sum_{y'} \pi_{y'|y} u_c(wy' + (1+r)a' - a'') \tag{16}$$

And this is known as the Euler Equation. Given that we have two specifications of the Euler Equation:

- **Quadratic utility:**

$$\Big(wy + (1+r)a - a' - \bar{c}\Big) = \beta(1+r) \sum_{y'} \pi_{y'|y} \Big(wy' + (1+r)a' - a'' - \bar{c}\Big) \tag{17}$$

- **CRRA:**

$$1 = \beta(1+r) \sum_{y'} \pi_{y'|y} \left(\frac{wy' + (1+r)a' - a''}{wy + (1+r)a - a'}\right)^{-\sigma} \tag{18}$$

2 The infinitely-lived households economy

For $T = \infty$ write a computer program that computes the value function v(a, y) and the decision rules a'(a,y) and c(a,y) for a given choice of the utility function as well as a given parameterization of the income process. Solve this economy using both, discrete and continuous methods to approximate functions v, a' and c. That is, first, discretize state space:

$$A \times Y = \{(a,y) : a \in A\ \ and\ \ y \in Y\}$$

where A=$\{a_1, \ldots, a_n\}$ and Y=$\{y_1, y_2\}$ with $a_1 = -A$ and use value function or euler equation methods. Make sure that you define the grid wide enough, i.e. for the last point in the grid an it should be the case that $a'(a_n, y_2) < a_n$. Second, use a continuous method of your choice, I suggest you to start with piecewise linear splines (there will be potential kinks in the decision rules). Also include in your program a subroutine that simulates paths of consumption and asset holdings for the first 45 periods of an agents life, starting from arbitrary asset position $a_0 \in A\ \ and\ \ y_0 \in Y$.

I have generated the evenly spaced grids and I have done VFI (Value Function Interactions), when agents encounter the natural borrowing limit, we observe that the utility reaches a large negative value. I have chosen random values for the parameters such as:

```
r=0.04              #interest rate
rho=0.06
w=1                 #following the pdf, we normalize the wage to 1
beta=1/(1+rho)      #discount factor
sigma=2             #coefficient of relative risk aversion
periods=100         #number of periods
sigma_y=0.2         #parameter for income shock
gamma=0.5           #correlation between y' and y
```

The code for this exercise and the following one can be seen in "Life-cycle economy and infinitely-lived HH economy". When we generate the return matrix: $Y \times A \times A'$, we have taken into account all possible combinations for current and future assets, given the shock y. If the preferences are quadratic we need 413 iterations of the value function to reach convergence and the time needed is 0.078 seconds, on the other hand, for CRRA preferences the iterations needed are 489 and the time needed 0.11 seconds. The following step consists of creating the vector of policy function, as we did in the previous problem set, first we compute the optimal decision for assets. Once we know this, it is easy to compute the optimal decision for consumption. The results for both types of preferences are displayed in figures 1 [1]and 2:

- **CRRA:**

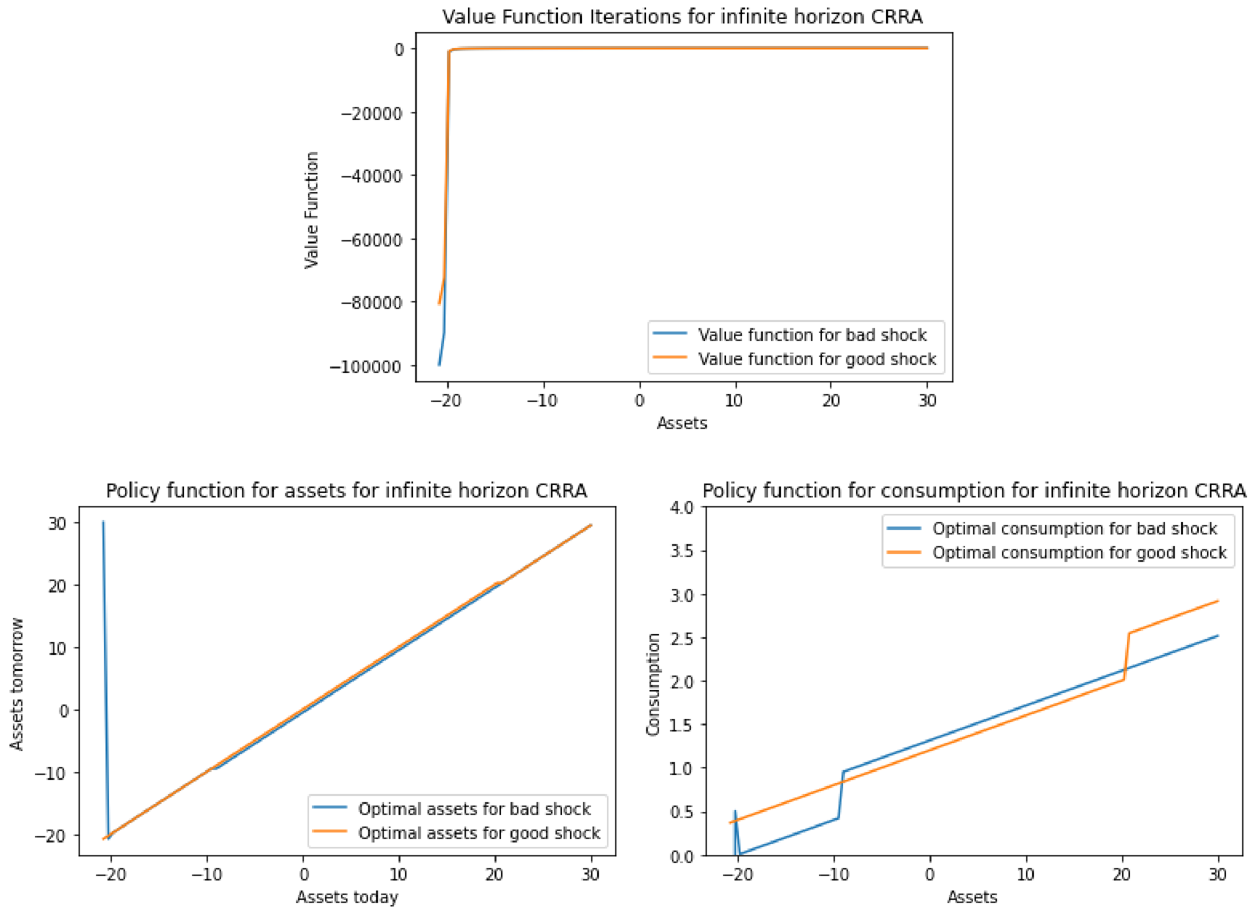


Figure 1: Value function, policy functions for assets and consumption for CRRA utility function and a given parameterization for infinite horizon.

