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Hyper-edges and multidimensional centrality☆

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Abstract

Many social transactions are supra-dyadic; they either involve more than two participants (buyer, seller, broker) or they involve important aspects of the interaction's setting, tike its timing or its location. Standard network techniques do not adequately plumb these networks. Using the concept of a hypergraph [Berge, C. 1973, Graphs and Hypergraphs], this paper shows how the concept of network centrality can be adapted to supra-dyadic networks. Use of the technique is illustrated with data on attacks by inhabitants of Caribbean islands on Spanish settlements in the period 1509–1700.

MSC: C00 General Mathematical and Quantitative Methods

Keywords: Centrality; Hypergraphs

1. Introduction

Networks typically involve dyadic relations between two actors. However, there are occasions in which relationships have more than two principals. Some non-dyadic actions, for example, may require the coordinated actions of three or more actors rather than two: a buyer, a seller, and an agent, for example[1]. Other principals can be introduced when actions are fruitfully best described with the addition of characteristics other than the identities of the two actors. For example, interactions may occur in certain places or at certain times. These additional characteristics can be as essential to the description of the act as the identities of the actors.

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[1] There can be a distortion if such non-dyadic relations are resolved into a set of dyads. For example, if A is a buyer, B a seller, and C an agent, A and B, B and C, and A and C can all have transacted without having transacted together as one triad.

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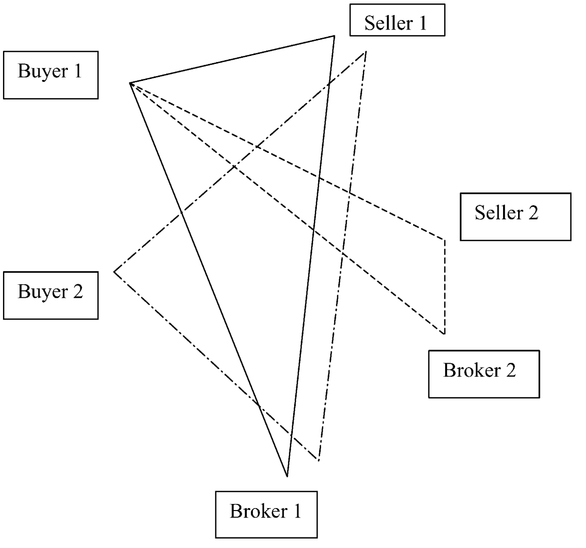

Fig. 1. Three buyer–seller–broker transactions.

Throughout this paper we will be considering the following simple examples. In Fig. 1 there are three interactions each involving a buyer, a seller, and a broker. The three interactions are indicated by three triangles. In Fig. 2 there are four individuals at three time periods. In the Fig. 3 there are eight individuals who can meet in three locations.

In Fig. 1 we note that Buyer 1, Seller 1, and Broker 1 are each involved in two of the three transactions, but that Buyer 1's importance might be diluted by his involvement with two of the three less involved actors. What we notice in Fig. 2 is that actor *A* is the most central in time 2, whereas in all the other time periods all actors are equally central. Thus, period 2 as well as actor *A* are distinguished. In Fig. 3 we notice that positions *C* and *E*

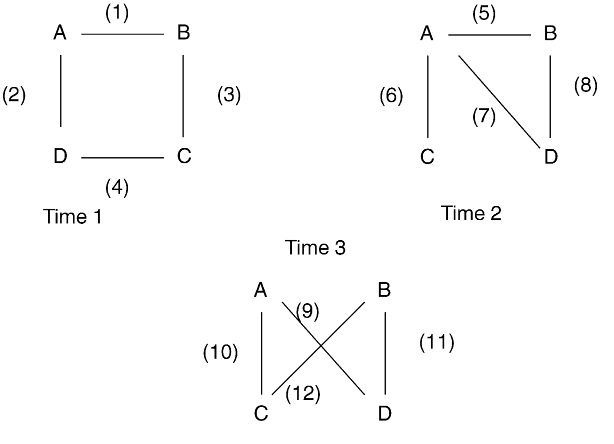

Fig. 2. Four actors at three points in time.

