ALGORITHM FOR ENUMERATING HYPERGRAPH TRANSVERSALS

by

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A THESIS

Presented to the College of Arts and Sciences: Computer and Information Science
and the Graduate School of the University of Oregon
in partial fulfillment of the requirements
for the degree of
Masters of Computer Information Science

June 2017

THESIS APPROVAL PAGE

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Title: Algorithm for Enumerating Hypergraph Transversals

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Degree awarded June 2017

THESIS ABSTRACT

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Masters of Computer Information Science

Computer and Information Science

June 2017

Title: Algorithm for Enumerating Hypergraph Transversals

This paper introduces the hypergraph traversal problem along with the known solutions: Naive, Branch and bound, and polynomial-space iteratrive. version of the algorithm. Lists instead of sets are used to define Odometers and Hypergraphs. All familiar set operations on Odometers are redefined as list operations for succinctness. Lastly future research directions are examined.

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ACKNOWLEDGEMENTS

For my parents, my wife, my kids, and all future generations.

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CHAPTER I

INTRODUCTION

"The theory of hypergraphs is seen to be a very useful tool for the solution of integer optimization problems when the matrix has certain special properties" - Berge [1984]

Hypergraphs are a recent mathematical and computational discovery. The number of edges in a *normal* hypergraph is potentially 2^N. Abstracting from a *normal* to a *unrestricted* hypergraph allows the edges to unbounded N^∞. An *unrestricted* hypergraph can still be reasoned about. Odometers Fuchs [2016] are defined and the functions that operate on them. Then hyperedges are shown to be mutually interchangeable with odometers given a hypergraph and transformation functions.

The NP-Complete problem enumeration of all minimal traversals of a hypergraph is defined Eiter [1991]. There are multiple corresponding NP-Complete problems that are shown to be equivalent Eiter and Gottlob [1995]. Finding all minimal transversals is a significant and worthy problem in computation and especially in AI Eiter and Gottlob [2002].

There are objectively better minimal transversals such as the total count of vertexes in the traversal Bailey et al. [2003]. Traversing a hypergraph is the first step to efficient traversal Boros et al. [2003]. Optimal traversals of a hypergraph are fundamentally similar to optimization of a property and NP-Hard. It is possible to at least generate efficient traversals that generate .

CHAPTER II

DEFINITIONS

Lists, Queues, Stacks, N-Trees, etc...

This paper assumes the reader is familiar with the basic data structures such as lists, arrays, stacks, queues, trees, graphs etc. Let a list l be an ordered list of things $\{t, i\}$ where each thing t must be the same type, and t is only distinguishable by its index i. For the purposes of this paper, $x.push(y)$ will insert y at the last index of the list, $x.pop()$ will remove from the last index of the list. $x.enque(y)$ will insert y at the last index of the list. $x.dequeue()$ will remove from the first index of the list. A list-of-lists structure is used in this paper to represent the traversal of an N way branching tree. The internal C++ implementation is an array representation with $O(1)$ index lookup time, $O(N)$ seek, insert, and remove times.

Odometers

An odometer is an ordered multiset of integer numbers. Let an odometer o be an list of integers n and indexes $\{n, i\}$. The ith indexable integer of an odometer can be written $n_i = o[i]$. Integers n can be repeated, they are distinguished via their index. Indexes i are unique non-repeating whole numbers from $[0, \infty]$. The size of the odometer is written as $o.size()$, is the count of $\{n, i\}$.

Instead of reasons about hyperedges, odometers and used in their place. In the next section is it shown they are mutually exchangeable. Bit vectors are commonly used instead for set operations, but in this case the number of vertexes is usually far larger then system bit sizes (32,64 etc). When using full integers

for items in the list there are N^M values are available, where N is the count of integers, and M is the number of values that integers can take on. Thus $O(N^{(2^{bits})})$ is the general complexity without restrictions. Fuchs [2016]

An odometer is a construct used extensively throughout this paper as it can be treated as an ordered set of numbers, an unordered bag of numbers, as an instance container to store state. As an unrestricted list of numbers the odometer is similar to an instance of a turing machine tape.

The following common functions are defined in Code Appendix C: $Union$, $Intersection$, $Minus$, $StrictEqual$, $SetEqual$. The following functions are implemented, additionally they are short circuit versions when possible: $DoesACoverB$, $DoesAHitB$, $DoesACoverBorBCoverA$, $DoesAHitAll$, $DoesAnyHitA$. Please note that all functions are polynomial in both space and time. Also note that $DoesAHitAll$ implements the hitting set test. $GenerateNMinusOne(o)$ is used for both minimal hitting set and other functions.