- **QUADRATIC:**

[1]The figures have cross references along the document

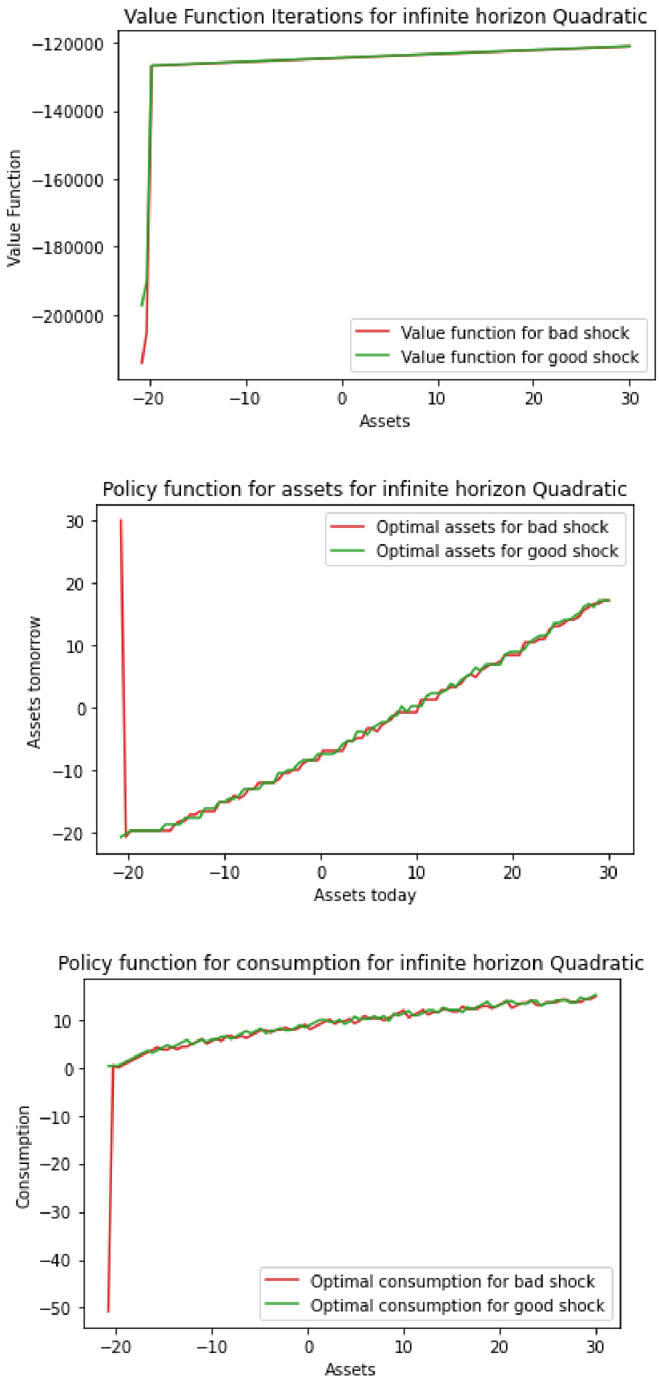


Figure 2: Value function, policy functions for assets and consumption for quadratic utility function and a given parameterization for infinite horizon.

In the previous figures, we observe the effect on the natural borrowing limit in the bottom left corner/top left corner of the figures. As an important fact, is that the policy functions for assets and consumption is higher for good shocks.

3 The life-cycle economy

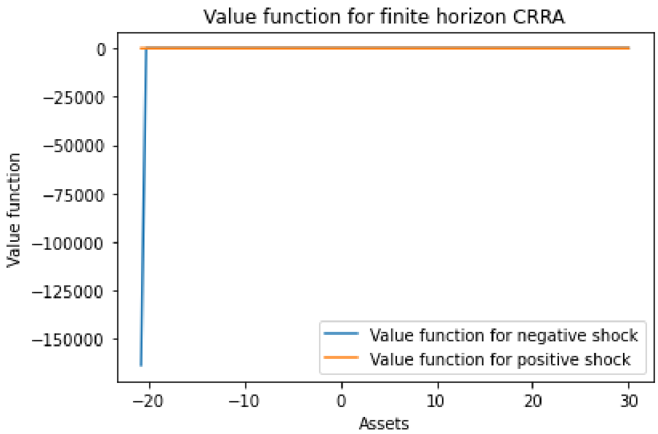
Repeat the exercise for T = 45, where now we aim for a

$$\{v_t(a,y), a_t'(a,y), c_t(a,y)\}_{t=0}^{T=45} \tag{19}$$

Remember that here you can iterate backwards from $v_{t+1}(a,y) = 0$ for all (a,y)$\in A \times Y$.

When we have a finite problem, we need to solve the problem by backward induction (a process of reasoning backwards in time, from the end of a problem or situation, to determine a sequence of optimal actions) as we have seen several times. For that reason, we impose a limit of 45 (since this is the last period), then we set the value function at T+1 to zero, recall that the value function at the final period is the utility function, in this case, given by the matrix M. The results are displayed here (for the first period, each period has its own policies and value function) :

- CRRA:



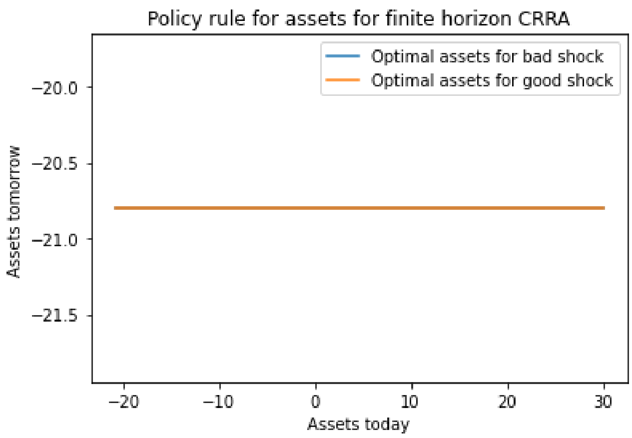


Figure 3: Value function, policy functions for assets and consumption for CRRA utility function and a given parameterization for finite horizon.

- QUADRATIC:

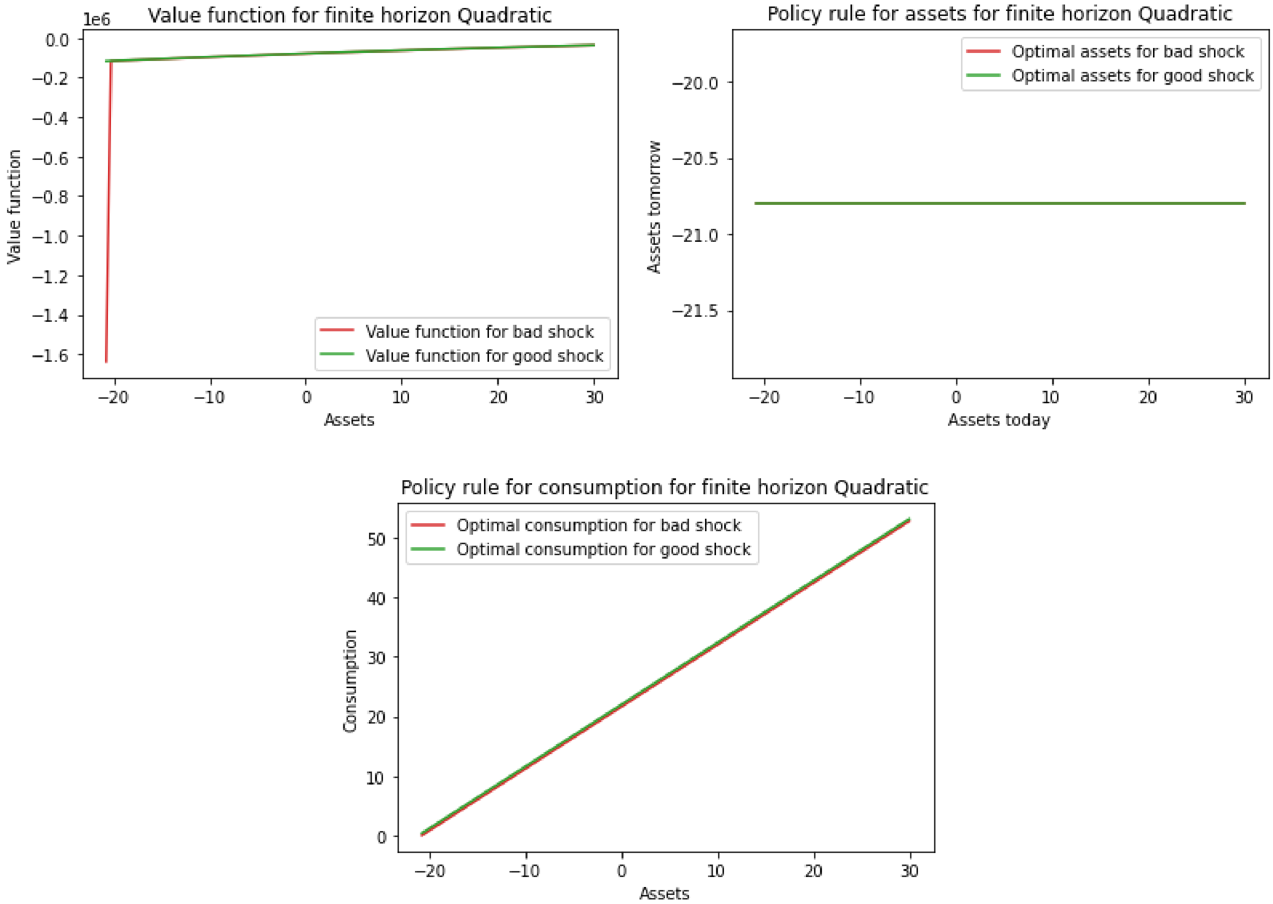


Figure 4: Value function, policy functions for assets and consumption for quadratic utility function and a given parameterization for finite horizon.