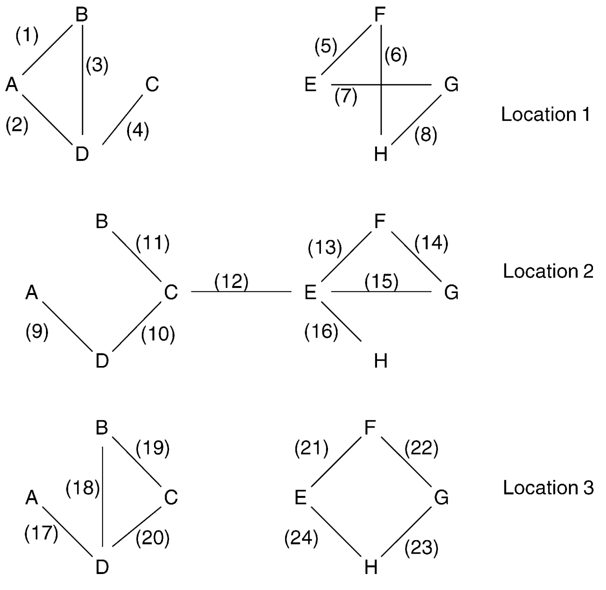

Fig. 3. Eight actors at three locations.

are bridges between two otherwise unconnected groups only in location 2. Location 2 is where positions *C* and *E* achieve their status as bridges. Location 2 might, for example, be a restaurant where powerful individuals from different domains meet. Lunch at Spago is different from lunch at Denny's. This distinguishes location 2. We will return to these two examples later.

With regard to these latter two examples, simultaneity and proximity of interactions can matter. If important people dine at a restaurant all other diners, and their transactions, gain in prominence. The restaurant becomes a medium for transmitting this prominence. Similarly, simultaneous events can generate a prominence that can be transmitted. When protest groups stage nation-wide simultaneous demonstrations, events in big cities like New York and Los Angeles contribute indirectly to the prominence of events in smaller cities.[2]

Measurement of the centrality of a set of unrelated individuals, who are not part of a status-transmitting group, is a different problem. We may wish to characterize the centrality of these sets (Everett and Borgatt, 2004) without believing that these sets transmit status. One might want to compare the centralities of automobile-owning students in a high school classroom to that of other students. Suppose, for example, that group *A* consists of individuals 1, 2, 5, and 6, and group *B* of individuals 3 and 4 in Fig. 4. Clearly, by any definition individuals 3 and 4 are most central. We may wish to create an overall measure of the relative centralities of the two groups.

[2] Papers by Feld (1981) and Pattison and Robins (2002) describe approaches for examining the effects of the contexts within which networks exist on the creation of new ties. Taking a Homansian approach, Feld hypothesizes that common activities or locations (*foci*) encourage ties. Pattison and Robins suggest that different *settings* in a network limit the statistical dependence between ties.

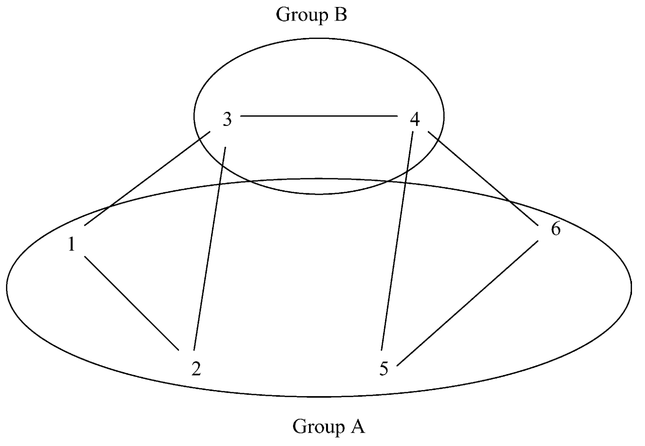

Fig. 4. Six vertices and two groups.

