Heat Transfer – Homework 2

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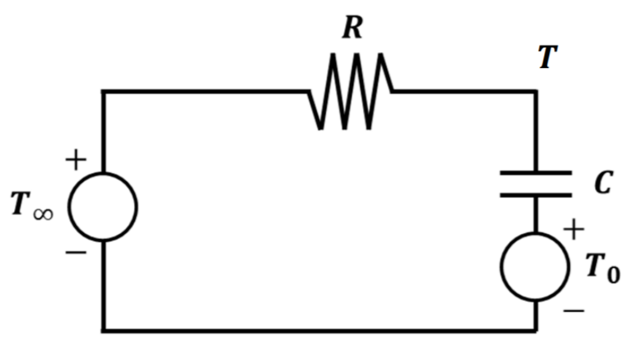
1 Base Case Model

Problem Statement: The circuit below models an iron block cooling in a room:



Consider the model with the values m = 200 kg, c = 0.45 kJ/kgK, R = °C/kW, T_∞ = 25°C, T_0 = 300°C. Using these values, plot temperature of the iron block and heat transfer as functions of time.

Results and Analysis:

$\mathrm{T_\infty - T_0 = Z_R\dot{q} + Z_C\dot{q}}$ (Mesh Analysis)

$\mathcal{L}\{\mathrm{T_\infty - T_0}\} = \mathcal{L}\{\mathrm{Z_R\dot{q} + Z_C\dot{q}}\}$

$$\frac{\mathrm{T_\infty - T_0}}{\mathrm{s}} = \mathrm{R\dot{Q}} + \frac{1}{\mathrm{sC}}\dot{\mathrm{Q}}\ (*)$$

$$\frac{\mathrm{T_\infty - T_0}}{\mathrm{R}} = \dot{\mathrm{q}}\ \text{(Ohm's Law)}$$

$$\mathcal{L}\{\frac{\mathrm{T_\infty - T_0}}{\mathrm{R}}\} = \mathcal{L}\{\dot{\mathrm{q}}\}$$

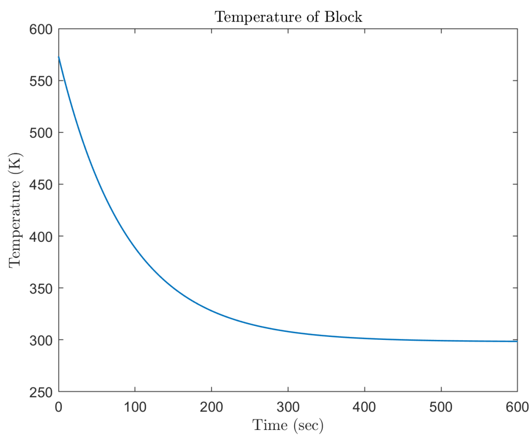
$$\frac{\mathrm{T_\infty}}{\mathrm{s}} - \mathrm{T} = \mathrm{R\dot{Q}}\ (**)$$

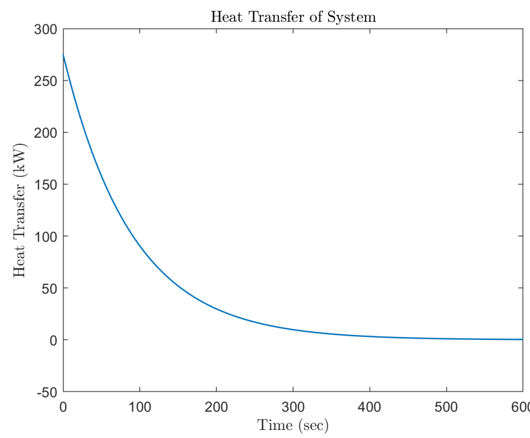
From (*), $\dot{\mathrm{Q}}\left(\mathrm{R} + \frac{1}{\mathrm{sC}}\right) = \frac{\mathrm{T_\infty - T_0}}{\mathrm{s}}$

From (**), $\frac{\mathrm{T_\infty}}{\mathrm{s}} - \mathrm{T} = \mathrm{R}\left(\frac{\frac{\mathrm{T_\infty}}{\mathrm{s}} - \frac{\mathrm{T_0}}{\mathrm{s}}}{\mathrm{R} + \frac{1}{\mathrm{sC}}}\right)$

$$\mathrm{T} = \frac{\mathrm{T_\infty}}{\mathrm{s}} - (\mathrm{T_\infty - T_0})\left(\frac{1}{\mathrm{s} - \frac{-1}{\mathrm{RC}}}\right)$$