For both types of utility the policy function for assets is inelastic, this means that the asset accumulation of the individuals is constant along their lives.

4 Partial equilibrium

Use the code from above to answer the following: Let σ= 2 and $\bar{c}$ = 100, and the borrowing constraint equal to the natural borrowing limit.

The interest rate process is exogenously given in the partial equilibrium. As we saw in class, this implies relaxing two assumptions underlying the martingale hypothesis:

1. Incorporate a precautionary saving motive into the model (relax the assumption of linear marginal utility under uncertainty) so that agents reduce current consumption (increase savings) as a reaction to an increase of uncertainty w.r.t. future labor income.

2. Incorporate some potentially binding borrowing constraint (a liquidity constraint) that may prevent agents to borrow as much as desired to smooth consumption over time.

4.1 With certainty

Fist, let γ = 0 and σ_y = 0, that is, there is no uncertainty.

The consumption function under certainty equivalence states that only the conditional (on t information) fist moment of the y_{t+s}'s matters for the consumption choice, but not the conditional variance of future labor income.

1. For T $= \infty$ the consumption function(s). On the x-axis should be a, on the y-axis c(a, y_1) and c(a, y_2) for both preference specifications. Also generate a time profile of consumption by choosing a_0 as starting assets and by using the policy functions c(a,y) and a'(a,y).
2. Do the same as in the previous question, but now with T = 45. For the consumption function plots pick two ages, say plot $c_5(a, y)$ and $c_{40}(a, y)$.

We have done the same as in the previous exercise, each period has associated a value function and a policy rule for assets and consumption. In the finite horizon, we store each value function of the backwards induction loop. In both types of preferences, the agent starts with a huge amount of assets, or in other words, he/she starts borrowing. For that reason, in order to return his/her debt, the consumption is equal to zero, it is smooth over all periods. I have considered the case where there is a natural borrowing limit but it could be the case where the agents cannot borrow and we would consider the second case (I am not going to do this, the homework will be so long). We observe that the policy for consumption is the same for good and back shocks since there is no uncertainty. As a remarkable fact, we focus on the policy rule for consumption in the finite horizon when we compare the ages of 5 and 40. This hardly any change with quadratic preferences, however, there is a big difference for CRRA preferences, being higher for the age of 5. We need to take into account that agents of different ages would encounter different decisions. The code for this exercise is in “Partial equilibrium with certainty.py”.

- QUADRATIC:

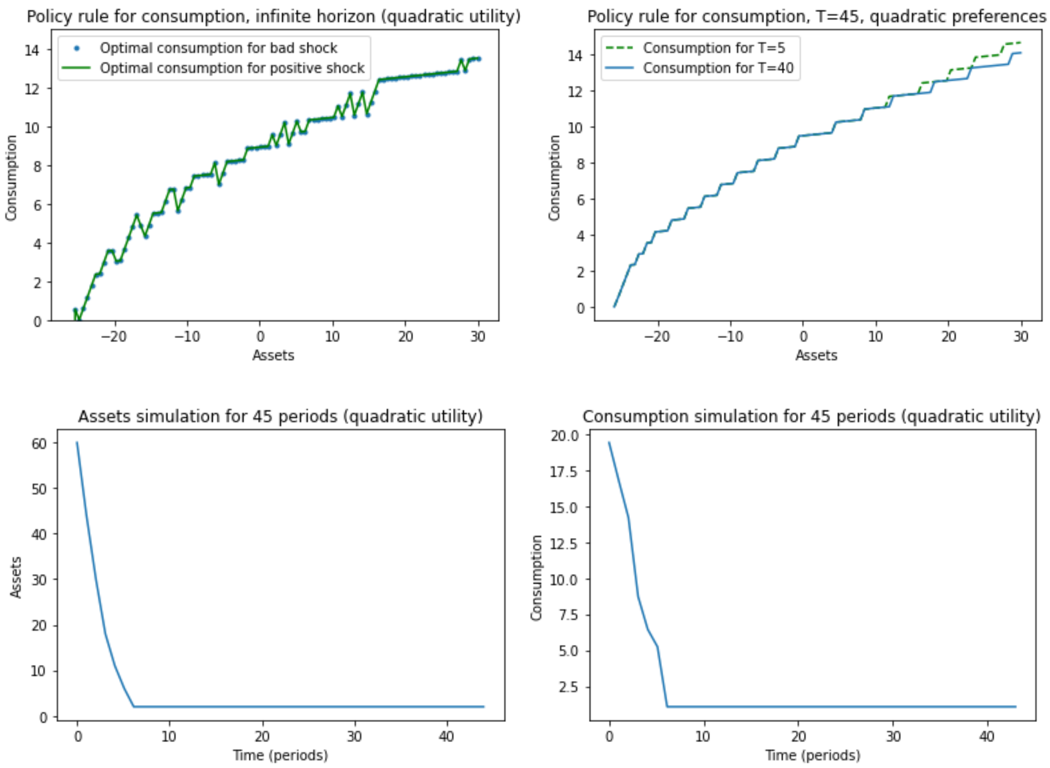


Figure 5: Policy rule for consumption when T=∞, and time profiles for assets and consumption when T=45, with quadratic preferences.

- CRRA:

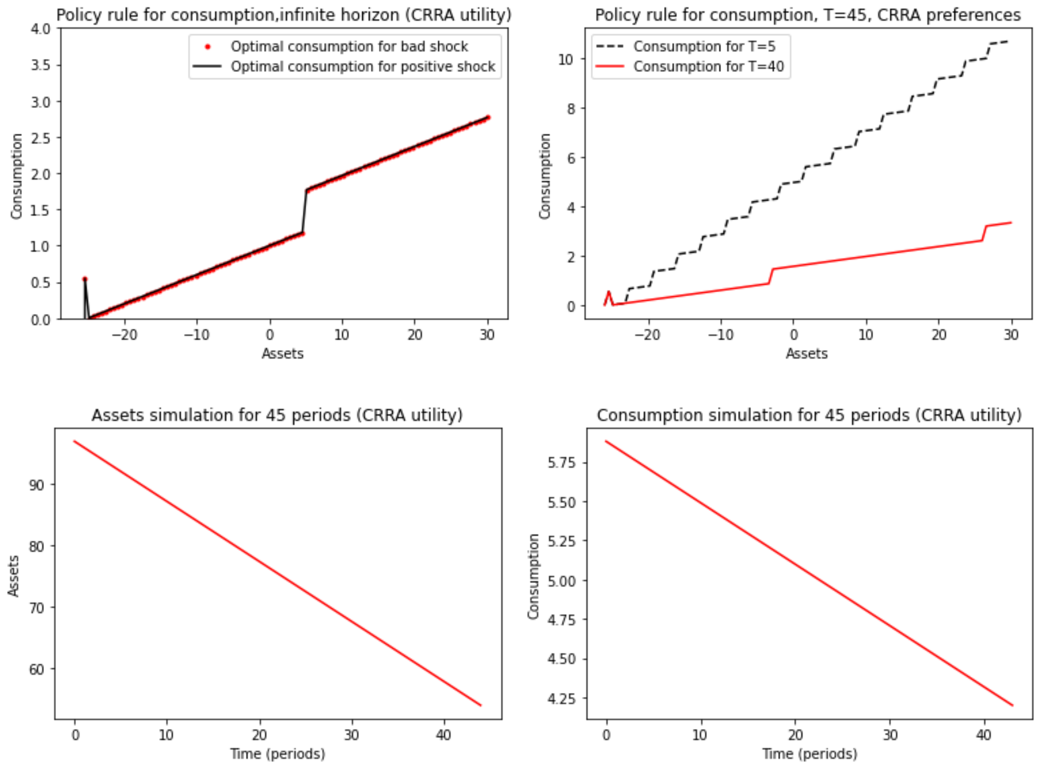


Figure 6: Policy rule for consumption when T=∞, and time profiles for assets and consumption when T=45 with CRRA preferences.

4.2 With uncertainty

Before going to analyze the code and the results, it would be interesting having a general overview of the precautionary savings motive. Kimball 1990 defines “prudence” as the intensity of the precautionary savings motive: the “propensity to prepare and forearm oneself in the face of uncertainty”. We need to take into account two considerations:

- The term prudence defines a property of the utility function that generates precautionary savings, but the term prudence is not precautionary savings. We could generate precautionary savings behavior without a prudence motive.
- Prudence (precautionary saving motive) is controlled by the convexity of the marginal utility while risk aversion by the concavity of the utility function.

Now let $\gamma = 0$ and $\sigma_y = 0.1$.

1. Plot and compare the consumption functions (for each y plot c(a; y) against a) under certainty equivalence (quadratic case) with the consumption function derived in the presence of a precautionary saving motive. Are the differences more pronounced for T = ∞ or T = 45 and why? How do they compare to what you found in the case of certainty.

As an important fact, the CRRA utility introduces a precautionary savings motive that tilts the consumption profile upward in expectation. That is, consumption growth is higher in expectation in the uncertainty case. The code for this exercise is in “Partial equilibrium with uncertainty.py”.

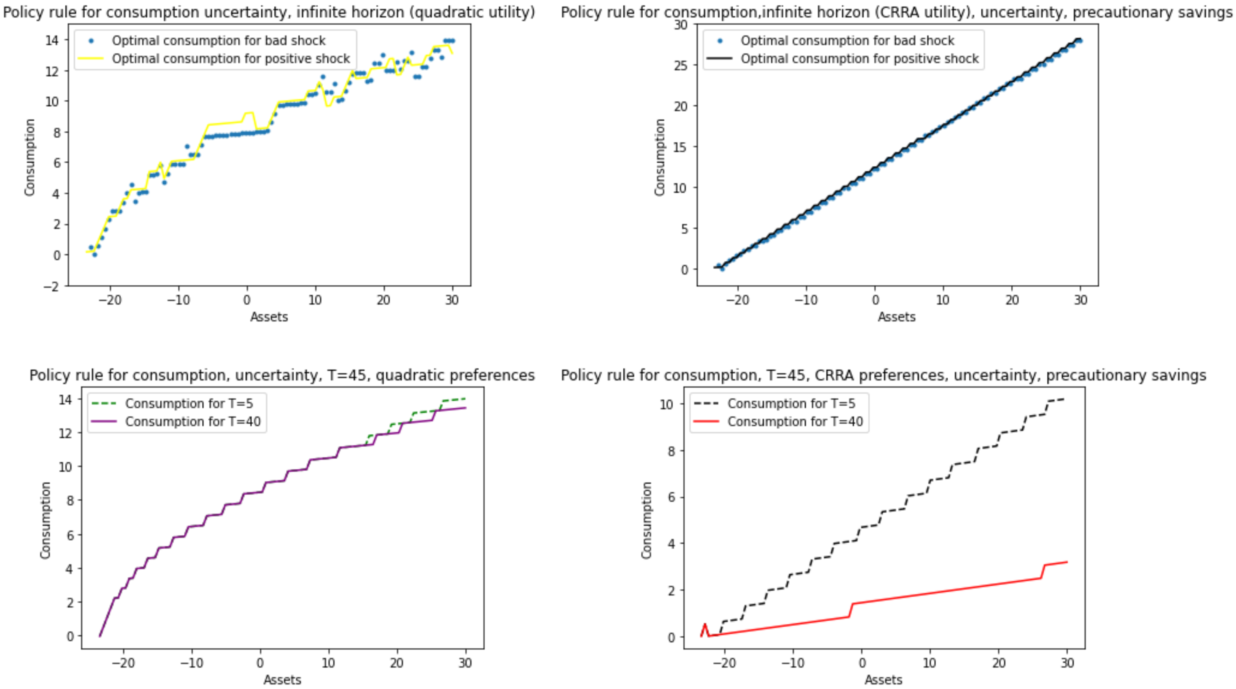


Figure 7: Policy rule for consumption when T=∞, and when T=45 picking two ages.

2. Present and compare representative simulated time paths of consumption for the certainty equivalence and precautionary saving economy. On the x-axis should be time, on the y-axis the income shock and the consumption realization. You may limit yourself to the T = 45 case.

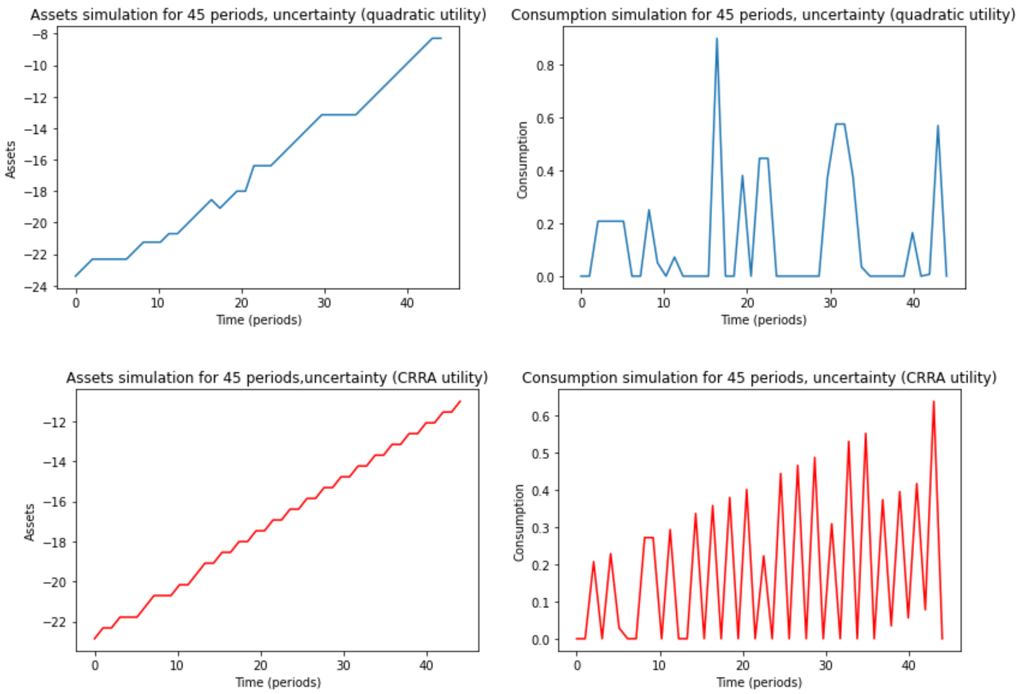


Figure 8: Simulated time paths of consumption and assets when T=45.