In these situations more than two principals are involved in each relationship: there are more than two actors (buyer, seller, broker); time is important; place is important; group membership is important. This paper is devoted to extending one measure of centrality, eigenvector centrality (Bonacich, 1972; Bonacich, 1987; Bonacich and Lloyd, 2001) to this type of situation. There will be four steps in presenting this extension. First, eigenvector centrality for adjacency matrices will be reviewed. Second, existing work on eigenvector measures of centrality for two-mode data will be described. Third, two-mode eigenvector centrality measures will be applied to incidence matrices of edges and vertices to assign centrality scores to relationships as well as actors. Finally, eigenvector centrality will be used with incidence matrices of hypergraphs whose hyperedges can contain any number of vertices.

1.1. *Eigenvector centrality for adjacency matrices, a review*

Centrality refers to the importance of a position within a network. The specific type of importance varies from measure to measure. A position can be important because information passes through it (*betweenness* centrality (Freeman, 1979)), or because it can easily communicate with other members of the network (*closeness* centrality (Freeman, 1979)), or because it is itself connected to other central positions. In this paper we will be concerned with generalizing the latter form of centrality, eigenvector centrality.

In many circumstances the value of a position in a network is a function of the values of the positions to which it is connected. For example, the status of a high school student may be a function of the status of the students with whom he is a friend. The information possessed by a member of an organization will be related to the information possessed by those with whom he communicates.

Let A be an adjacency matrix representing the existence of relationships among a set of n actors: $a_{ij} = 1$ indicates the presence of a relationship between i and j while $a_{ij} = 0$ indicates the absence of a relationship. Let x be a vector of centrality scores. In one conception of centrality everyone's centrality is supposed to be a linear combination of the centralities of those to whom they are connected.[3]

$$\lambda x_i = \sum_{j=1}^{n} a_{ij} x_j \tag{1}$$

This has the following expression in terms of matrices:

$$Ax = \lambda x \tag{2}$$

1.2. Eigenvector centrality for two-mode data, a review

In two-mode data the network is bipartite, there are two disjoint sets of vertices and all relations are between members of the two sets (Borgatti and Everett, 1997). Examples would be dating relationships between men and women, or memberships of individuals (one mode) in groups (the other mode). Two-mode network data can be represented by an adjacency matrix in which the rows represent one set and the columns the other.

One can calculate simultaneous eigenvector centrality scores for the row and column elements of two-mode data (Bonacich, 1991). For example, the rows of the matrix could represent individuals and the columns the events they may attend. Individuals acquire their centrality by attending important events; important events are attended by central individuals. Let x and y be the centrality scores for the row and column elements. We can then define the following pair of equations for x and y.

$$A^t x = \lambda y, \qquad Ay = \lambda x \tag{3}$$

The vectors x and y are eigenvectors of different matrices, although both are associated with the same eigenvalue λ^2.

$$AA^t x = \lambda^2 x, \qquad A^t A y = \lambda^2 y \tag{4}$$

1.3. Centrality for incidence matrices

In *incidence matrix* is an alternative to an adjacency matrix as a representation of a graph. Each edge is represented by a row and each vertex by a column. The two ones in each row show the vertices connected by each edge. For example, the incidence matrix for the graph in Fig. 4 is

$$E = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \tag{5}$$

[3] Lambda is the largest eigenvalue of the matrix A. It guarantees a solution to the equation in which all centrality scores are positive.

An incidence matrix can be thought of as two-mode data, where the modes are the edges and vertices of the original graph. Thought of in this way, one can compute simultaneous centrality scores using the same eigenvalue for edges and vertices using the eigenvector approach for two-mode data ((3) and (4)). The centralities of the vertices in Fig. 4 are 1, 1, 2, 2, 1, and 1, while the centralities of the edges are 1, 1.5, 1.5, 2, 1.5, 1.5, and 1. Notice that the most central edge (4) connects the most central vertices (3 and 4). A two-mode analysis of incidence matrices produces not only the usual centrality scores for vertices but, as an added bonus, scores that enable us to identity important edges.[4]

1.4. Centrality for hypergraphs and hyperedges

A *hypergraph* (Berge, 1973), with *hyperedges*, can be used to describe these more-than-dyadic situations involving either three or more actors or two actors plus some characteristics of the situation, like time or place. An edge of an ordinary graph is an ordered pair (when the relationship is not symmetric) or an unordered pair (if the relation is symmetric). A hyperedge is an ordered or unordered set of two or more vertices. A hyperedge exists apart from its constituent edges. For example, *A* and *B*, *B* and *C*, and *A* and *C* can exist as sub-parts of three separate triadic relations without themselves forming a triadic relation, represented by a hyper-edge. *A* and *B* can have lunch together, *A* can have had lunch at restaurant *C*, *B* can have had lunch at restaurant *C*, without *A* and *B* having had lunch at restaurant *C*. Yet, it may be the coincidence of these three events that may be important to us.

