The Three Body Problem

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For my final project, I modeled the motion of orbiting objects. I started with two bodies (our Sun and the Earth) before adding a third body (Jupiter) to create a restricted three-body system. I then investigated a true three-body system, including the effects on the sun from Earth and Jupiter.

1 The Two Body Problem

To start this problem, I created a system that consisted of the Sun and Earth. As the Sun is much more massive than Earth, there is little effect on the Sun, and I assumed that the Sun will not change position as the Earth orbits. For calculations, I only considered the force on the Earth due to the Sun, using equation 1, where Ms is the solar mass, ME is the Earth's mass, and r is the distance between them.

$$Fg = -\frac{GM_S M_E}{r^2} \tag{1}$$

I created a program, twobody.py, that solves for the position of the Earth as a function of time, solving the equations of motion:

$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_E} \tag{2}$$

$$\frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_E} \tag{3}$$

I divided equations 2 and 3 into a set of four first order ordinary differential equations.

$$\frac{dv_x}{dt} = \frac{F_{G,x}}{M_E} \tag{4}$$

$$\frac{dx}{dt} = v_x \tag{5}$$

$$\frac{dv_y}{dt} = \frac{F_{G,y}}{M_E} \tag{6}$$

$$\frac{dy}{dt} = v_y \tag{7}$$

To update velocity and position, I used the Euler-Cromer method, updating velocity in equations 4 and 6 with the previous values and then updating position in 5 and 7 with the new velocities.

For units, I solved the problem with distance in AU, time in years, and mass in solar mass. With these units, here are my initial conditions.

1. $G = 4\pi^2$ Gravitational constant ($\frac{AU^3}{M_S * year^2}$)

2. $M_E = 3.0027 \times 10^{-6}$ Mass of Earth (Solar Mass)

3. $M_S = 1$ Mass of Sun (Solar Mass)

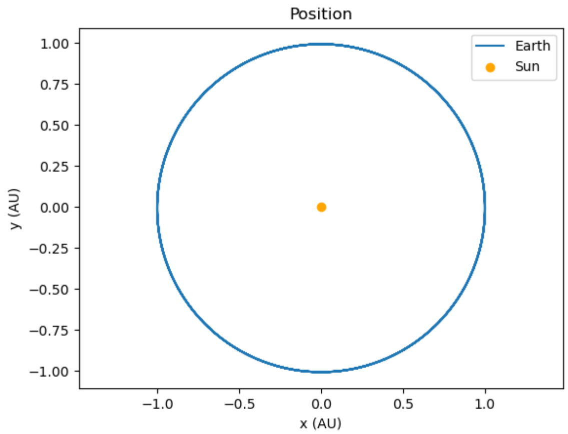

Figure 1: Position, x, vs. Position, y, showing the circular orbit of a two-body orbital model.

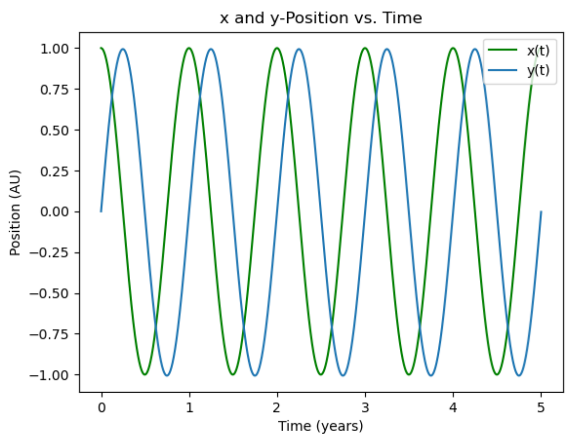

Figure 2: Positions, x and y, plotted as a function of time, showing the uniform sinusoidal motion of a circular orbit over time.

4. $dt = 0.002$ Time step (years)
5. $t_{Final} = 5$ End time (years)
6. $sunX, sunY = 0, 0$ Sun position (AU)
7. $earthX, earthY = 1, 0$ Earth initial position (AU)
8. $vEarthX, vEarthY = 0, 2\pi$ Earth initial velocity $(\frac{AU}{year})$
9. $r = 1$ Orbital radius (AU)

The updated velocities and positions are stored in an array and I plotted the orbit, $earthY$ vs. $earthX$, in figure 1 and the positions earthX and earthY as functions of time in figure 2. These graphs show the uninterrupted, circular motion of earth in this two-body model.