3. Increase prudence by increasing σ = 2 to σ= 5 and σ= 20. How much do your answers change and why?

Once the previous questions are done, I only need to adjust the code ('Partial equilibrium with uncertainty.py".) to specific parameters for this and the following questions, and compare them. The size of precautionary savings is determined by the parameter σ controlling prudence for the CRRA utility function.

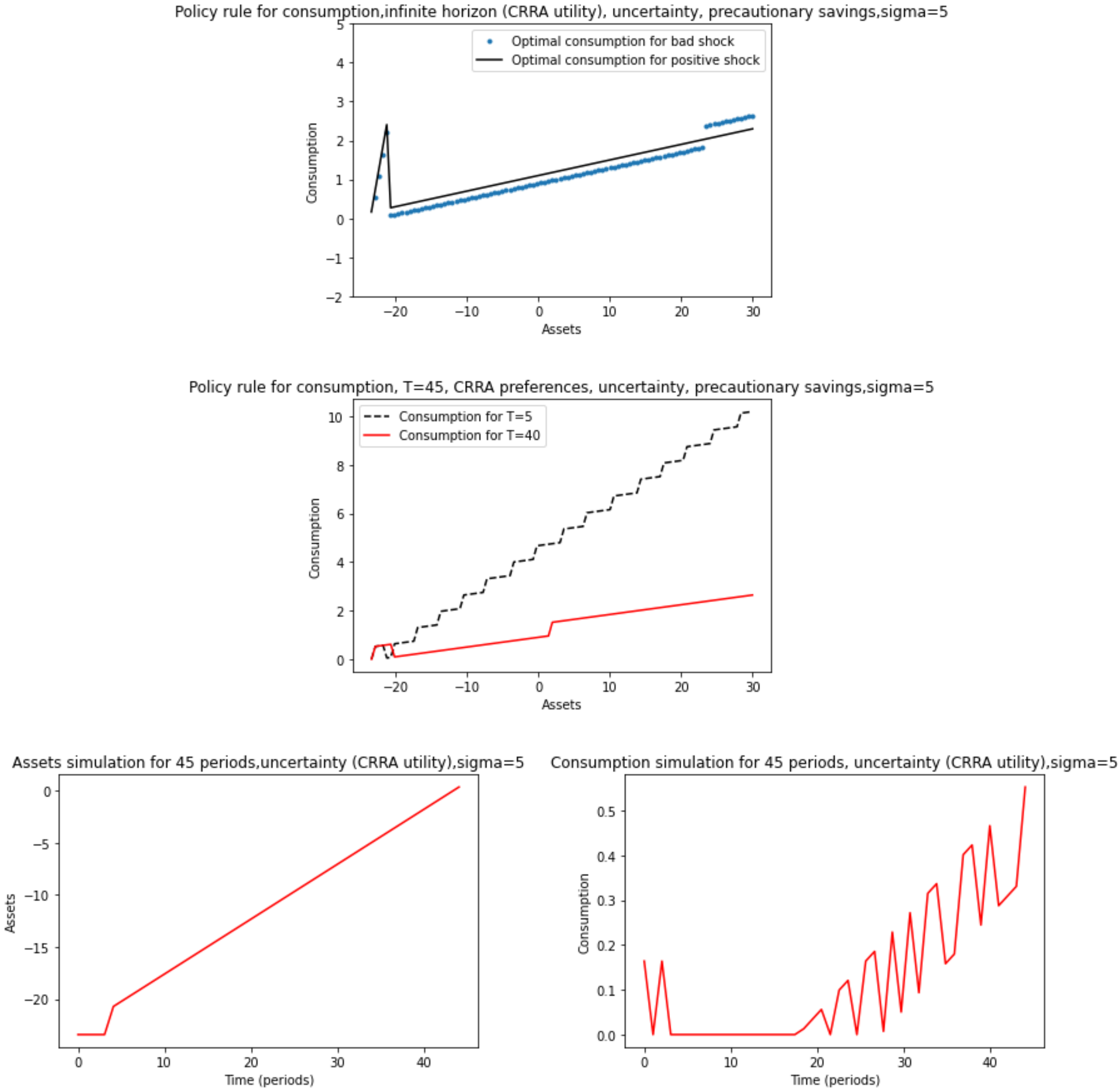


Figure 9: Exercise 1 and 2 repeated for $\sigma = 5$ to the precautionary saving economy (CRRA).

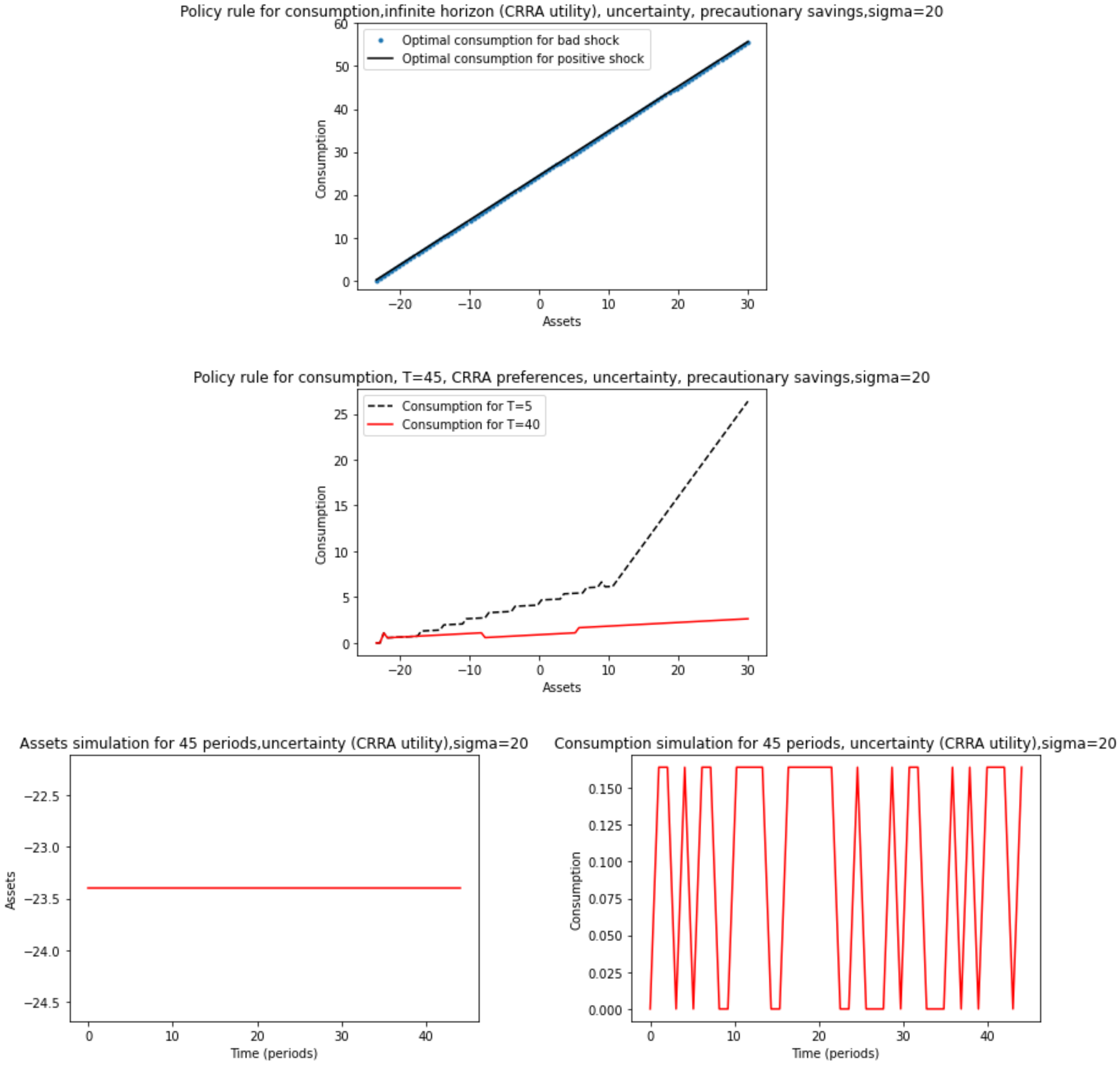


Figure 10: Exercise 1 and 2 repeated for $\sigma = 20$ to the precautionary saving economy (CRRA).