Incidence matrices can be used to represent hypergraphs. Whereas in the incidence matrix for an ordinary graph each row will have exactly two ones, the rows of the incidence matrix for a hypergraph, representing the hyperedges, may have more than two ones. As an illustration, the incidence matrix for the network in Fig. 1 is given in (6). The first two columns are the buyers, the next two are the sellers, and the last two are the brokers.

$$E = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \tag{6}$$

Exactly the same manipulations that were used to produce centrality scores for both the rows (edges) and vertices (columns) of an incidence matrix can be used to produce scores for hyperedges and multiple principals in a network. Seller 1 and Broker 2 turn out to be the most central in this analysis, as we expected; the most central transaction was the first, involving the three most active participants.

The incidence matrix for the network in Fig. 2 is given in (7). Three sets of four rows represent the three time periods. The first four columns represent the four actors, while the remaining three columns are for the three time periods.

[4] Unfortunately, we now have two different ways of calculating the centralities of vertices. We can use (1) for the adjacency matrix or (3) using the two mode incidence matrix *E*. Our experience and some mathematical analysis shows that the two measures do not differ significantly.

Table 1
Centrality scores for edges and vertices of the network in Fig. 2

	Centrality
Positions (E^tE)	
A	1
B	0.799
C	0.634
D	0.829
Times (E^tE)	
1	0.533
2	0.563
3	0.533
Edges (EE^t)	
1	1.000
2	1.000
3	0.835
4	0.846
5	0.999
6	0.929
7	1.011
8	0.929
9	0.999
10	0.976
11	0.916
12	0.833

$$E = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{7}$$

For the network in Fig. 2 and (7) the largest eigenvalue of EE^t and E^tE is 16.22.[5] The eigenvectors of E^tE and EE^t are given in Table 1.

As expected, position A is the most and position C is the least central. Time 2, in which the centrality of A is accomplished, is also the most central. The most central hyperedge, 7, connects the two most central positions in the most central period.

[5] If E is an m by n matrix (n vertices and m edges) EE^t and E^tE will share minimum(n,m) eigenvalues.

Table 2
Centrality scores for edges and vertices of the network in Fig. 3

	Centrality
Position	
A	1.000
B	1.262
C	1.706
D	2.223
E	2.417
F	1.710
G	1.645
H	1.234
Location	
1	2.132
2	2.323
3	2.188
Edge	
1	1
2	1.219
3	1.278
4	1.379
5	1.424
6	1.176
7	1.410
8	1.161
9	1.262
10	1.423
11	1.204
12	1.467
13	1.468
14	1.292
15	1.453
16	1.380
17	1.231
18	1.291
19	1.173
20	1.392
21	1.437
22	1.261
23	1.174
24	1.349

The complete set of centrality scores for the network of Fig. 3 is given in Table 2.

Location 2, where the two groups are connected, is the most central, as expected. The hyperedge connecting them, 12, is the second most central, and the positions serving as bridges between the two groups, *C* and *E*, are among the most central.

Finally, consider the issue of comparing the centrality scores of sets of vertices in a graph. Consider again Fig. 4, where we want to compare the centralities of set $A = \{1, 2, 5, 6\}$ and set $B = \{3, 4\}$. The approach we are going to suggest is to add two new hyperedges corresponding to the two sets. However, these edges are measurements of the data, not data themselves; the centralities of vertices and hyperedges should not be affected by the measurement process. For example, adding a 3–4 measurement hyperedge would reduce the centrality of the existing 3–4 data edge. These measurement hyperedges must be treated differently. They must be responsive to the centrality patterns existing in the data but not affect them.

