ME493 Drone Control - Assignment 2

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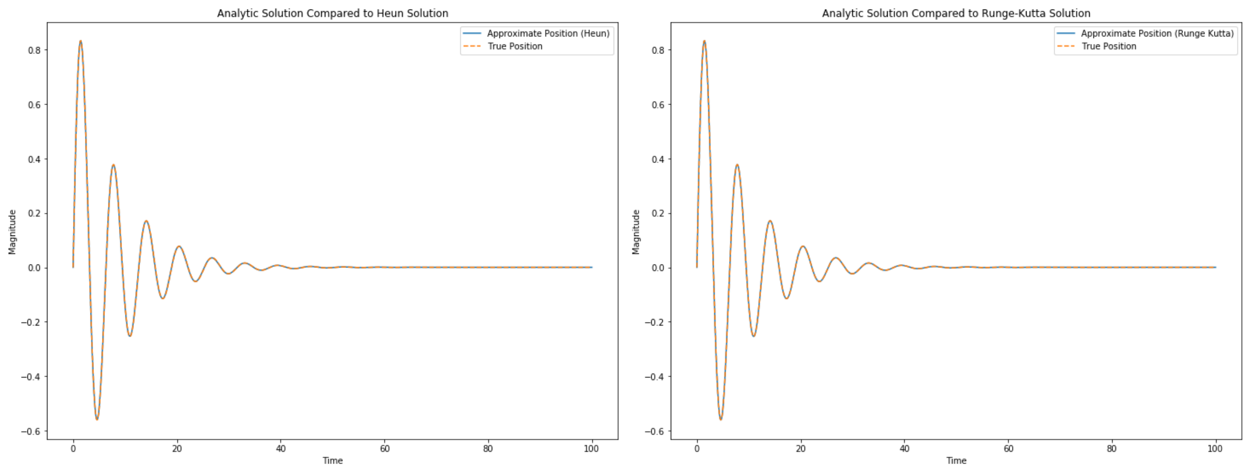
Exercise 1: Download Python files for numerical integration from MS Teams. For this question you will Implement a Runge-Kutta 4 integration routine.

(a) First, run the downloaded simulation and make sure it works. Done.

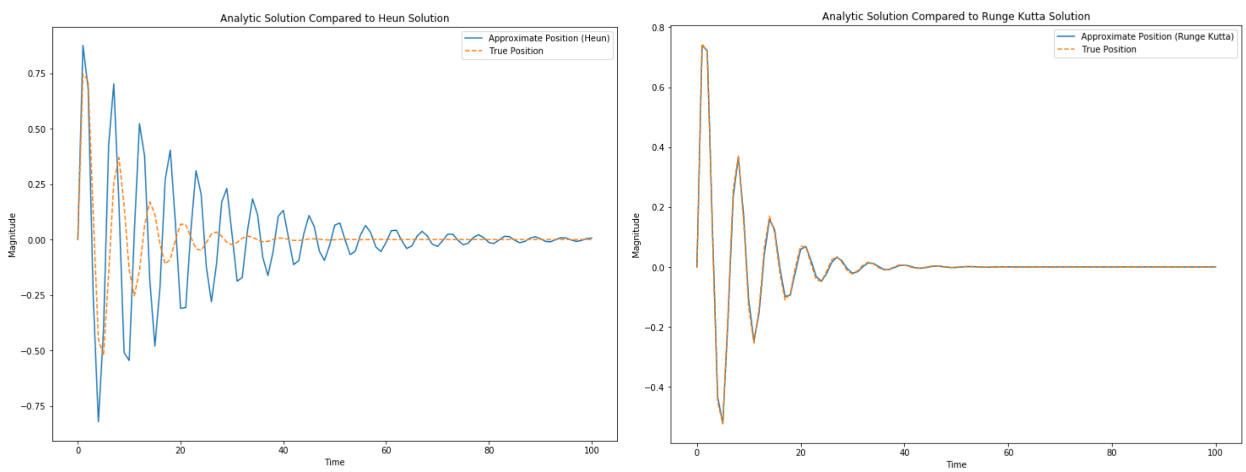
(b) Next, implement your version of the Runge Kutta 4 integrator in the file integrators.py.

```python
class Integrator:
    def __init__(self, dt, f):
        self.dt = dt
        self.f = f

    def step(self, t, x, u):
        raise NotImplementedError

class Euler(Integrator):
    def step(self, t, x, u):
        return x + self.dt * self.f(t, x, u)

class Heun(Integrator):
    def step(self, t, x, u):
        intg = Euler(self.dt, self.f)
        xe = intg.step(t, x, u) # Euler predictor step
        return x + 0.5*self.dt * (self.f(t, x, u) + self.f(t+self.dt, xe, u))

class RK4(Integrator):
    def step(self, t, x, u):
        k_1 = self.f(t, x, u)
        k_2 = self.f(t + self.dt/2, x + self.dt*k_1/2, u)
        k_3 = self.f(t + self.dt/2, x + self.dt*k_2/2, u)
        k_4 = self.f(t + self.dt, x + self.dt*k_3, u)
        return x + self.dt*(k_1 + 2*k_2 + 2*k_3 + k_4)/6
```