(Inverse Laplace) $\mathrm{T(t) = T_\infty - (T_\infty - T_0)e^{\frac{-t}{RC}}}$

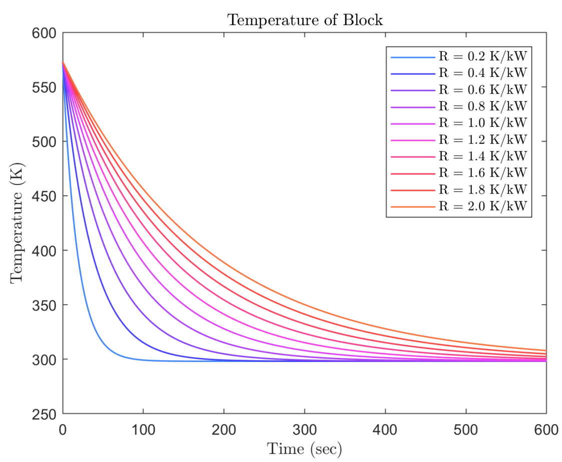


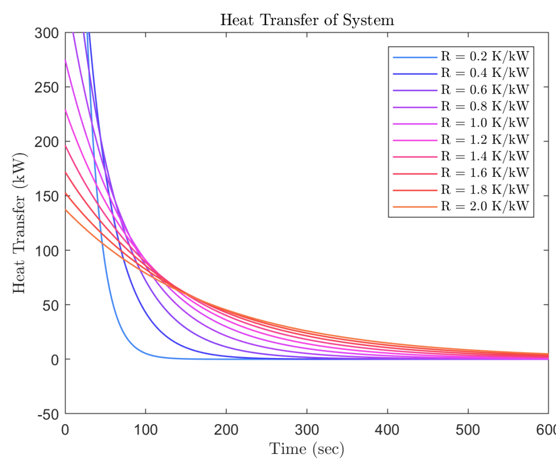


2 Resistance Variation

Problem Statement: Plot temperature and heat transfer as functions of time using several different values of R. Show each plot on one figure using the same axis limits used in the previous section. What is the effect of varying R? What does this mean physically?

Results and Analysis:





For smaller R values, the temperature of the block approaches its steady-state value quicker. For larger R values, it takes longer for the block to change temperature.

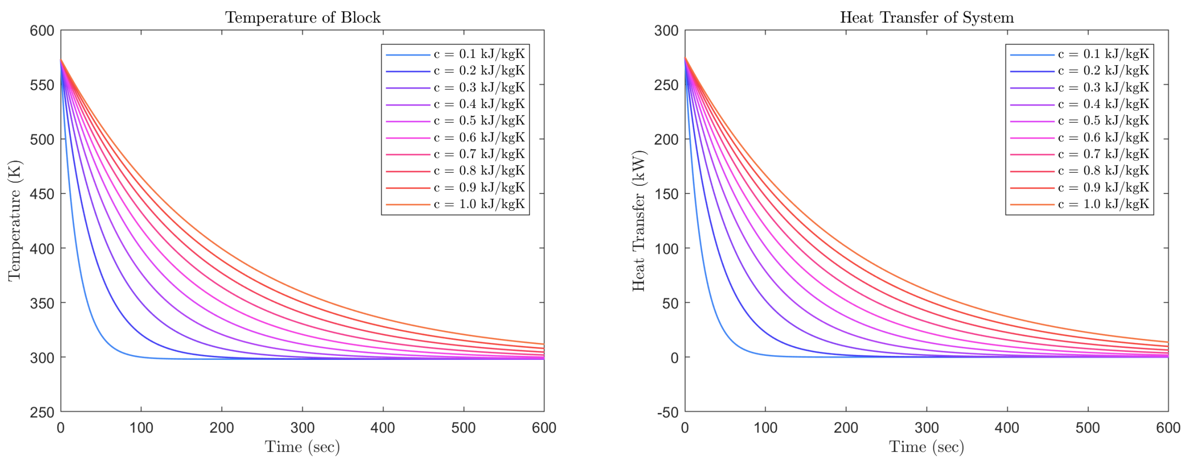
Likewise, for smaller R values, the initial rate of heat transfer is very high and it approaches zero relatively quickly. For larger R values, the initial rate of heat transfer is low and it takes longer to approach zero.