Hypergraphs

Unrestricted Hypergraphs

The traditional hypergraph definition $H = (V, E)$ is terse for implementers. Traditionally a hypergraph is defined as a collection of sets where there is no ordering and repeated elements are not allowed. The following definitions were used to implement the hypergraph interface. The odometer is of particular interest as it can be used independently from hypergraphs for linear integer optimization techniques.

Let a hyperedge e be a list of vertexes: $e = \{v, i\}$. The ith indexable vertex of e can be written $v_i = e[i]$. Vertexes v can be repeated, they are distinguished

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via their index. Indexes i are unique non-repeating whole numbers from $[0, \infty]$. The size of the hyperedge written as $e.size()$ is the count of $\{v, i\}$.

Let an unrestricted hypergraph U be a single hyperedge *nodes* and the two functions *OtoE* and *EtoO*. *OtoE* is the surjective function to map a given odometer to a hyperedge. *EtoO* is the injective function to map a given hyperedge to an odometer. The hyperedge *U.nodes* cannot repeat any vertexes v for the function *EtoO* to behave correctly.

Given these definitions, the following is now possible given a hypergraph: A hyperedge can be constructed from an odometer. An odometer can be constructed from a hypergraph. While the functions in the paper use hyperedges the code uses odometers in place of hyperedges. Thus every instance of a hyperedges can be converted to an instance of an odometer, and every instance of an odometer can be converted to an instance of a hyperedge.

Specifically the odometer is an instance of a set of integer numbers that can be reasoned about independently of a hypergraph. The code implements some common set functions that allow the constraints to be expressed, such as union, minus, include short circuit versions of functions for faster performance.

Algorithm 1 OdometerToHyperedge

1: **function** OTOE(U, o)
2:     $e \leftarrow \emptyset$
3:     $size \leftarrow U.nodes.size()$
4:     **for all** $\{n, i\} \in o$ **do**
5:         // where -2 mod 7 = 5 .
6:         $e[i] \leftarrow U.nodes[n\%size]$ // convert an integer number to index.
7:     **return** e

Notice that these functions provide polynomial time access to all permutations, combinations, repeats, patterns etc. Thus reasoning about a

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Algorithm 2 HyperedgeToOdometer

1: **function** EToO(U, e)
2:     $o \leftarrow \emptyset$
3:     **for all** $\{v_e, i_e\} \in e$ **do** // use hashmap of v to i
4:         **for all** $\{v_n, i_n\} \in U.nodes$ **do** // to reduce $O(n^2)$ to $O(n)$
5:             **if** $v_e = v_n$ **then**
6:                 $o[i_e] \leftarrow i_n$ // lookup index and save as integer.
7:     **return** o
---

hyperedge is equivalent to reasoning about its corresponding odometer, and vice

versa. Vertex data can be complex and large, thus reasoning about the odometer in

place of the hyperedge is for performance and interesting reasons noted later.

Normal Hypergraphs **?**

Let a *normal* hypergraph be $H = (V, E)$ where V is a list of vertexes v, i, E is a list of hyperedges e, i where each hyperedge e is a subset of V. The following trivial restrictions must be imposed to get the expected behavior out of a *normal* hypergraph given the unrestricted list definitions. No hyperedge contains a duplicated vertex. Every vertex in all hyperedges is contained in the hypergraph list of vertexes. There are no duplicate hyperedges. The maximal size of a hyperedge is the size of all hypergraph vertexes. Every vertex exists in at least one hyperedge. There are no duplicate vertexes in the hypergraph.

$$\forall e \in E, \forall v, v' \in e | v \neq v'$$

$$\forall e \in E, \forall v \in e | v \in V$$

$$\forall e \in E, \nexists e' \in E | e = e'$$

$$\forall e \in E | |e| \leq |V|$$

$$\forall v \in V, \exists e \in E | v \in e$$

$$\forall v \in V, \nexists v' \in V | v = v'$$

Simple Hypergraphs

Let a *simple* hypergraph be $H = (V, E)$ as *normal* hypergraph with the additional restriction that no hyperedge fully contains any other hyperedge.

$$\forall e, e' \in E | e \not\subseteq e' \wedge e' \not\subseteq e$$

Minimal Transversal of a Hypergraph

Let the transversal of a hypergraph $T \subseteq H.V$ be a hitting set of all the hyperedges of a hypergraph such that $DoesAHitAll(T, H.E) = true$. Using the definitions of $GenerateNMinusOne$ the following implementation determines if an odometer hits ever odometer in a list.