4. Increase the variance of the income shock from $\sigma_y = 0.1$ to $\sigma_y = 0.5$. What happens to the consumption function in the certainty equivalence case (you should know the theoretical answer to that question). Also plot the new consumption functions for the precautionary savings case. Are the differences between certainty equivalence and precautionary savings consumption functions bigger or smaller now? Explain. Support your explanations with simulated time paths of consumption. Again limit yourself to T = 45. How much do your answers change and why?

When the variance of future income increases, the low realizations of income are more likely. Then, saving increases in reaction to increases in uncertainty about future income, because agents, afraid of future contingencies of low consumption smooth low-income shocks via borrowing, increase their precautionary savings. If we increase the variance, there is more volatility in the economy due to the increase of the idiosyncratic risk in the economy.

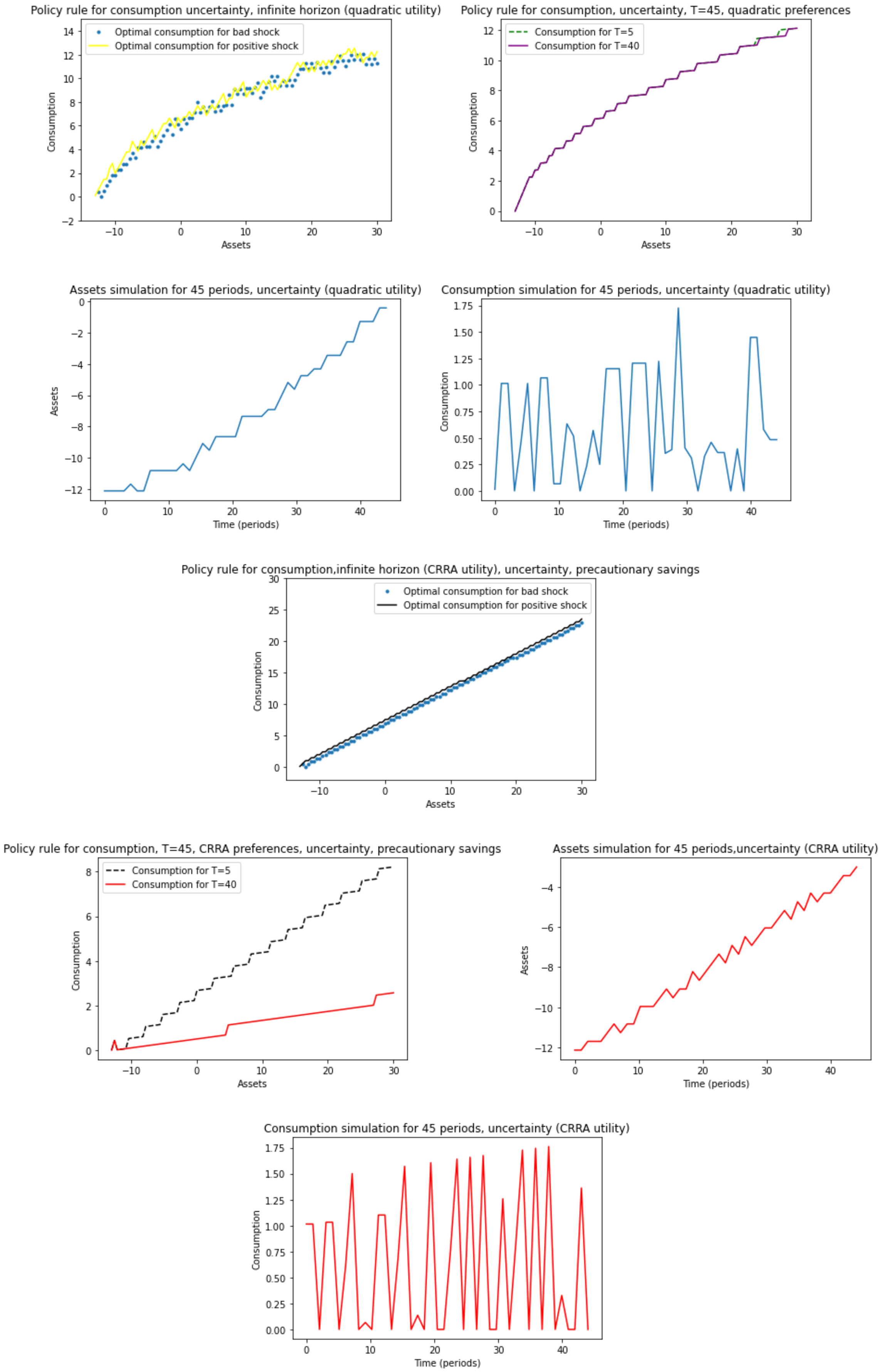
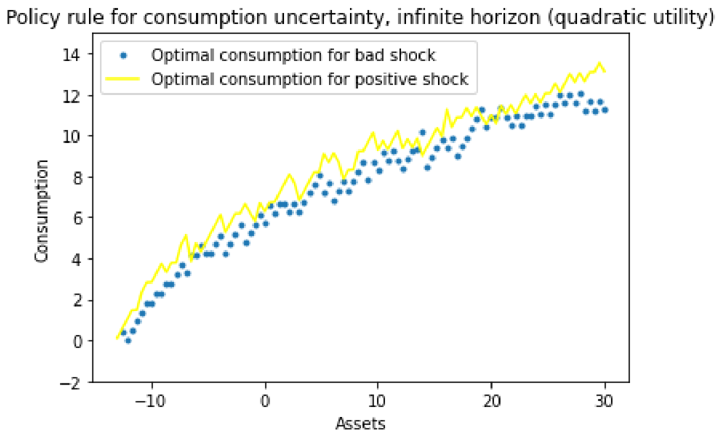
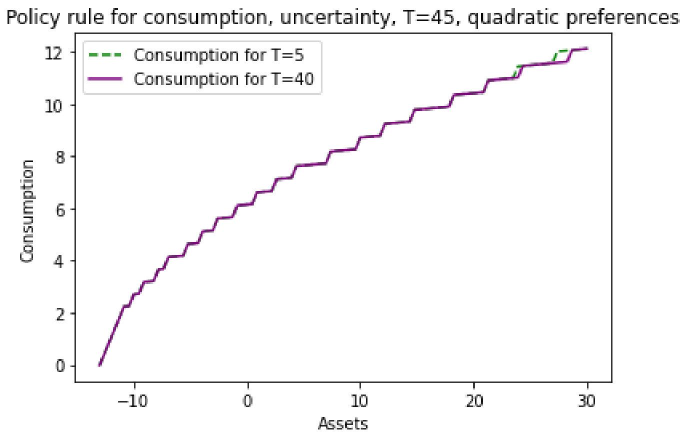


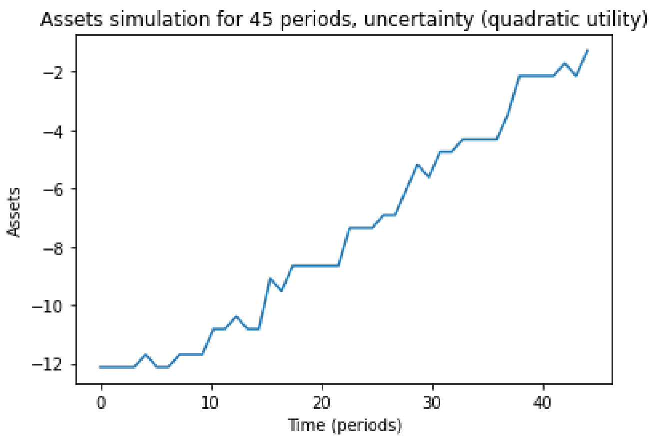
Figure 11: Exercise 1 and 2 repeated for $\sigma_y = 0.5$ and $\sigma = 2$).