2 The Three Body Problem (Restricted)

Now, I want to add Jupiter to the model of the solar system. I created a new program called threebody.py and modified the function I made to calculate the force to be more generic for two bodies in space, shown

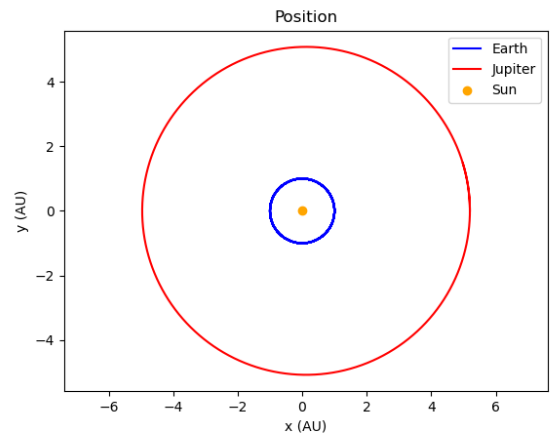

Figure 3: Position x vs y for Earth and Jupiter in a restricted three-body simulation.

in equations 8 and 9. I also modified the function for equations 4 and 6 to account for more than one force.

$$F = -\frac{GM_1M_2dx}{r^3} \tag{8}$$

$$F = -\frac{GM_1M_2dy}{r^3} \tag{9}$$

Using my new functions, I updated the velocity and position for both Earth and Jupiter for each time step (dx now 0.001 years) and ran the program for a simulated time of 12 years. This program simulating a restricted three-body problem, the Sun remained stationary at x, y = 0.0 AU. Earth's initial conditions remained the same, Jupiter's defined as: x(0) = 5.2AU, y(0) = 0, vx (0) = 0, vy(0) = 2 x(0)/12.

I then plotted x verses y for Earth and Jupiter, shown in figure 3, and the x and y-positions for both Earth and Jupiter as a function of time, showing in figure 4. The orbital positions are still close to perfectly circular with these conditions.

To investigate how the positions of each body are related to each other, I experimented with changing the mass of Jupiter. Figures 5 and 6 are the same objects plotted, but Jupiter is 10 times as massive. It does not seem to make a significant difference in the motion of Earth, as seen in figures 5 and 6. When I changed the mass of Jupiter to be 1000 times as massive, there was significant change in the motion of Earth, as seen in figures 7 and 8.

3 The Three Body Problem

Now it's time to add in the effects of Earth and Jupiter on the Sun. I modified threebody.py to calculate the position of the Sun as a function of time. Since the Sun is now allowed to move, its position cannot be the origin. Instead, the origin should be the center of mass of the solar system:

$$X_{CM} = \frac{x_S M_S + x_E M_E + x_J M_J}{M_S + M_E + M_E} \tag{10}$$

I used $x_S = 0$ AU for the initial calculation and then adjusted the position for each body so the center of mass, X_{CM} was at 0.0 AU. I then calculated the momentum for Each body to apply conservation of momentum on the system. I did this to find the initial velocity of the sun to be -0.0026179363 AU/year in the y-direction. I then altered my force equations and updated the positions for each body to account for the sun moving as well. I plotted the orbits and the positions over time, running the simulation to 40 years, seen in figures **??** and 10. It is clear that, the Sun does move as the system moves, but because the center of mass is close to the sun, it does not have a large orbital radius, staying in relatively the same spot compared to Earth and Jupiter.

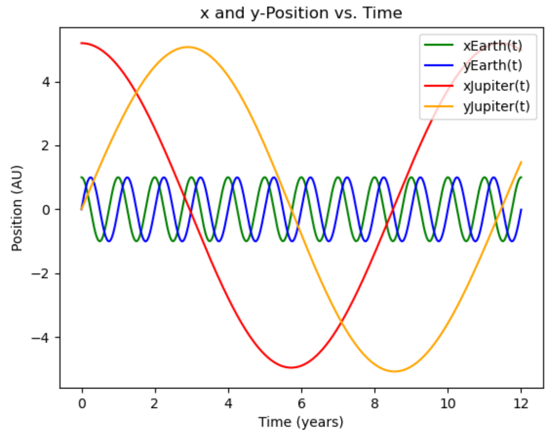

Figure 4: x and y positions as a function of time for Earth and Jupiter.