Algorithm 3 IsMinimalTransversal

1: **function** IsMinimalTransversal($o, list\_of\_o$)
2:     **if** $DoesAHitAll(o, list\_of\_o) = false$ **then**
3:         **return** $false$
4:     **for all** $\{o_n, i_n\} \in GenerateNMinusOne(o)$ **do**
5:         **if** $DoesAHitAll(o_n, list\_of\_o)$ **then**
6:             **return** $false$
7:     **return** $true$

All minimal transversals of a hypergraph

There are $2^{|V|}$ possible combination sets that can be derived from the hypergraph $H = (V, E)$ and therefore $2^{|V|}$ transversals that need to be enumerated in the worst case. Tractable scalable algorithms fundamentally need to use polynomial space storage and exponential time to enumerate the traversals efficiently.

CHAPTER III

NAIVE SOLUTION

Algorithm 4 NaiveAllPotentialTransversals

1: **function** $NaiveAllPotentialTransversals(H, CallbackFunc)$
2:     $count \leftarrow H.E.size()$
3:     $o \leftarrow \emptyset$
4:     $o.push(0)$
5:     **while** $o.size() > 0$ **do**
6:         **if** $IsMinimalTransversal(o, H.E)$ **then**
7:             $CallbackFunc(o, OtoE(H, o))$
8:         $cur \leftarrow o[o.size() - 1];$
9:         $next \leftarrow cur + 1;$
10:        **if** $next < count$ **then**
11:            $o.push(next)$
12:        **else**
13:            $o.pop()$
14:           **if** $o.size() > 0$ **then**
15:               $o[o.size() - 1] \leftarrow o[o.size() - 1] + 1$

CHAPTER IV

BRANCH AND BOUND

Algorithm 5 BranchAndBoundTransversals

1: **function** $BranchAndBoundTransversals(H, CallbackFunc)$
2:     $count \leftarrow H.E.size()$
3:     $o \leftarrow \emptyset$
4:     $o.push(0)$
5:     **while** $o.size() > 0$ **do**
6:         **if** $IsMinimalTransversal(o, H.E)$ **then**
7:             $CallbackFunc(o, OtoE(H, o))$
8:             **if** $o[o.size() - 1] < count - 1$ **then**
9:                 $o[o.size() - 1] \leftarrow o[o.size() - 1] + 1$
10:             **else**
11:                 $o.pop()$
12:                 **if** $o.size() > 0$ **then**
13:                     $o[o.size() - 1] \leftarrow o[o.size() - 1] + 1$
14:         **else**
15:             $cur \leftarrow o[o.size() - 1];$
16:             $next \leftarrow cur + 1;$
17:             **if** $next < count$ **then**
18:                 $o.push(next)$
19:             **else**
20:                 $o.pop()$
21:                 **if** $o.size() > 0$ **then**
22:                     $o[o.size() - 1] \leftarrow o[o.size() - 1] + 1$

CHAPTER V

RECURSIVE POLYNOMIAL SPACE

The recursive polynomial space solution can be found in the previous works: Kavvadias and Stavropoulos [1999], Kavvadias and Stavropoulos [2005],

CHAPTER VI

ITERATIVE POLYNOMIAL SPACE SOLUTION

This paper now introduces the iterative psuedo-polynomial space solution to enumerating all minimal hypergraph traversals. First the depth control is used to expand the tree to the leaf and store the next nodes to be processed. Each node is then removed and processed, if the node is a leaf then the minimal transversal is visited, if the node is not a leaf then generate a new children to process, if no children are generated then this minimal transversal does not have any children after the next edge.

Define: Generalized variable

Let a generalized variable G be an odometer of vertex indexes. such that each element of the odometer

Define: Gamma

A Gamma is the piecewise segmentation of an individual generalized variable intersecting parts with the incoming edge. $G = (XMinusY, XIntersectY, YMinusX)$.

Define: IHGResult

An IHGResult $ihg\_result = (Alphas, Betas, Gammas, new\_alpha)$ is a collection where $Alphas$ is a list of generalized variables (Odometers), $Betas$ is a list of generalized variables (odometers), Gammas is a list of Gammas from the previous definition, and new_alpha is the incoming edge minus all intersections.