Regardless of the R value, the total heat transfer for an infinite amount of time will be the same, so the area under each plot will be the same. However, for lower R values, the heat transfer is happening very quickly early on and reaches an approximate steady-state very quickly. For higher R values, the heat transfer is slower but longer lasting. Physically, large R values seem to correspond to the block taking a longer time to change temperature, or, a larger resistance to heat transfer.

3 Capacitance Variation

Problem Statement: Plot temperature and heat transfer as functions of time using several different values of C. Show each plot on one figure using the same axis limits used in the first section. What is the effect of varying C? What does this mean physically?

Results and Analysis:



Note: The legend shows the c values used (specific heat capacity) rather that the C values used (heat capacity). To instead get heat capacity, one can multiply by the mass, 200 kg.

For small C values, the temperature of the block approaches its steady-state value quickly, while for large C values, the temperature of the block takes a long time to approach steady-state.

Similarly, for small C values the heat transfer rate approaches zero very quickly, while for large C values, it takes longer for the heat transfer rate to approach zero. However, unlike the heat transfer rates for varying values of R, regardless of the value of C the initial heat transfer rate stays the same.

Physically, C values seem to correspond to the block's ability to store thermal energy. A large C value indicates a greater ability to store thermal energy, while small C value indicates a smaller ability to store thermal energy.

4 Discussion

Final Results: In the model used, the following behavior can be observed:

- The temperature starts at T_0 (573.15 K or 300 °C) and approaches T_∞ as time goes to infinity.
- The heat transfer is initially relatively large and decreases to zero as time goes to infinity.
- As the resistance increases, it takes longer for the block to reach steady-state temperature.
- As the resistance increases, the initial rate of heat transfer starts slower and takes longer to decay.
- As the heat capacity increases, it takes longer for the block to reach steady-state temperature.
- As the heat capacity increases, the initial rate of heat transfer stays the same, but it takes longer for the rate to decay.

Concluding Statements: To model the temperature of an iron block cooling in a room, an analogous circuit was created and analyzed in the time domain. In this circuit, one power source models a constant ambient temperature, and another power source models an initial temperature. The capacitor models the heat capacity of the iron block. Initially, the capacitor is uncharged, so the temperature of the block is T_0. As time increases, the capacitor charges up, creating a temperature differential across the capacitor. The resistor models the resistance to heat transfer from the iron block to the atmosphere. From this model and basic circuit analysis, the behavior of the system over time can be predicted and the change in behavior due to varying parameters of the system can be predicted.

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```
clear all; close all; clc;
```

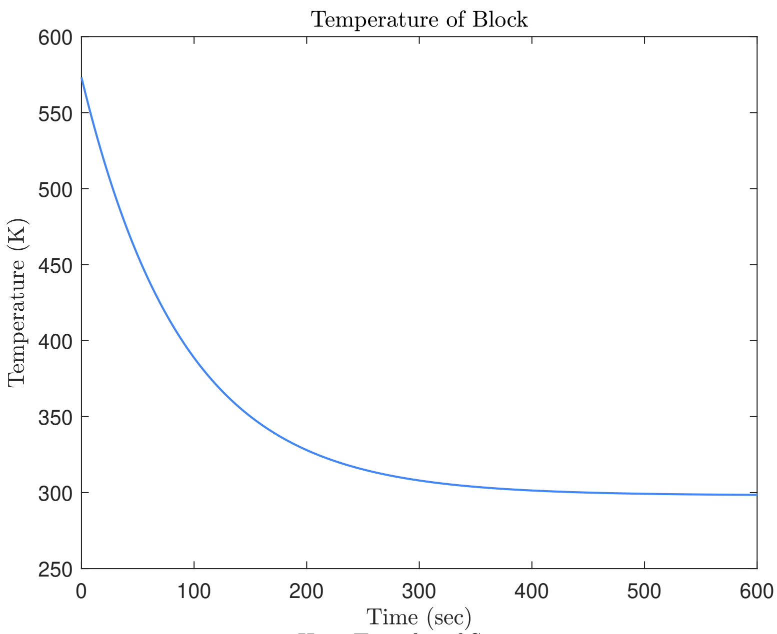
Define parameters used

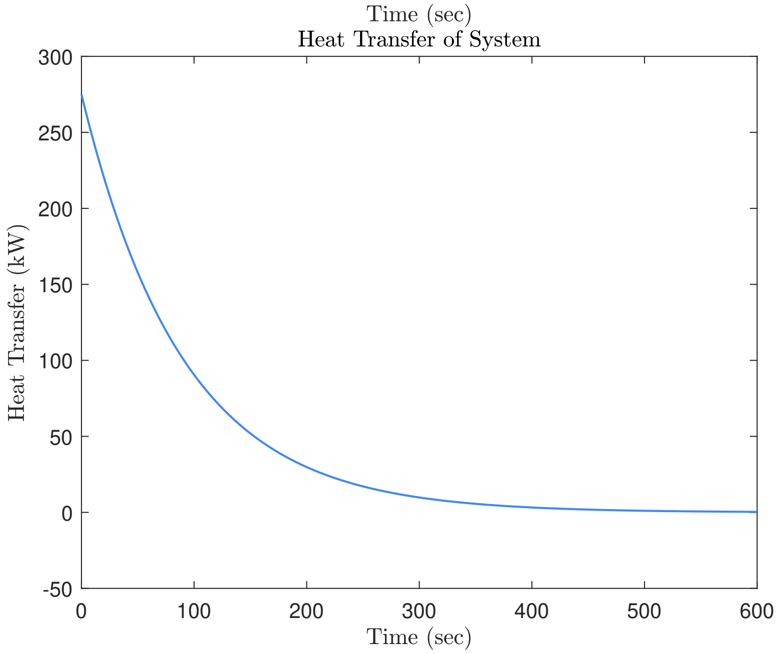
```
% Define system
m = 200; % kg
c = 0.45; % kJ/kgK
R = 1; % K/kW
C = m*c; % kJ/K
T_inf = 25 + 273.15; % K
T_0 = 300 + 273.15; % K

