Deriving the Update Equation for Membership Values of PCM

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1 Derivation

Consider the Possibilistic C-Means (PCM) objective function:

$$J = \sum_{i=1}^{N}\sum_{j=1}^{M} u_{ij}^{m} d(x_i, \theta_j) + \sum_{j=1}^{M} \eta_j \sum_{i=1}^{N} (1 - u_{ij})^m \tag{1}$$

Let's find the u_{ij}s which optimize J by differentiating (1) with respect to u_{ij} and setting the result equal to zero:

$$\begin{aligned}
\frac{\partial J}{\partial u_{ij}} &= \frac{\partial}{\partial u_{ij}} \left(\sum_{i=1}^{N}\sum_{j=1}^{M} u_{ij}^{m} d(x_i, \theta_j) + \sum_{j=1}^{M} \eta_j \sum_{i=1}^{N} (1 - u_{ij})^m \right) \\
0 &= \cancel{m} u_{ij}^{m-1} d(x_i, \theta_j) + \eta_j \cancel{m} (1 - u_{ij})^{m-1} (-1) \\
\eta_j (1 - u_{ij})^{m-1} &= u_{ij}^{m-1} d(x_i, \theta_j)
\end{aligned}$$

Now raise both sides to the power $\frac{1}{m-1}$:

$$\begin{aligned}
\eta_j^{\frac{1}{m-1}} (1 - u_{ij}) &= u_{ij} d(x_i, \theta_j)^{\frac{1}{m-1}} \\
1 - u_{ij} &= u_{ij} \left(\frac{d(x_i, \theta_j)}{\eta_j} \right)^{\frac{1}{m-1}} \\
1 &= u_{ij} \left(1 + \frac{d(x_i, \theta_j)}{\eta_j}^{\frac{1}{m-1}} \right) \\
\implies u_{ij} &= \frac{1}{1 + \left(\frac{d(x_i, \theta_j)}{\eta_j} \right)^{\frac{1}{m-1}}}
\end{aligned}$$

QED

Also this is my first LaTeX document!