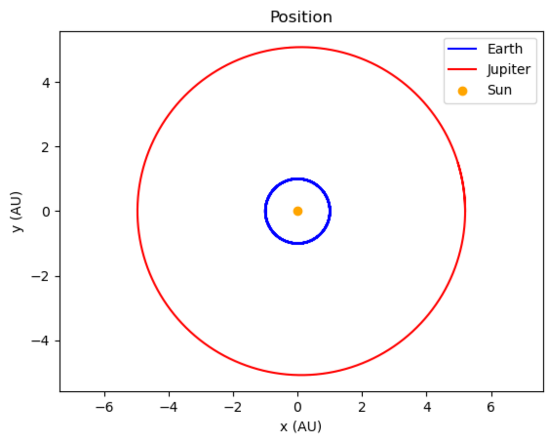

Figure 5: Position x vs y for Earth and Jupiter in a restricted three-body simulation, where Jupiter is 10 times as massive.

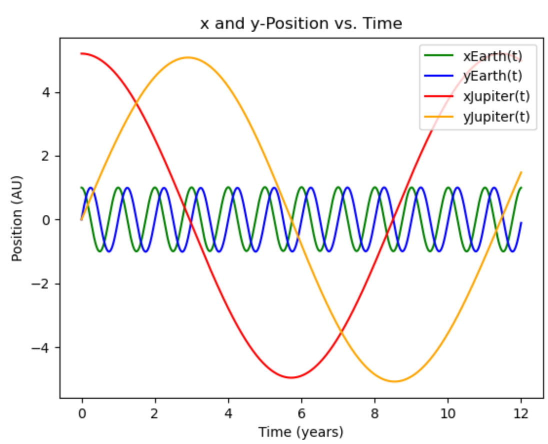

Figure 6: x and y positions as a function of time for Earth and Jupiter, where Jupiter is 10 times as Massive.

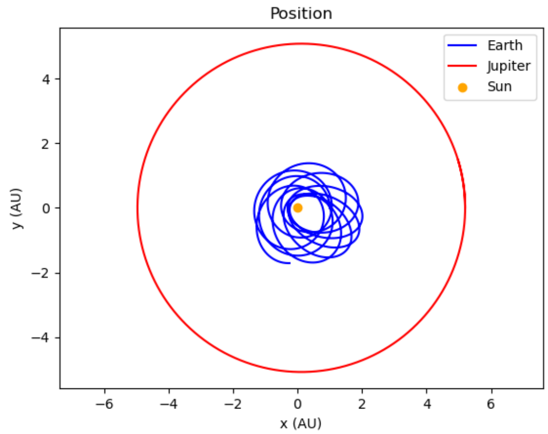

Figure 7: Position x vs y for Earth and Jupiter in a restricted three-body simulation, where Jupiter is 1000 times as massive.

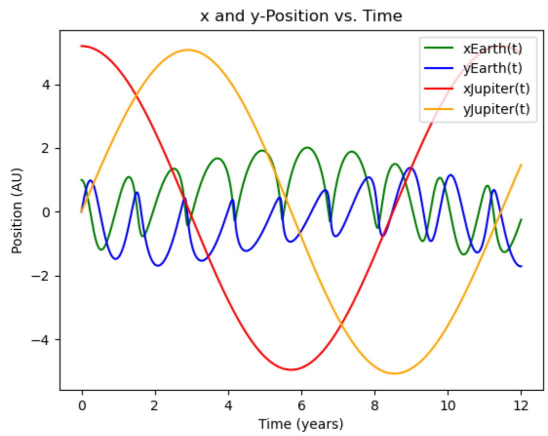

Figure 8: x and y positions as a function of time for Earth and Jupiter, where Jupiter is 1000 times as Massive.