Define: Partial transversal stack frame

A partial transversal stack frame $PTSF = (Transversals, Negations)$ is a collection where $Transversals$ is a list of generalized variables (odometers), $Negations$ is a list of generalized variables (odometers) for $IsAppropriate$.

This is the structure that contains the data needed for the current work item to be processed into the next list of partial transversal stack frames.

Generate IHGResult from Transversals and Edge

Using the previous definitions the function to break a transversals generalized variables down into the constituent types and pieces. The function $IntersectTransversalWithEdge$ breaks apart the entire intersection of a minimal transversal with a new edge.

Algorithm 6 IntersectTransversalWithEdge

1: **function** $IntersectTransversalWithEdge(list\_of\_transversals, edge)$
2:      $return\_value \leftarrow \emptyset$ // IHGResult.
3:      $new\_alpha \leftarrow edge$ // copy incoming edge.
4:      **for all** $\{g_t, i_t\} \in list\_of\_transversals$ **do**
5:          $intersect = Intersection(g_t, edge)$
6:          $new\_alpha \leftarrow Minus(new\_alpha, interset)$
7:          **if** $intersect.size() = 0$ **then**
8:              $return\_value.Alphas.push(g_t)$
9:          **else**
10:              **if** $intersect.size() = g_t.size()$ **then**
11:                  $return\_value.Betas.push(g_t)$
12:              **else**
13:                  $Gamma \leftarrow \emptyset$ // Gamma type.
14:                  $Gamma.XMinusY = Minus(g_t, edge)$
15:                  $Gamma.XIntersectY = interset$
16:                  $Gamma.YMinusX = Minus(edge, g_t)$
17:                  $return\_value.Gammas.push(gamma)$
18:      $return\_value.new\_alpha = new\_alpha$
19:      **return** $return\_value$

Generate Next Depth

Algorithm 7 GenerateNextDepth
___

 1: **function** $GenerateNextDepth(HSF, edge)$
 2:     $new\_frame \leftarrow \emptyset$ // hypergraph stack frame
 3:     $return\_value \leftarrow \emptyset$ // list of hypergraph stack frames.
 4:     $result \leftarrow IntersectTransversalWithEdge(HSF.Transversals, edge)$
 5:     **if** $result.Gammas.size() == 0$ **then**
 6:         $new\_frame \leftarrow HSF$
 7:         **if** $result.Betas.size() > 0$ **then**
 8:             **for all** $\{b, i\} \in result.Beta$ **do**
 9:                 $new\_frame.push(b)$
10:         **else**
11:             $new\_frame.push(edge)$
12:         **if** $IsAppropriate(new\_frame, edge)$ **then**
13:             $return\_value.push(new\_frame)$
14:     **else**
15:         **for all** $list\_of\_bool \in Gen2expNtruefalse(result.Gammas.size())$ **do**
16:             $new\_frame.Transversals \leftarrow result.Alphas$
17:             $new\_frame.Negations \leftarrow HSF.Negations$
18:             **for all** $\{tf, j\} \in list\_of\_bool$ **do**
19:                 $gamma \leftarrow result.Gammas[j]$
20:                 **if** $tf[j] = false$ **then**
21:                     $new\_frame.Transversals.push(gamma.XMinusY)$
22:                     $new\_frame.Negations.push(gamma.XIntersectY)$
23:                 **else**
24:                     $new\_frame.Transversals.push(gamma.XIntersectY)$
25:             **if** $IsAllTrue(tf) = true$ **then**
26:                 **if** $result.new\_alpha.size() > 0$ **then**
27:                     $new\_frame.Transversals.push(result.new\_alpha)$
28:             **else**
29:                 **for all** $beta \in result.Betas$ **do**
30:                     $new\_frame.Transverals.push(beta)$
31:                 **if** $IsAllFalse(tf) = true$ **then**
32:                     **for all** $gamma \in result.Gammas$ **do**
33:                         $new\_frame.Negations.push(gamma.XMinusY)$
34:             **if** $IsAppropriate(new\_frame, edge) = true$ **then**
35:                 $returnValue.push(new\_frame)$
36:                 $new\_frame \leftarrow \emptyset$
37:     **return** $return\_value$
___

