تمرین سری ۱ DIP

$\det(R) = 1$ & $R^{-1} = R^{T}$

سوال ۱)

$\text{dft} \rightarrow X(n) = \sum^{N-1} x(k)\, e^{-\frac{2\pi}{N} knj}$

الف)

$$R = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-2\pi j/N} & \cdots & e^{-\frac{2\pi j(N-1)}{N}} \\ \vdots & e^{-2\pi j\, 2/N} & \ddots & \\ \vdots & \vdots & & \\ 1 & e^{-2\pi j \frac{N-1}{N}} & \cdots & e^{-\frac{2\pi j (N-1)^2}{N}} \end{bmatrix}$$

در n و k متقارن است ... شود

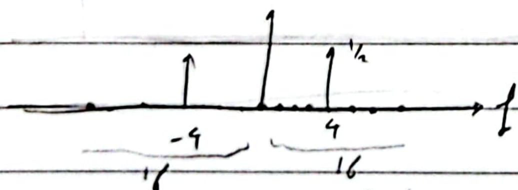
$R_{\text{orthonormalize}} = \frac{1}{\sqrt{N}} R$ برای نرمالایز کردن $\det R_{\text{norm}} = 1$

$R^{T} = R^{-1}$

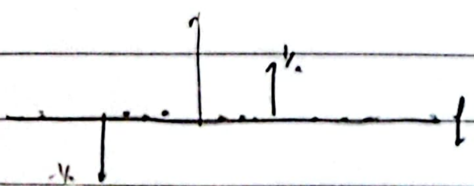
سوال ۲)

a) $\sum^{31} \cos(\frac{\pi}{4} n)\, e^{-\frac{2\pi}{32} f n j}$ $\rightarrow \cos(\frac{\pi}{4} n) = \cos(\frac{2\pi \cdot 4 n}{32})$

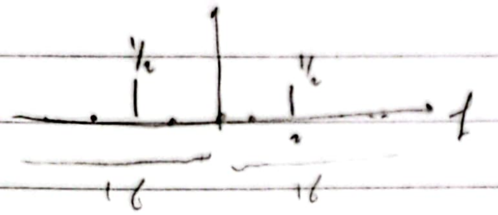
$f_s\ 4 \rightarrow X(k) = \frac{1}{2}(\delta(f-4) + \delta(f+4))$



b) $\sin(\frac{\pi}{4} n) = \sin(\frac{2\pi 4 n}{32})$ $\quad X(k) = \frac{1}{2}(\delta(f-4) - \delta(f+4))$



c) $\cos\left(\frac{\pi}{8} n\right) = \cos\left(\frac{2\pi\, 2\, n}{32}\right) \longrightarrow X(k) = \frac{1}{2}\left(\delta(k-2) + \delta(k+2)\right)$



d) $\left(\frac{1}{2}\right)^n u(n) \rightarrow X(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \Rightarrow X[k] = \frac{1}{1-\frac{1}{2}e^{-\frac{j2\pi k}{32}}} \Bigg|_{n=}^{31}$

$$g = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \rightarrow Har(g) = \tilde{A}$$

سوال 13

$$H_{4T} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt2 & -\sqrt2 & 0 & 0 \\ 0 & 0 & \sqrt2 & -\sqrt2 \end{bmatrix} \qquad H_r^{-1} = H^T = \begin{bmatrix} 1 & 1 & \sqrt2 & 0 \\ 1 & 1 & -\sqrt2 & 0 \\ 1 & -1 & 0 & \sqrt2 \\ 1 & -1 & 0 & -\sqrt2 \end{bmatrix}$$

$$\tilde{A} = H_r T g = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt2 & -\sqrt2 & 0 & 0 \\ 0 & 0 & \sqrt2 & -\sqrt2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ -\sqrt2 & \sqrt2 & \sqrt2 & -\sqrt2 \\ \sqrt2 & -\sqrt2 & -\sqrt2 & \cancel{\sqrt2}\ 0 \end{bmatrix}$$

$$A_{rec} = H^{-1}\tilde{A} = \begin{bmatrix} 1 & 1 & \sqrt2 & 0 \\ 1 & 1 & -\sqrt2 & 0 \\ 1 & -1 & 0 & \sqrt2 \\ 1 & -1 & 0 & -\sqrt2 \end{bmatrix}\begin{bmatrix} 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ -\sqrt2 & \sqrt2 & \sqrt2 & -\sqrt2 \\ \sqrt2 & -\sqrt2 & -\sqrt2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 4 & 4 \\ 4 & 0 & 0 & 4 \\ 4 & 0 & 0 & 2 \\ 0 & 4 & 4 & 2 \end{bmatrix}$$

$$\sqrt{\sum\sum |A_{rec} - g|^2} < 70 \qquad A_{rec} - g = \begin{bmatrix} 0 & 3 & 3 & 4 \\ 3 & 0 & 0 & 3 \\ 3 & 0 & 0 & 1 \\ 0 & 3 & 3 & 2 \end{bmatrix} \Rightarrow \sigma = \sqrt{85} = 9.22$$