(c) Integrate the mass-spring system with both the Heun and Runge Kutta 4 method. Experiment with suitable step sizes dt. Compare the numerical solutions with the analytical solution. Attach your plots and describe your findings.



The plots above show the mass-spring system for 1000 time-steps with a step size of 0.1. In this case, both integrators work very well. In the plots below, 100 times steps are used with a step size of 1. In this case, Heun stops working, but Runge Kutta continues to work.



Exercise 2: Let the body-fixed frame of an aircraft be chosen such that it coincides with the principal axes, i.e. the mass moment of inertia matrix is diagonal.

(a) Write down the three equations that follow from application of Euler's second law in the body frame.

Given that

$$I = \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & J_3 \end{bmatrix}$$

$$\omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

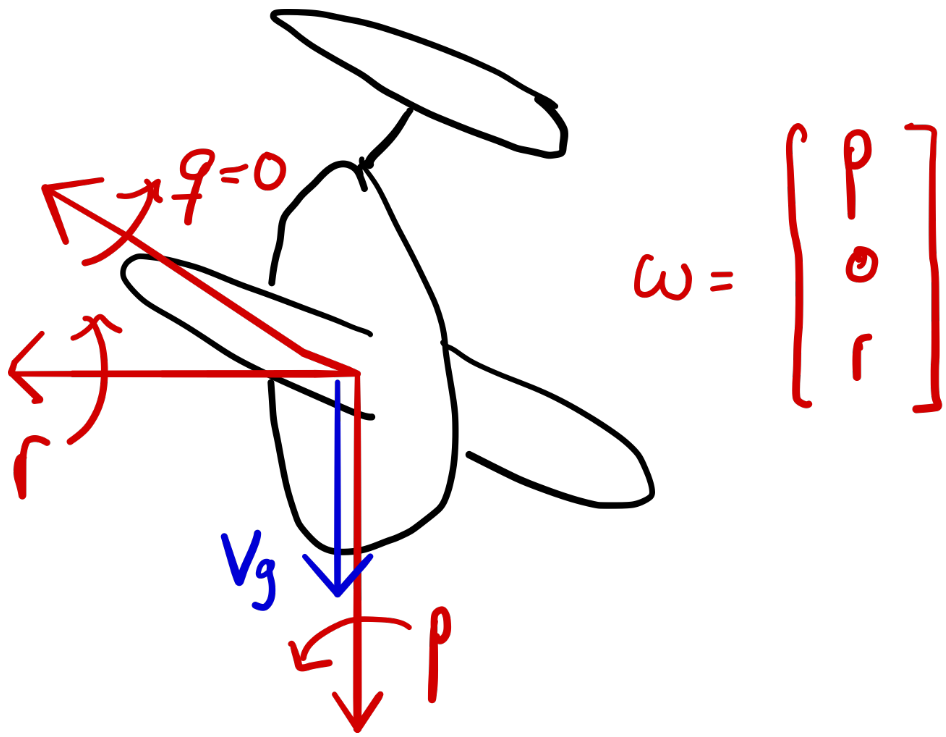
$$H = I\omega$$

$$M = \frac{dH}{dt} + \omega \times H$$

One gets the following moment vector components:

$$M_x = J_1\dot{p} + qr(J_3 - J_2)$$
$$M_y = J_2\dot{q} + pr(J_1 - J_3)$$
$$M_z = J_3\dot{r} + pq(J_2 - J_1)$$

(b) Sketch an aircraft with body frame, and indicate V_g, ω, p and r.



(c) Assume stationary vertical spin motion. What is needed to maintain the spin motion? Which part of the aircraft can provide this?

Spin motion is caused by a large yaw rate. This happens when one wing becomes more stalled than these other, which can be caused by the rudder. (Source: https://www.skybrary.aero/index.php/Spin)