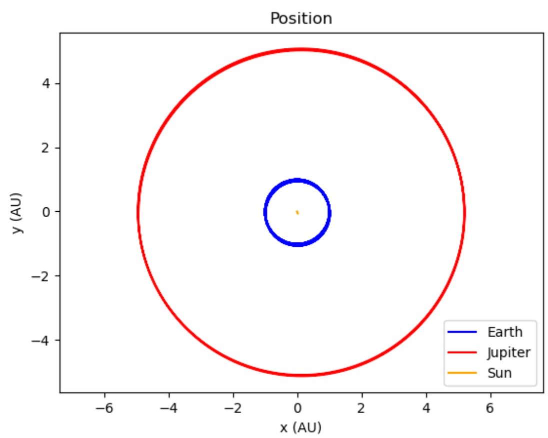

Figure 9: Position x verses y for the Sun, Earth, and Jupiter in an unrestricted three-body simulation.

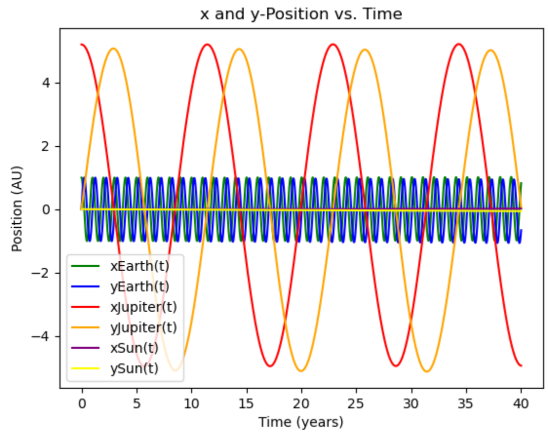

Figure 10: Positions x and y as a function of time for the Sun, Earth, and Jupiter in an unrestricted three-body simulation.

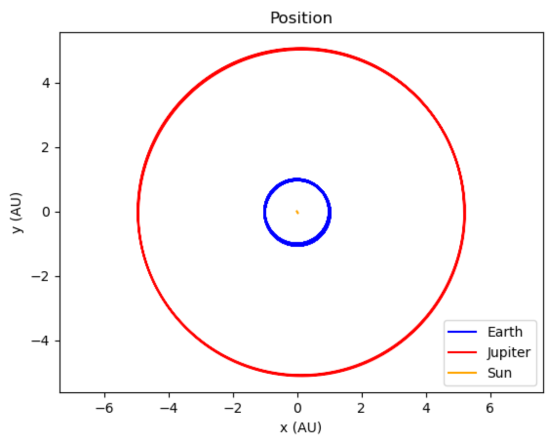

Figure 11: Position x verses y for the Sun, Earth, and Jupiter in an unrestricted three-body simulation, when Jupiter is 10 times as massive.

To, again, investigate how the positions of each body are related to each other, I experimented with changing the mass of Jupiter. Figures 9 and 10 are the same objects plotted, but Jupiter is 10 times as massive. It does not seem to make a significant difference in the motion of Earth, 11 and 12. When I changed the mass of Jupiter to be 1000 times as massive, there was significant change in the motion of Earth, as seen in figures 13 and 14.

To then investigate changing another variable, I decreaced the time step, dt, to 0.0001 years and ran the simulation with Jupiter 1000 times as massive, as seen in 15 and 16. From my graphs, there does not seem to be a noticible difference in outcome.

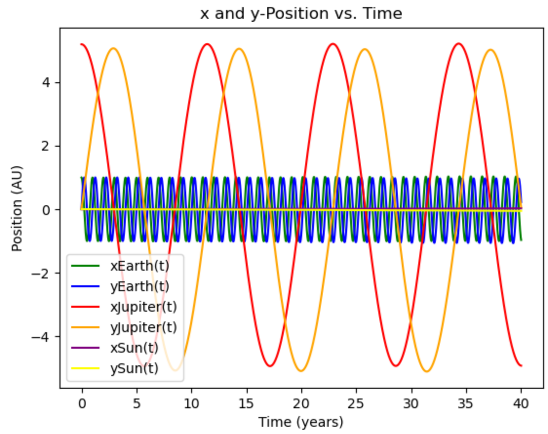

Figure 12: Positions x and y as a function of time for the Sun, Earth, and Jupiter in an unrestricted three-body simulation, when Jupiter is 10 times as massive.