The solution is to make these measurement hyperedges have vanishing small values. E^* is a modification of E in (5) where which two edges for the two sets have been added.

$$E^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ \Delta & \Delta & 0 & 0 & \Delta & \Delta \\ 0 & 0 & \Delta & \Delta & 0 & 0 \end{pmatrix} \tag{8}$$

As Δ approaches zero the eigenvalues of E^*E^{*t} approach those of EE^t (plus some additional values of zero). Letting x and y be the eigenvectors of EE^t and E^tE associated with their largest eigenvalue, the following is true in the limit.

$$E^*x = \lambda y \tag{9}$$

Let A and B be the groups whose centralities we wish to compare. Then, using (9) for small values of Δ:

$$\Delta\sum_{i\epsilon A} x_i = \lambda y_A, \qquad \Delta\sum_{i\epsilon B} x_i = \lambda y_B \tag{10}$$

The result is that the centrality of each group is proportional to the sum of the centralities of its members. For Fig. 4 the two groups are equal in total centrality, but the average centrality per member is twice as high for group B as for group A. This measure of group centrality, the sum of the centralities of its members, is not surprising or subtle. What is novel is its justification in terms of hypergraphs and hyperedges.

2. The Caraïbe

Columbus's first voyage to the West Indies told the European world of "the Caraïbe".[6] Early European explorers and chroniclers identified the "Caraïbe Islands" as encompassing the Lesser Antilles, especially the Windward Islands, what is now considered to be the Eastern Caribbean (see Fig. 5). Variations of the name "Caraïbe" referred to a vaguely defined people in the southern islands. Modern history and anthropology have persisted in

[6] All data come from Holdren (1998).

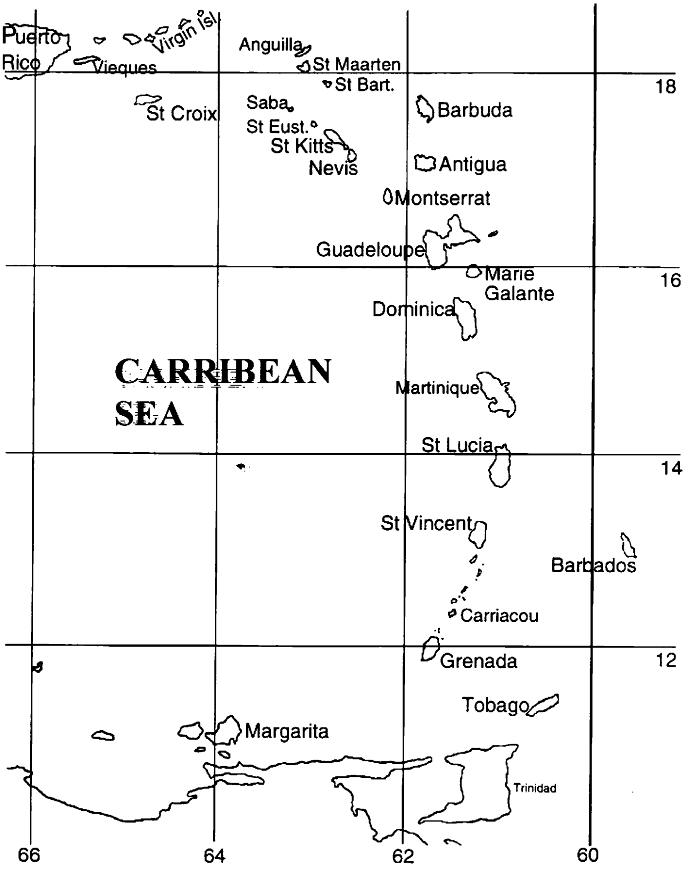

Fig. 5. Map of the Carribe region.

analyzing the Caraïbe as a single unit, as the conquerors of island Arawaks, as the terrorists of early colonies, as the original practitioners of guerrilla warfare.

