Digital image processing
Prof. Shohreh Kasaei
Sharif University of Technology

HW3 **Deadline**: 1403/2/4 **Support mail**: arazlighi@gmail.com

Theoretical Questions:

1. In this question, we want to show that DFT can be interpreted as a complex rotation. First, we define a rotation matrix (R) and the DFT of a signal $x(n)$ as follows:

$$R \text{ is rotation matrix } iff\ \mathbf{det}(\boldsymbol{R}) = +\mathbf{1}\ \&\ \boldsymbol{R}^{-1} = \boldsymbol{R}^{T}\ \&\ \boldsymbol{rows\ of\ R\ are\ orthogonal}$$
$$X(n) = \Sigma_{k=0}^{N-1} x(k).e^{-i.\frac{2\pi}{N}kn}$$

Where $\mathrm{X(n)}$ is the transformed signal from spectral domain to frequency domain. Prove that DFT (as expressed as follows) can be written in matrix form, such that the specified matrix has all of the properties of a rotation matrix and thus, is a complex rotation! If the final matrix is not orthonormal, multiply the required factor to make it orthonormal.

2. Compute DFT for following signals (suppose analysis width is equal to $N = 32$):

a) $x_1[n] = \cos\left(\frac{\pi}{4}n\right)$
b) $x_2[n] = \sin\left(\frac{\pi}{4}n\right)$
c) $x_3[n] = \cos\left(\frac{\pi}{8}n\right)$
d) $x_4[n] = \left(\frac{1}{2}\right)^n u(n)$

3. Calculate the Haar transform of below image. Call the transformed image as $\tilde{A}$. Then, reconstruct the original image by setting the bottom right element of $\tilde{A}$ to 0 and then doing inverse transformation. Finally, calculate the square error between reconstructed image ($A_{reconstructed}$) and original image (A).

Digital image processing
Prof. Shohreh Kasaei
Sharif University of Technology

$$g = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Practical Questions:

4. **Implementing Convolution and Circular Convolution:** refer to the notebook provided!

5. **Implementing DCT image Compression:** refer to the notebook provided!