Depth First N-Way Tree Control

Algorithm 8 HypergraphTransversals

1: **function** $HypergraphTransversals(H, CallbackFunc)$
2:     $edge\_count \leftarrow H.E.size()$
3:     $control\_stack \leftarrow list(edge\_count)$ // list of stacks pre-sized.
4:     $HSF \leftarrow \emptyset$ // current hypergraph stack frame
5:     $HSF.Transversals.push(edge)$
6:     $control \leftarrow 0$ // depth control variable.
7:     $control\_stack[control].push(HSF)$ // load the process.
8:     **while** $control \geq 0$ **do**
9:         **if** $control\_stack[control].size() = 0$ **then**
10:             $control \leftarrow control - 1$
11:         **else**
12:             $frame \leftarrow control\_stack[control].pop()$
13:             **if** $control = edge\_count - 1$ **then**
14:                 $CallbackFunc(frame.Transversals)$ // min transversal reached.
15:             **else**
16:                 $control \leftarrow control + 1$
17:                 $next\_edge \leftarrow H.E[control]$
18:                 $children \leftarrow GenerateNextDepth(frame, next\_edge)$
19:                 **for all** $\{c, i\} \in children$ **do**
20:                     $control\_stack[control].push(c)$ // next to be processed

---

IsAppropriate

The following algorithm is used to determine if the new transversals

---

Algorithm 9 IsAppropriate

1: **function** $IsAppropriate(HSF, edge)$
2:     $list\_of\_new_traversals \leftarrow \emptyset$
3:     **for all** $\{o, i\} \in HSF.Transversals$ **do**
4:         $gv \leftarrow o$
5:         **for all** $\{n, i\} \in HSF.Transversals$ **do**
6:             **if** $DoesACoverB(n, gv) = true$ **then**
7:                 $gv \leftarrow Minus(gv, n)$
8:         **if** $gv.size() > 0$ **then**
9:             $list\_of\_new_traversals.push(gv)$
10:     **if** $DoesAnyHitA(list\_of\_new_traversals, edge) = false$ **then**
11:         **return** $false$
12:     **return** $true$

CHAPTER VII

TESTING

Generating Hypergraphs

Hypergraphs are a relatively new data structure; large datasets are not
currently modeled as hypergraphs so ensuring correctness is the onus of the
implementers. We seek to prove the algorithm is correct for a large set of small
hypergraphs and a small set of large hypergraphs. The following algorithm
will generate all simple hypergraphs with node count N. The complexity is
exponentially exponential on the order of $N!^{N-1!^{N-2!^{\cdots}}}$.

Algorithm 10 GenHypergraphs
___

1: **function** $GenHypergraphs(Nodes, CallbackFunc)$
2:      $V = Nodes$ // new hyperedge for a hypergraph of all the nodes.
3:      $E = Nodes$ // single hyperedge of all the nodes.
4:      $H = (V, E)$ // hypergraph with the above.
5:      $CurrentQueue.push(H)$
6:      $WorkQueue.push(CurrentQueue)$
7:      **while** $!WorkQueue.empty()$ **do**
8:          $CurrentQueue \leftarrow WorkQueue.pop()$
9:          $H \leftarrow CurrentQueue.pop()$
10:          **if** $!CurrentQueue.empty()$ **then**
11:              $WorkQueue.push(CurrentQueue)$
12:          $CallbackFunc(H)$ // process generated hypergraph
13:          $CurrentQueue = GenerateHypergraphChildren(H)$
14:          **if** $!CurrentQueue.empty()$ **then**
15:              $WorkQueue.push(CurrentQueue)$
16: **function** $GenerateHypergraphChildren(H)$
17:      $Children \leftarrow \emptyset$
18:      **for all** $edge \in H.E$ **do**
19:          **for all** $\{edges, i\} \in GenerateNMinusOne(edge)$ **do**
20:              $new\_edges \leftarrow H.E \setminus edge)$ // every edge but the new ones
21:              **for all** $\{e_n, i_n\} \in edges$ **do**
22:                  $new\_edges.push(e_n)$ // add new broken down edges.
23:              $Children.push(Hypergraph(H.V, new\_edges))$
24:          **return** $Children$
___

CHAPTER VIII

FUTURE DIRECTIONS

Distributed Computation via Iterative Containers

Fundamentally this paper re-introduces a polynomial space algorithm for computing hypergraph transversals as an iterative procedure. Each work item that is processed is a partial transversal with the included negative sets and a new hyperedge. The abstraction completely captures all the data needed to generate the next set of work items. The N-Way-Tree algorithm that iteratively constructs the next work items or processes completed transversals can be easily replaced.