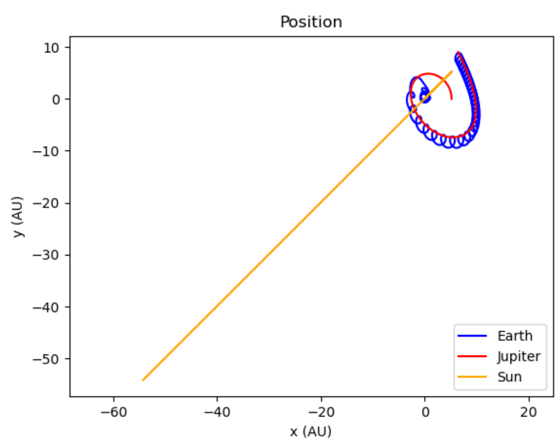

Figure 13: Position x verses y for the Sun, Earth, and Jupiter in an unrestricted three-body simulation, when Jupiter is 1000 times as massive.

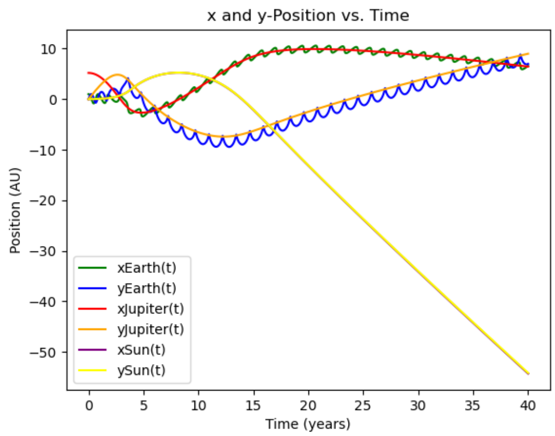

Figure 14: Positions x and y as a function of time for the Sun, Earth, and Jupiter in an unrestricted three-body simulation, when Jupiter is 1000 times as massive.

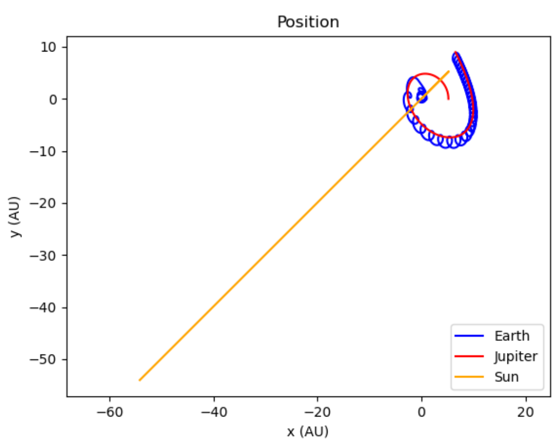

Figure 15: Position x verses y for the Sun, Earth, and Jupiter in an unrestricted three-body simulation, when Jupiter is 1000 times as massive. The new time step is 0.0001 years.

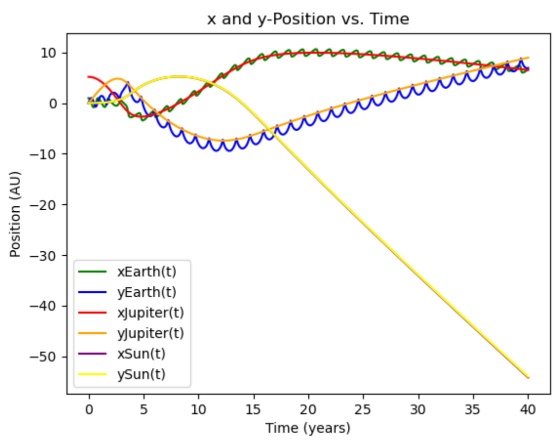

Figure 16: Positions x and y as a function of time for the Sun, Earth, and Jupiter in an unrestricted three-body simulation, when Jupiter is 1000 times as massive. The new time step is 0.0001 years.