In point of fact, however, the bloc of people known historically as the “Caraïbe” likely did not stem from a single family tree but instead were composed of multiple, varyingly related, politically autonomous groups before first contact with Columbus. The Caraïbe were a cluster of smaller-scale, reticulately-related ethnic groups that shared a habitus, the practical mastery of an underlying, common code (see Bourdieu, 1977, p. 78), that was maintained through social exchange networks. The Caraïbe polity was non-hierarchical. Alliance systems provided the foundation for political institutions. Local groups were connected through marriages and other exchanges, and networks of local groups maintained complimentary relationships through raiding and trading expeditions. Caraïbe leaders developed their influence through prowess in warfare and trade.

European conquest attempts were systemized when Spain declared royal control over the island of Hispaniola in 1502, with establishment of a system of *encomienda* grants that effectively enslaved the indigenous Taino population for work in the fields and gold mines. The system rapidly exhausted the gold and extinguished the indigenous people of Hispaniola. To procure more gold, and more labor, the Spanish crown colonized Puerto Rico (1508), Jamaica (and 1509), and Cuba (1511). In early 1511, the indigenous people of Puerto Rico revolted "with the help of more warlike terms who had entered the island room St. Croix" (Rogozinski 1992, p. 29). The raids, together with depleted gold mines, encouraged most colonists to leave Puerto Rico by 1518. At the other end of the island chain, Trinidad often served as a source of slaves, often procured by Caraïbes (Whitehead, 1995). The indigenous people began to resist further Spanish encroachment, however, and Spanish attempts to colonize Trinidad in 1532–1534 and 1569 failed. Successful colonization did not occur until 1592, but since Trinidad did not lie on a Spanish shipping route, few Spaniards lived there.

The first Northern European colonies were established in the 1620s by individual adventurers, often acting for groups of merchants who invested in the establishment of tobacco plantations. They settled uninhabited islands of the Lesser Antilles: Barbados, St. Kitts, Nevis, Montserrat, Martinique, and Guadeloupe. By the middle of the seventeenth century, white populations in these islands reached their peak. In large part, the population peak was due to economic changes associated with a change in agricultural production. Before 1650, most European colonists grew tobacco with the assistance of indentured servants on small farms, but, by 1639, the European markets were glutted with the leaf. In the 1640s, the large landowners began raising sugar cane on slave plantations, and small holders began to migrate out to less developed islands, such as Antigua, Dominica, St. Vincent, and Grenada.

When confronted by the warlike, conquest-minded Europeans, the formerly autonomous ethnic groups known as the Caraïbe developed a larger sense of identity. Integrating mechanisms for the network included long-distance raiding and trading expeditions carried out jointly. Further inter-regional and collective action reinforced the social solidarity. Alliances threaded through the reticulately-related groups, and these threads pulled tighter as the need for larger scale collective action grew. European conquest attempts were catalysts that led the multiple, heterogeneous native groups to confederate, thereby transforming themselves into the multi-ethnic polity known historically as "Caraïbe".

The increased intensity of English and French colonization attempts in the Lesser Antilles prompted a 25-year war with the "Caraïbe". In 1660, both sides signed a treaty, which gave the Caraïbe retention of two islands, Dominica and St. Vincent, in return for their renunciation of all the others. The islands remained relatively undeveloped until after the 1763 Treaty of Paris, in which Britain gained ownership of St. Vincent and Dominica, Granada and the Grenadines, and Tobago.

3. Data and analysis

The data describe 56 attacks on European settlements by the Caraïbe between the years 1509 and 1700. These attacks involved Caraïbe from twenty-two islands and occurred in 29 different years. The data were analyzed in the form of a 56 by 51 matrix E, where the rows referred to events, the first 22 columns to islands, and the last 29 columns to years. These data are doubly supra-dyadic: attacks could involve more than two islands; and each attach involved a year. Eigenvectors of EE^t and E^tE corresponding to the largest eigenvalue of these two matrices were extracted to give centrality scores for events, islands, and years.

