Demystifying the Impact Parameter (d_0)

J. Rosenzweig

2020-01-30

Consider a reconstructed muon track projected onto the xy plane, transverse to the beam pipe. If the muon was a prompt muon, then it truly originated from the primary vertex (PV). However, due to inefficiencies in reconstructing the muon track, the best-fit track may not intersect the PV.

Looking at Figure 1 (Left), the (very exaggerated) muon track is represented by the black circle. This track could either be a μ^+ (blue arrowheads) travelling around clockwise, since the magnetic field points along the $+\hat{z}$ direction, or it could represent the track of a μ^- (orange arrowheads) travelling anticlockwise.

It is convenient to define a few variables:

- $\vec{s}$ = the field point vector which begins at the PV (the origin) and ends at the point-of-closest-approach (PCA) along the muon track.
- ϕ_s = the azimuthal angle of $\vec{s}$ as measured from the x-axis.
- $\phi_{\mu^\pm}$ = the azimuthal angle of the $\vec{p}_{T,\mu^\pm}$, tangent to the track at the PCA, measured from the x-axis.

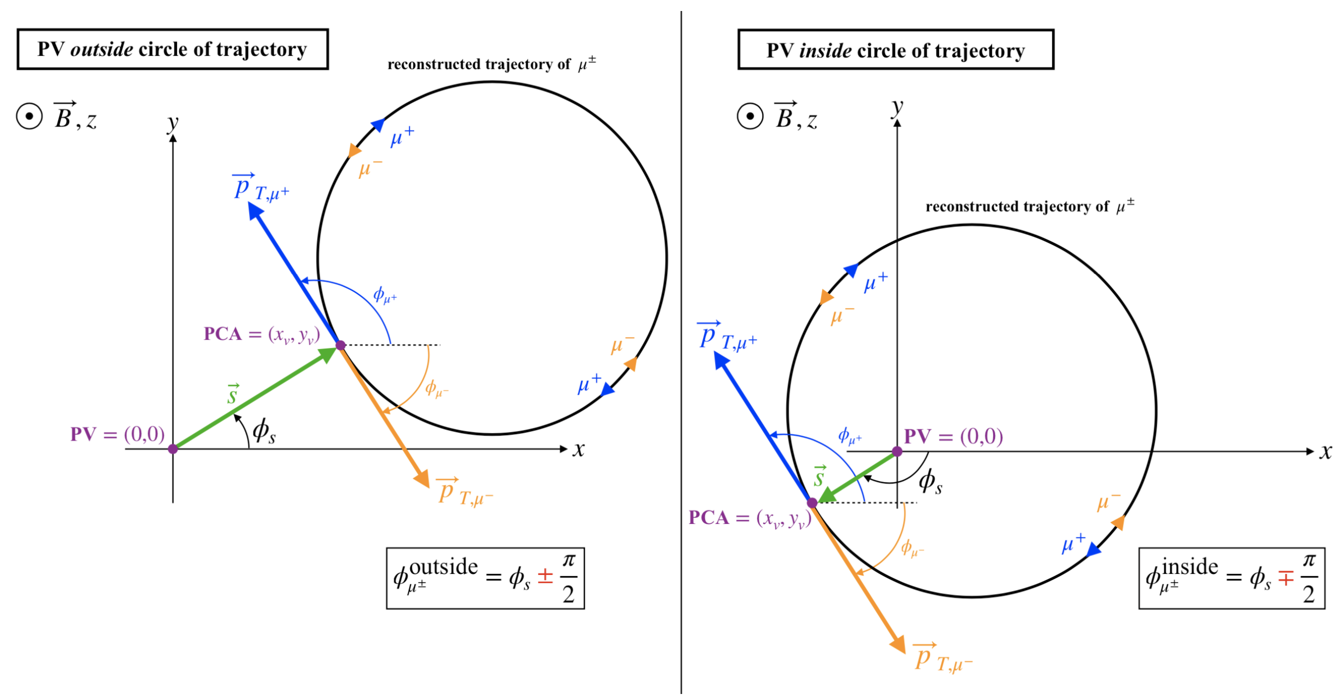

Figure 1: (Left) The case in which the true primary vertex (PV) is *outside* the circular trajectory of a muon. (Right) The opposite case in which the PV is *inside* the circular trajectory.