The replacement for the N-Way-Tree would be a distributed processing dispatcher. Each work item would be put in a queue for execution on a compute node. All data to process the node is present in the work item. All the future work items can be generated on a compute node, then enqueued in the distributed dispatcher processor.

Currently hyperedges are always encountered in the same order at each depth of the tree. A pipeline of compute nodes with hyperedges can be built to distribute the work. A compute node could be paired with a specific hyperedge and a work queue. As each work item is created by the previous compute node it would be enqueued in the next compute node work queue.

Controlling work item generation to keep the pipeline full becomes the new problem. A cascading exponential number of work items generated by the early compute nodes will flood the later parts. The expansion of work items is currently exponential in the maximal worst case. Fundamentally another odometer state can

be added to the partial transversal frame and used to iteratively generate the next set of work items. As the pipeline of work items becomes empty the work items would be iterated, generated, and added to the pipeline to keep it full until the current work item is processed.

Hyperedge Visitation Manipulation

An interesting and notable effect of encountering a new hyperedge, is when the partial transversal negates all new transversals, causing the complete removal of all future work items. Simply a new hyperedge is encountered that causes a minimal transversal to either become invalid or redundant. Eliminating future minimal transversals before they generate children that need to be eliminated is a possible optimization.

Reordering the visitation of hyperedges would need to be provably sound such that an dispatch algorithm could reorder the visitation of hyperedges to eliminate minimal transversals as quickly as possible during computation. As each work item has a list of negation odometers, it is possible to look for a hyperedge contained in the negation odometer union with the transversal odometer to generate either 1 new minimal transversal child or not being an appropriate minimal transversal and being eliminated from the work queue.

Advanced Algorithmic Modifications

The inter-section, outer-section, and cross product sections each generate a work item. Currently all work items for a given level are expanded to the next level (exponential in the worst case), then processing the first one in the same way expanding the tree in a depth first search. Control of the expansion can be done

via additional odometer states in each work item. Replacement of the function
$Gen2expNtruefalse$ with an iterative generator allows for a more controlled
expansion.

The generation of the next list of work items is currently a fixed ordering
that generates the same traversals in the same order every time. Obviously this
is desirable from a completeness aspect, but from a practical perspective iterating
the traversals which satisfy optimality constraints 'better' is of particular interest.
'Better' is a term that needs to be defined in terms of the particular problem that
is being solved. Khachiyan et al. [2006]

Experimental Research

A possible direction of research is to modify the core algorithms to look
for ways to collapse the work items being generated at each depth. The compact
transversal representations generated by this algorithm store exponential traversals
in polynomial space. Removal of the negation sets in conjunction with collapsing
work items at each level would result in a polynomial reduction by storing
traversals in an exponential encoding. This is the partial transversal frame that
is enumerated to generate its transversals

It is not possible to enumerate all transversals of a hypergraph in polynomial
space and polynomial time. It may be possible to enumerate all exponential
encodings of transversals of a hypergraph in polynomial space and polynomial time.

The current algorithm *already* uses a polynomial space encoding of an
exponential transversal. The partial transversal frame is interpreted as an encoding
that generates exponential transversals. A partial transversals can be merged with
another partial transversal if and only if the following holds: They contain the same

number of generalized variables. There is only one generalized variable in both transversals that are not equivalent. Notice that both collapsing and comparing against all transversals at a given level is polynomial in space and time. Collapsing all partial transversals at every depth would need to be proven to work correctly.

The output of such an algorithm would be a polynomial space encoding, the same as today. Extracting all of the transversals via interpretation of the encoding is an exponential time operation. Instead of extracting all the transversals, the encodings themselves could represent such things as the clique of cliques, central clusters in clusters, or the eigenvalues that are of the same size.