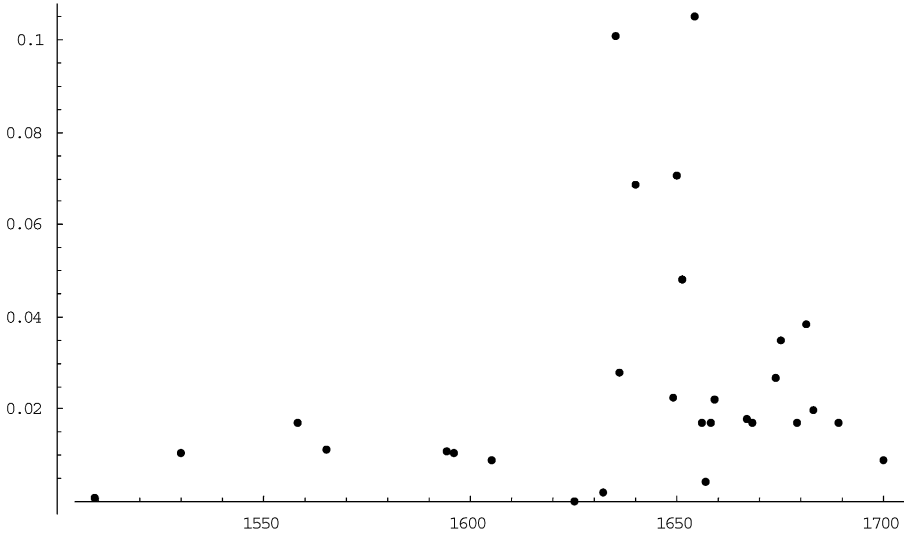

Fig. 6. Centrality scores for years.

3.1. *Centrality scores for islands*

The 22 islands were highly unequally involved in the attacks. Dominica and St. Vincent were involved in 39 and 38 of the attacks, while the next most active islands, Grenada and Martinique, were involved in fourteen and thirteen attacks, respectively. The islands involved in more attacks were simply the ones that were colonized at a later time. Dominica, St. Vincent, and Granada were the last Caraïbe "strongholds". They were the islands to which the other Amerindians fled as their own islands were threatened and then colonized by the Europeans.

3.2. *Centrality scores for years*

Centrality scores for years show an interesting pattern, as revealed in Fig. 6.

Year centrality increases dramatically just before the end of the period under examination (Fig. 7). However, without further examination it is unclear what this signifies. How can we interpret this finding? What aspect of the developing struggle between the Caraïbe and the Spanish does this reflect? Does it mean that there was any kind of growing integration or leadership among the islands?

3.3. *Interpreting effects*

The general question addressed here is how to interpret variations in the centrality of the non-actor principals, be they years or restaurants or some other important aspect that defines interaction in the network. In the following description we will assume that it is time. To

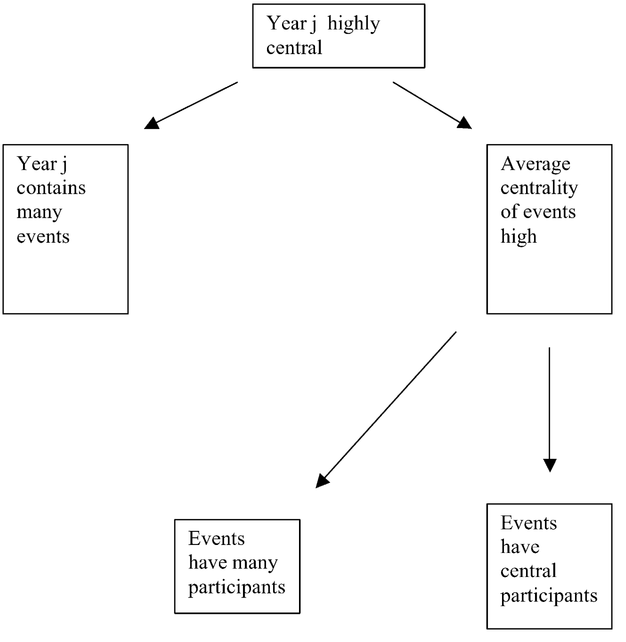

Fig. 7. Interpreting high centrality scores for years.

interpret these centralities we will introduce some additional symbols and equations. As before, E is the incidence matrix for a hypergraph in which the first set of columns describe actors and the second set describe time (or place). This implies a partition of E into two sub-matrices, A for the actor columns and T for the time columns.

$$E = (A|T) \tag{11}$$

The vectors x and y are centrality scores for the rows (hyperedges) and columns (actors and time) of E. The vector y is further divided into two sub-vectors, y_A and y_T for actors and time, respectively.