From Fig. 1 (Left) we see that:

$$\phi_{\mu^\pm} = \phi_s \pm \pi/2. \tag{1}$$

Using one possible definition of d_0 and Equation 1 shows that the d_0 for $\mu^\pm$ which came from a PV outside the circle trajectory is:

$$
\begin{aligned}
d_{0,\mu^\pm}^{\text{PV,outside}} &= -x\sin(\phi_{\mu^\pm}) + y\cos(\phi_{\mu^\pm}) \\
&= -x\sin(\phi_s \pm \pi/2) + y\cos(\phi_s \pm \pi/2) \\
&= -x\big[\sin(\phi_s)\cos(\pi/2) \pm \sin(\pi/2)\cos(\phi_s)\big] + y\big[\cos(\phi_s)\cos(\pi/2) \mp \sin(\phi_s)\sin(\pi/2)\big] \\
&= -x[\pm\cos(\phi_s)] + y[\mp\sin(\phi_s)] \\
&= \mp[x\cos(\phi_s) + y\sin(\phi_s)] \\
&= \mp\big[x\hat{x} + y\hat{y}\big] \cdot \big[\cos(\phi_s)\hat{x} + \sin(\phi_s)\hat{y}\big] \\
&= \mp\vec{s} \cdot \hat{s} \\
&= \mp|\vec{s}||\hat{s}|\cos(0)
\end{aligned}
$$

$$\implies d_{0,\mu^\pm}^{\text{PV,outside}} = \mp|\vec{s}|. \tag{2}$$

The case for the PV being *inside* the circle trajectory (Fig. 1, Right) simply leads to a sign change in Eqn. 1:

$$\phi_{\mu^\pm} = \phi_s \mp \pi/2. \tag{3}$$

Starting again from the definition of d_0, but this time using Eqn. 3, ultimately gives:

$$\implies d_{0,\mu^\pm}^{\text{PV,inside}} = \pm|\vec{s}|. \tag{4}$$

Indeed we see that the magnitude of d_0 *is the transverse impact parameter* ($|\vec{s}|$), as expected! The sign of d_0, however, is not possible to interpret at this point: for a $d_0 > 0$ could either mean a μ^- coming from a PV outside the circlular trajectory *or* could mean a μ^+ coming from a PV found inside the circle.

Since the sign of d_0 is not useful by itself, consider multiplying d_0 by the charge of its corresponding muon. We then see that Eqn. 2 becomes:

$$
\begin{aligned}
\text{charge}(\mu^\pm) \cdot d_{0,\mu^\pm}^{\text{PV,outside}} &= \pm 1 \cdot \mp|\vec{s}| \\
&= -|\vec{s}| < 0,
\end{aligned}
$$

which is always negative. Similarly, Eqn. 4 gives the opposite result:

$$\text{charge}(\mu^\pm) \cdot d_{0,\mu^\pm}^{\text{PV,inside}} = +|\vec{s}| > 0,$$

which of course is always positive.

Therefore, if we know the sign of the muon (say, negative) and the sign of its d_0 (say, positive), then we can simply take the product (negative in this case) and infer that the PV must have been *outside* the muon trajectory!

To summarize: it is the *product* of the charge and d_0 that contains useful information about the muon track.

$$\text{PV} = \begin{cases} \text{inside of circle,} & \text{if charge}(\mu^\pm) \cdot d_{0,\mu^\pm} > 0 \\ \text{outside of circle,} & \text{if charge}(\mu^\pm) \cdot d_{0,\mu^\pm} < 0. \end{cases}$$