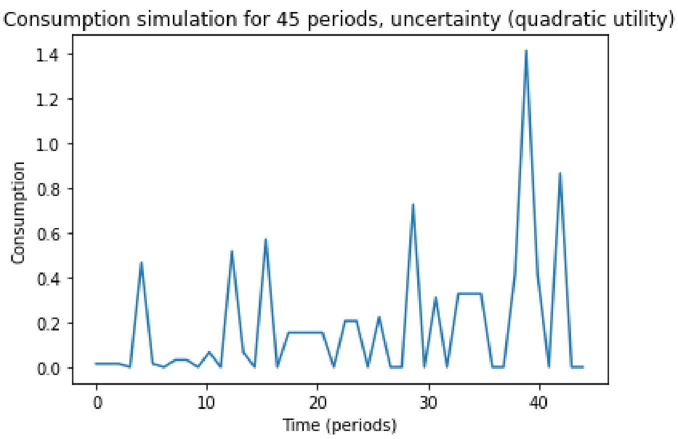
5. Increase the persistence of the income shocks from $\gamma = 0$ to γ =0.95 (keep $\sigma_y = 0.5$ as well as all other parameters constant). How much do your answers change and why?.

In this case, we observe some important facts commented before. The positive shock always leads to higher consumption and asset accumulation with respect to the bad shock. Now, volatility is not as extreme as in the previous exercise due to the persistence (if there is a bad shock today is more likely to have a bad shock tomorrow) but as we have studied in class, due to the precautionary savings reason, agents tend to borrow when there are bad shocks in the economy and save in the opposite case.









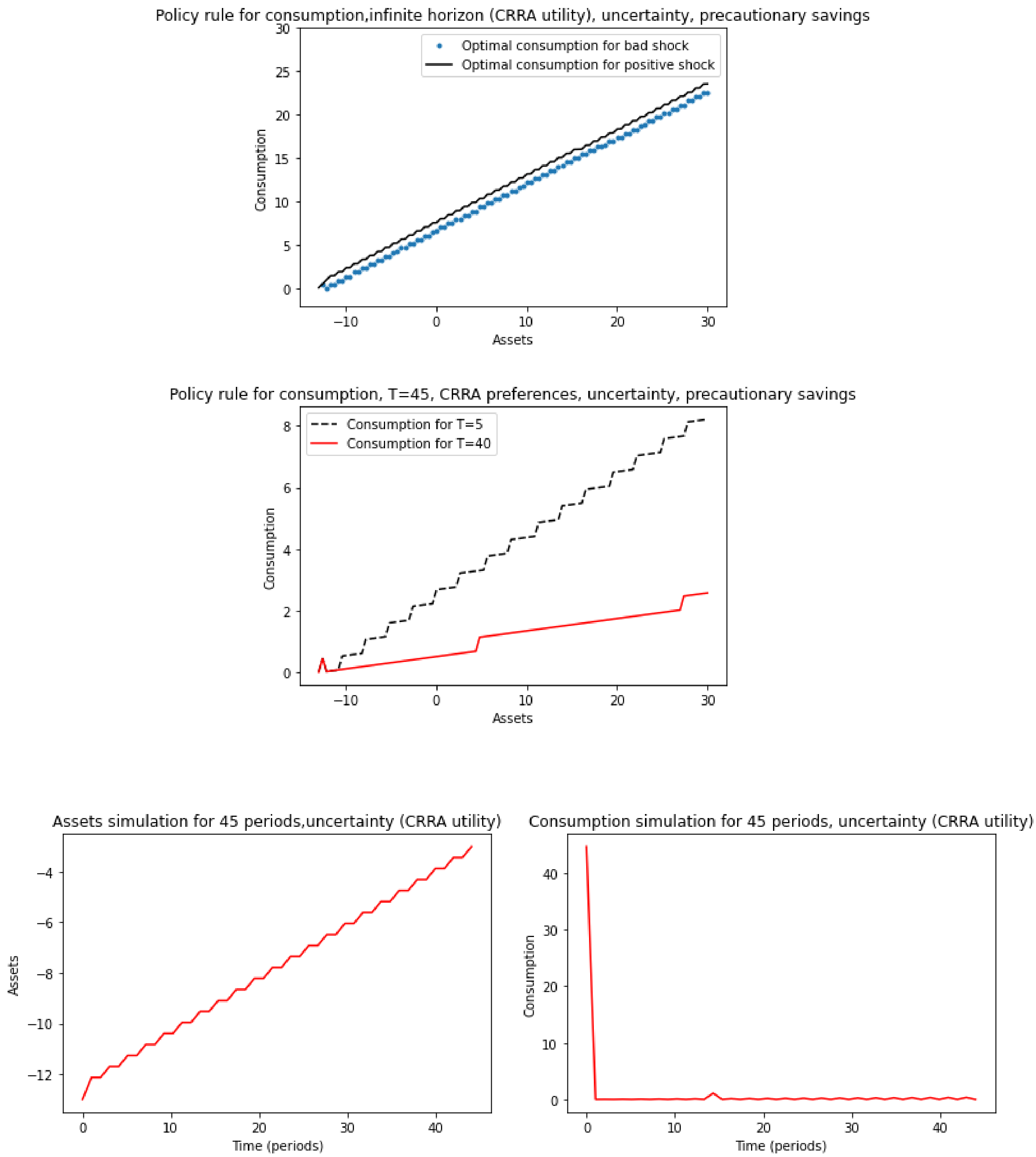


Figure 12: Exercise 1 and 2 repeated for $\gamma = 0.95$, $\sigma_y = 0.5$ and $\sigma = 2$).

5 General equilibrium

5.1 The simple ABHI model

Report the endogenous distribution of consumption, income and wealth and compare them to the corresponding data distributions reported in the Handbook Chapter by Krueger, Mitman and Perri. Compare also the joint distribution of consumption and wealth in the model and the data.

The General Equilibrium implies that now the agents are still price-takers but prices (w_t, r_t) are determined endogenously within the model. The prices will be determined at the systemic level, depending on the aggregate quantity of capital, and therefore, on the asset distribution. The modification concerning the previous exercise is introducing the production environment. The production function of the economy is a Cobb-Douglas, then, it has the following form:

$$Y = AK^{\alpha}N^{1-\alpha}$$

The maximization problem of the firm can be written as:

$$max\ \ \pi = max\ \ AK^{\alpha}N^{1-\alpha} - (r+\delta)K - wN$$

where firms choose labor and capital to maximize the present discounted value of profits. Recall that with constant returns to scale and perfect competition, the number of firms is indeterminate, and without loss of generality we can assert the existence of a single representative firm. Then, the FOC's of the firm are:

$$\frac{\partial \pi}{\partial K} = A\alpha K^{\alpha-1}N^{1-\alpha} - (r+\delta) = 0 \tag{20}$$

$$\frac{\partial \pi}{\partial L} = A(1-\alpha)K^{\alpha}N^{-\alpha} - w = 0 \tag{21}$$

If I isolate the interest rate of expression 20, I get the inverse demand function:

$$r = A\alpha\left(\frac{N}{K}\right)^{1-\alpha} - \delta \tag{22}$$

I do some arrangements that are going to be useful in the FOC for labor:

$$\left(\frac{K}{N}\right)^{1-\alpha} = \frac{A\alpha}{r+\delta}$$

$$\left(\frac{K}{N}\right) = \left(\frac{A\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}$$