% Vectors of varying time, resistance, and capacitance
t = linspace(0,600,1000);

% Color
chromatic = [66, 135, 245; 69, 66, 245; 138, 66, 245; 176, 66, 245; 221, 66, 245;...
    245, 66, 227; 245, 66, 144; 245, 66, 93; 245, 75, 66; 245, 117, 66]/255;
set(groot,'defaultAxesColorOrder',chromatic)
```

1. Plot System in Time Domain

```
% Define temperature as a function of time
T = T_inf - (T_inf-T_0)*exp((-1*t)/(R*C));

% Define heat transfer as a function of time
Q_dot = (1/R)*(T - T_inf);

% Plot temperature
figure(1)
plot(t,T,'LineWidth',1)
title('Temperature of Block','interpreter','latex')
xlabel('Time (sec)','interpreter','latex')
ylabel('Temperature (K)','interpreter','latex')
```

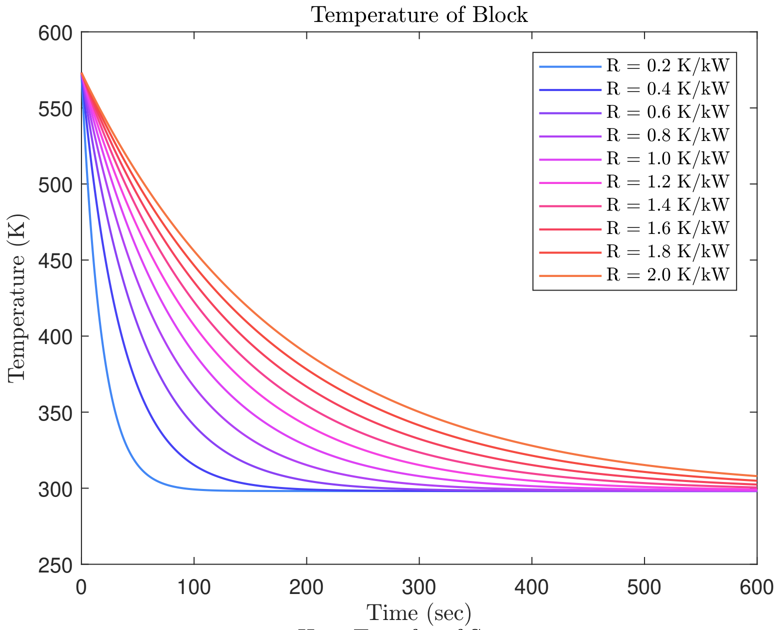
```
ylim([250,600]);