APPENDIX

ALGORITHMS REFERENCED IN PAPER

Algorithm 11 Union

1: **function** $Union(A, B)$
2:     $returnValue \leftarrow \emptyset$
3:     **for all** $\{n, i\} \in A$ **do**
4:       **if** $!returnValue.contains(n)$ **then**
5:         $returnValue.push(n)$
6:     **for all** $\{n, i\} \in B$ **do**
7:       **if** $!returnValue.contains(n)$ **then**
8:         $returnValue.push(n)$
9:     **return** $returnValue$

Algorithm 12 Intersection

1: **function** $Intersection(A, B)$
2:     $returnValue \leftarrow \emptyset$
3:     **for all** $\{n_A, i_A\} \in A$ **do**
4:       **for all** $\{n_B, i_B\} \in B$ **do**
5:         **if** $n_A = n_B$ **then**
6:           $returnValue.push(n_A)$
7:     **return** $returnValue$

Algorithm 13 Minus

1: **function** $Minus(A, B)$
2:     $returnValue \leftarrow \emptyset$
3:     **for all** $\{n_A, i_A\} \in A$ **do**
4:         $add \leftarrow true$
5:         **for all** $\{n_B, i_B\} \in B$ **do**
6:             **if** $n_A = n_B$ **then**
7:                 $add \leftarrow false$
8:         **if** $add = true$ **then**
9:             $returnValue.push(n_A)$
10:     **return** $returnValue$

Algorithm 14 StrictEqual

1: **function** $StrictEqual(A, B)$
2:     **for all** $\{n_A, i_A\} \in A$ **do**
3:         **for all** $\{n_B, i_B\} \in B$ **do**
4:             **if** $n_A! = n_B$ **then**
5:                 **return** $false$
6:     **return** $true$

Algorithm 15 SetEqual

1: **function** $SetEqual(A, B)$
2:     $A \leftarrow Sort(A);$
3:     $B \leftarrow Sort(B);$
4:     **return** $StrictEqual(A, B)$

Algorithm 16 DoesACoverB

1: **function** $DoesACoverB(A, B)$
2:     **for all** $\{n_A, i_A\} \in A$ **do**
3:         $found \leftarrow false$
4:         **for all** $\{n_B, i_B\} \in B$ **do**
5:             **if** $n_A = n_B$ **then**
6:                 $found \leftarrow true$
7:         **if** $found = false$ **then**
8:             **return** $false$
9:     **return** $true$

Algorithm 17 DoesACoverBorBCoverA

1: **function** $DoesACoverBorBCoverA(A, B)$
2:     **if** $DoesACoverB(A, B) = true$ **then**
3:         **return** $true$
4:     **if** $DoesACoverB(B, A) = true$ **then**
5:         **return** $true$
6:     **return** $false$

Algorithm 18 DoesAHitB

1: **function** $DoesAHitB(A, B)$
2:     **for all** $\{n_A, i_A\} \in A$ **do**
3:         **for all** $\{n_B, i_B\} \in B$ **do**
4:             **if** $n_A = n_B$ **then**
5:                 **return** $true$
6:     **return** $false$

Algorithm 19 DoesAHitAll

1: **function** $DoesAHitAll(A, list\_of\_o)$
2:     **for all** $\{o, i\} \in list\_of\_o$ **do**
3:         **if** $DoesAHitB(A, o) = false$ **then**
4:             **return** $false$
5:     **return** $true$

Algorithm 20 DoesAnyHitA

1: **function** $DoesAnyHitA(list\_of\_o, A)$
2:     **for all** $\{o, i\} \in list\_of\_o$ **do**
3:         **if** $DoesAHitB(o, A) = true$ **then**
4:             **return** $true$
5:     **return** $false$

Algorithm 21 GenerateNMinusOne

1: **function** $GenerateNMinusOne(o)$
2:     $returnValue \leftarrow \emptyset$ // list of odometers
3:     **for all** $\{n, i\} \in o$ **do**
4:         $add \leftarrow o$ // copy odometer
5:         $add.remove(n, i)$ // erase 1 value.
6:         $returnValue.push(add)$
7:     **return** $returnValue$ // N odometers, each with one item removed.

Algorithm 22 Gen2expNtruefalse

1: **function** $Gen2expNtruefalse(n)$
2:     $returnValue \leftarrow \emptyset$ // list of (list of true—false)
3:     $max = 1 << n$ // max is 2n̂ bit shifted.
4:     **for all** $i \in 0..max$ **do**
5:         $add \leftarrow \emptyset$ // list of true—false
6:         $counter \leftarrow 1$
7:         **while** $counter < max$ **do**
8:             **if** $counter \& i = counter$ **then**
9:                 $add.push(true)$
10:            **else**
11:                $add.push(false)$
12:            $counter \leftarrow counter << 1$ // bit shift left.
13:        $returnValue.push(add)$
14:    **return** $returnValue$ // N odometers, each with one item removed.

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