Then, I can substitute this in the expression 21 and I get:

$$w = A(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

$$w = A(1-\alpha)\left(\frac{A\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} \tag{23}$$

In this simple version of the model, households supply labor inelastically because they do not value leisure. As I have explained in the code, this has been imported from the website "https://python.quantecon.org" but I have modified it to adapt this to our problem. The code for this exercise is in "General Equilibrium.py". We observe that our result is near to Krueger et al. (2015) since they reported that the Gini Index is equal to 0.77 while in our case is equal to 0.7035, therefore, this is a society very unequal. In chapter 11 of the Handbook of Macroeconomic, they state the β=0.9899 and σ=1 but in our code σ=5 and β=0.98, if I state $\rho = 0.06$ and $\sigma = 2$ (as in some part of the previous exercise) we don‘t observe the equilibrium in the graph. One of the key mechanisms to change the stationary distribution is the transition matrix and the standard deviation of income shocks. In other words, how unequal the income is and how intense is the social mobility (between states). We observe that the top 10% shares 40% of the wealth, while the middle class the 52%. The general equilibrium provides a theory of wealth inequality, that is, it provides a theoretical framework potentially able to account for stylized facts of empirical wealth distributions.

```
The GINI coefficient is 0.7035
The 80/20 ratio is equal to 359.8189
The aggregate wealth is equal to 1.00
The average assets is equal to 0.0100
The standard deviation of assets is 0.0147
The top 1% shares 0.06
The top 10% shares 0.40
The bottom 50% shares 0.02
The middle class shares 0.52
```

The equilibrium interest rate is such that the asset market clears (the condition requires equality between the demand for capital by firms and the supply of capital by households). The code consists in a computation algorithm for solving the general equilibrium: first, guess an interest rate, use the FOC for the firm to determine K(r) and w(r), solve the household problem for given r and w(r), compute the demand and the supply of capital until both are equal. We observe that the equilibrium interest rate is close to 2.5%:

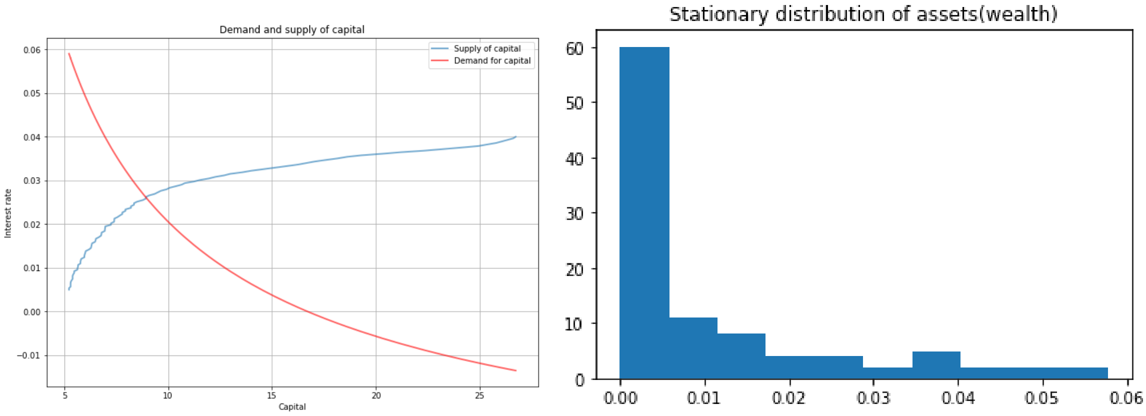


Figure 13: Demand and offer for capital and the stationary distribution of assets.

5.2 Solve Aiyagari (1994)

Use your programs to reproduce selected results on the size of precautionary savings from Aiyagari's (1994) Table 2. Note that this exercise uses the same code as the one you programmed for the previous section (with infinite horizon) except for adding more realistic features (e.g., a Markov chain of more than 2 state income shocks) and using some of Aiygari's choices for the parameters in utility functions, borrowing limits, and so on. Again, report the endogenous distribution of consumption, income and wealth and compare them to the corresponding data distributions reported in the Handbook Chapter by Krueger, Mitman and Perri. Compare also the joint distribution of consumption and wealth in the model and the data.

Aiyagari considered an economy where all the agents were ex-ante identical, but due to a random income process were ex-post heterogeneous. Additionally, he considered agents that were borrowing constrained so that they would be incentivized to increase savings for precautionary reasons. In this case, I adopt the specifications of Aiyagari (1994), the utility function has the form of a constant return relative risk aversion, the risk aversion parameter can take three different values (1, 3, 5), in the code, I have chosen the value of 3. Moreover, the discount factor is equal to 0.96, the capital share is equal to 0.36, while the depreciation rate is equal to 0.08. To conclude, nobody can borrow in this economy, therefore, the natural borrowing limit is equal to zero. One important difference is that I cannot introduce negative shocks.

The Gini coefficient is far away from the value reported by Krueger et al. (2015) since it is 0.5047 in this case. The same can be said about the wealth shares, according to Piketty's, the share of the top 10% in total wealth is much higher (0.72) whereas the middle-class share is much slower (0.2795). In our case is 0.26 and 0.56, respectively.

```
The GINI coefficient is 0.5047
The 80/20 ratio is equal to 9.2483
The aggregate wealth is equal to 1.00
The average assets is equal to 0.0100
The standard deviation of assets is 0.0094
The top 1% shares 0.04
The top 10% shares 0.26
The bottom 50% shares 0.13
The middle class shares 0.56
```

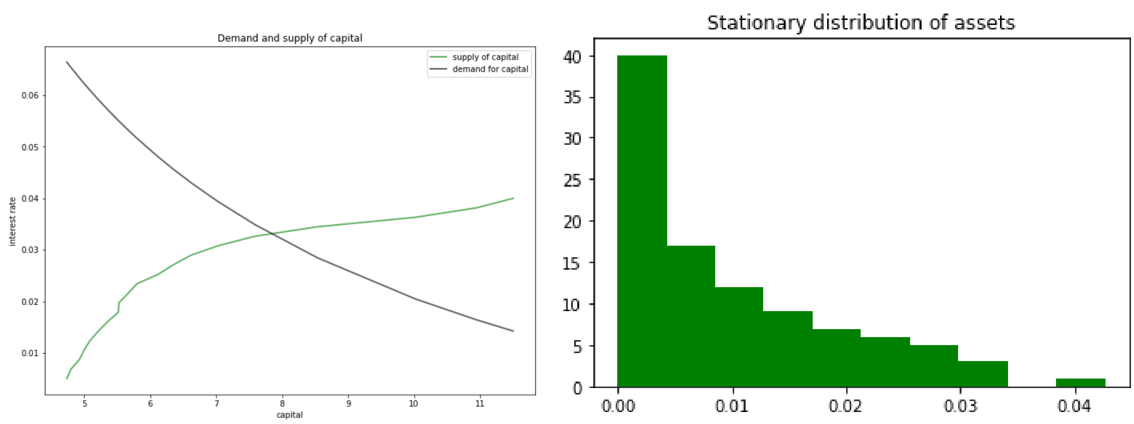


Figure 14: Demand and offer for capital and the stationary distribution of assets

To conclude with this question, the interest rate in equilibrium is close to 3% but a bit higher. I cannot extract the income and the consumption distribution, but we can state a general idea behind it.

Given the asset distribution, people would prefer consumption today instead of consuming tomorrow, therefore, they accumulate fewer assets over time. Moreover, in the second graph, we observe that most of the people are poor in stationary equilibrium, the frequency of people observed in the graph is decreasing when assets increase.

3 References

References

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, *109*(3), 659–684.

Krueger, D., Mitman, K., & Perri, F. (2015). Macroeconomics and heterogeneity, including inequality. *Handbook of Macroeconomics*.