% Plot heat transfer
figure(2)
plot(t,Q_dot,'LineWidth',1)
title('Heat Transfer of System','interpreter','latex')
xlabel('Time (sec)','interpreter','latex')
ylabel('Heat Transfer (kW)','interpreter','latex')
ylim([-50,300]);
```

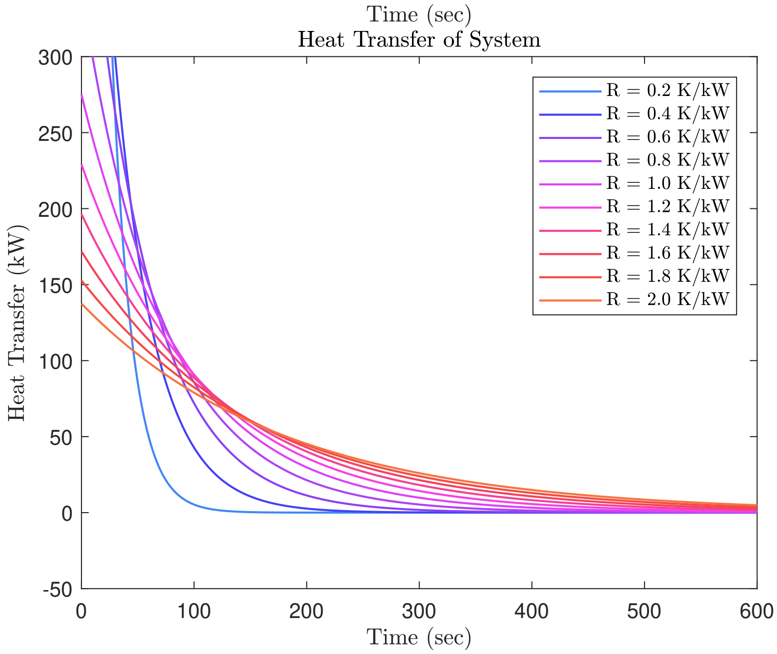




2. Vary resistance values

```
% Get temperature and heat transfer behavior for several resistances
T_R = zeros(length(t),10);
Q_R = zeros(length(t),10);
for index = 1:10
    T_R(:,index) = T_inf - (T_inf-T_0)*exp((-1*t)/(index*0.2*C));
    Q_R(:,index) = (1/(index*0.2))*(T_R(:,index) - T_inf);
end

% Plot temperature for several resistances
figure(3)
plot(t,T_R,'LineWidth',1)
title('Temperature of Block','interpreter','latex')
xlabel('Time (sec)','interpreter','latex')
ylabel('Temperature (K)','interpreter','latex')
legendTR = legend('R = 0.2 K/kW','R = 0.4 K/kW','R = 0.6 K/kW','R = 0.8 K/kW','R = 1.0 K/kW'...
    ,'R = 1.2 K/kW','R = 1.4 K/kW','R = 1.6 K/kW','R = 1.8 K/kW','R = 2.0 K/kW');
set(legendTR,'Interpreter','latex');
ylim([250,600]);