$$y = \begin{pmatrix} y_A \\ y_T \end{pmatrix} \tag{12}$$

Expanding (3),

$$Ey = (A|T)\begin{pmatrix} y_A \\ y_T \end{pmatrix} = Ay_A + Ty_T = \lambda x \tag{13}$$

$$E^t x = \begin{pmatrix} A^t \\ T^t \end{pmatrix} x = \lambda \begin{pmatrix} y_A \\ y_T \end{pmatrix} \tag{14}$$

Eq. (14) can be rewritten as two equations:

$$A^t x = \lambda y_A \tag{15}$$

$$T^t x = \lambda y_T \tag{16}$$

These equations can be used to express the centrality of years as a function of the centralities of the actors involved in events in those years.

$$\begin{aligned} y_T &= \frac{1}{\lambda}T^t x = \frac{1}{\lambda^2}T^t(Ay_A + Ty_T) = \frac{1}{\lambda^2}T^t Ay_A + \frac{1}{\lambda^2}T^t Ty_T \\ &= \frac{1}{\lambda^2}\left(I - \frac{1}{\lambda^2}T^t T\right)^{-1} T^t Ay_A \end{aligned} \tag{17}$$

Now, suppose we want to explain why the centralities in y_T differ, as they do for the Caraïbe data. Looking at (16), we see that the centrality of a year is a function of the centralities of the hyperedges (attack events) to which it is connected. A given year can be central because it contained many events or because the events it contained were particularly central. The first possibility is assessed by the sum of the columns of T, the second by the average centrality of the events of that year.If the centrality of a year is not explained by the sheer frequency of events but by the centrality of its events, we can use (17) to see whether the centrality of these events is explained (in part) by the fact that they had many participants or whether they had particularly central participants. Eq. (17) shows that the centrality of each year is (with a little distortion[7]) the sum of the centralities of the actors who are active in events in that year. All of this can be summarized by the tree in Figure 7.

This suggests that a year can be central for three reasons: it contains many events, the events it contained had many participants, the events it contained had particularly central and active participants. In the Caraïbe context, this would offer three different accounts of why the years around 1650 were particularly central: these years had more attacks; the attacks in these years had more participants; the attacks in these years had particularly active participants.

These three outcomes are ordered in their degree of sociological complexity. The simplest outcome is one in which more attacks occurred in later years. Increasing organization or coordination among the island communities would not be required. The next level of complexity would be if later attacks involved more islands. This would indicate an increasing degree of coordination among the island communities. Finally, increasing centrality of actors in later years could imply a growing leadership system in the islands.

The average centrality for the years 1640, 1650, 1651, and 1652 was 0.0733, much higher than the average centrality of 0.0268 for all years. The average centrality for the attacks occurring in that year, however, was no higher than the average centrality of all attacks. What was distinctive about those years was they had by far the greatest number of attacks. This alone accounts for the elevated centralities of those years. The attacks in those years did not involve more islands nor did they involve especially active islands. This means that the growing cohesiveness of the Caraïbe in their battle with the Spanish was of a particularly limited type.

[7] The distortion is introduced by the term $(I-1/\lambda^2 T\text{'}T)^{-1}$, which is not equal to I.

4. Conclusions

In this paper eigenvector measures of centrality are adapted to networks consisting of more-than-dyadic interaction. The setting of an action, its location in space and time, can play as meaningful a role as any human actor. Lohmann (1994) points out the key role that the Monday demonstrations at the Karl–Marx–Platz in Leipzig, Germany acquired in bringing down the East German government. The status of the demonstrations persisted long after the government had changed and other groups tried to borrow its status to add legitimacy to their causes. Even after they had lost all political significance the Monday demonstrations remained as a tourist attraction.

This paper describes methods for the assessment of centrality when settings can acquire and transmit centrality. The measures are modifications of the eigenvector of centrality in which settings are incorporated through hypergraphs. The result is a measure of centrality for settings as well as an improved measure of centrality for actors. Moreover, an approach is suggested for parceling out the centrality of settings by determining the extent to which they owe their centrality to the quantity or quality of their participants.

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