% Plot heat transfer for several resistances
figure(4)
plot(t,Q_R,'LineWidth',1)
title('Heat Transfer of System','interpreter','latex')
xlabel('Time (sec)','interpreter','latex')
ylabel('Heat Transfer (kW)','interpreter','latex')
legendQR = legend('R = 0.2 K/kW','R = 0.4 K/kW','R = 0.6 K/kW','R = 0.8 K/kW','R = 1.0 K/kW'...
    ,'R = 1.2 K/kW','R = 1.4 K/kW','R = 1.6 K/kW','R = 1.8 K/kW','R = 2.0 K/kW');
set(legendQR,'Interpreter','latex');
ylim([-50,300]);
```

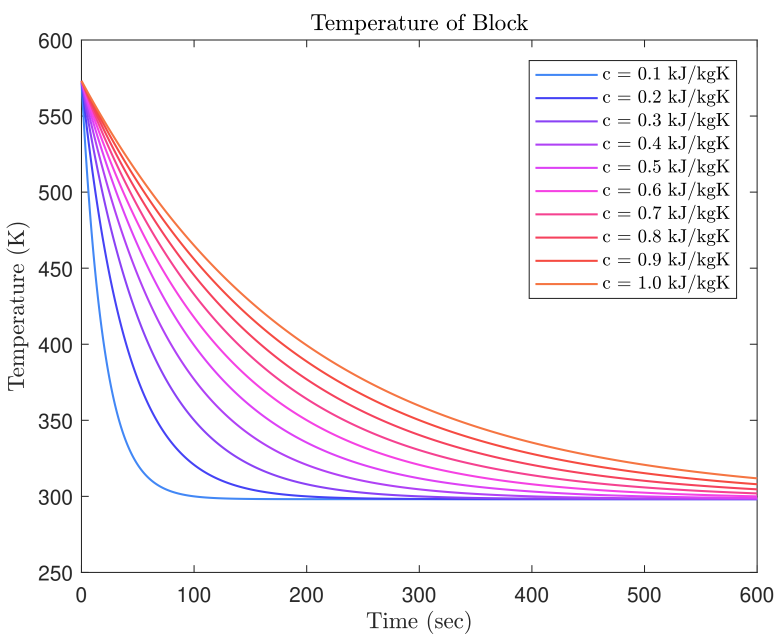


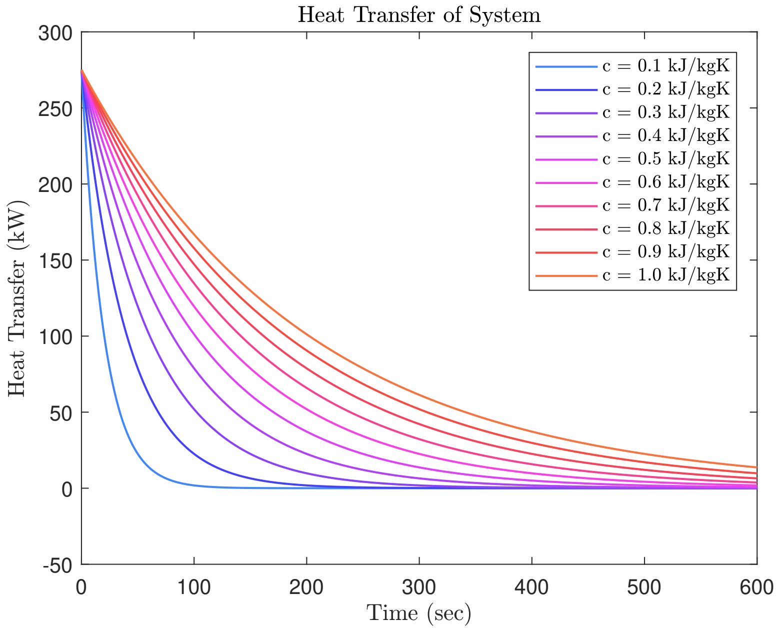


3. Vary capacitance values

```
% Get temperature and heat transfer behavior for several capacitances
T_C = zeros(length(t),10);
Q_C = zeros(length(t),10);
for index = 1:10
    T_C(:,index) = T_inf - (T_inf-T_0)*exp((-1*t)/(R*(index*20)));
    Q_C(:,index) = (1/R)*(T_C(:,index) - T_inf);
end

% Plot temperature for several resistances
```

```
figure(5)
plot(t,T_C,'LineWidth',1)
title('Temperature of Block','interpreter','latex')
xlabel('Time (sec)','interpreter','latex')
ylabel('Temperature (K)','interpreter','latex')
legendTC = legend('c = 0.1 kJ/kgK','c = 0.2 kJ/kgK','c = 0.3 kJ/kgK','c = 0.4 kJ/kgK','c = 0.5 kJ/kgK'.
    ,'c = 0.6 kJ/kgK','c = 0.7 kJ/kgK','c = 0.8 kJ/kgK','c = 0.9 kJ/kgK','c = 1.0 kJ/kgK');
set(legendTC,'Interpreter','latex');
ylim([250,600]);

% Plot heat transfer for several resistances
figure(6)
plot(t,Q_C,'LineWidth',1)
title('Heat Transfer of System','interpreter','latex')
xlabel('Time (sec)','interpreter','latex')
ylabel('Heat Transfer (kW)','interpreter','latex')
legendQC = legend('c = 0.1 kJ/kgK','c = 0.2 kJ/kgK','c = 0.3 kJ/kgK','c = 0.4 kJ/kgK','c = 0.5 kJ/kgK'.
    ,'c = 0.6 kJ/kgK','c = 0.7 kJ/kgK','c = 0.8 kJ/kgK','c = 0.9 kJ/kgK','c = 1.0 kJ/kgK');
set(legendQC,'Interpreter','latex');
ylim([-50,300]);
```




Heat Transfer of System
c = 0.1 kJ/kgK
c = 0.2 kJ/kgK
c = 0.3 kJ/kgK
c = 0.4 kJ/kgK
c = 0.5 kJ/kgK
c = 0.6 kJ/kgK
c = 0.7 kJ/kgK
c = 0.8 kJ/kgK
c = 0.9 kJ/kgK
c = 1.0 kJ/kgK
Heat Transfer (kW)
Time (sec)
300
250
200
150
100
50
0
-50
0
100
200
300
400
500
600