Heat Transfer - Homework 4

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1 Critical Radius of Insulation

Problem Statement: The outer surface of a 100m long electric wire with radius, $r_{wire} = 5$mm should not exceed 90 °C. The wire is exposed to air at $T_\infty = 0$°C with an effective convection coefficient of 15 W/m^{2}K.

(a) Calculate the steady-state rate of heat loss for a bare wire.

The heat transfer from the surface of the wire to the surrounding air is purely convective. So, Newton's Law of Cooling, $\dot{q} = hA(T_{fluid} - T_{surf})$, governs the heat transfer rate.

$$\dot{q} = hA(T_{fluid} - T_{surf})$$
$$\dot{q} = \left(15\frac{W}{m^2K}\right)(2\pi(0.005m)(100m))(90K)$$
$$\dot{q} = 4241.15W$$

(b) Next, a layer of insulation with thermal conductivity 0.15 W/mK is to be added. What insulation thickness, Δx, will maximize the heat transfer rate between the pipe and ambient atmosphere? What is the corresponding heat transfer rate and outer surface temperature of the insulation?

The heat transfer from the surface of the wire to the ambient air temperature is both conductive and convective. Convection occurs from the surface of the wire to the surface of the insulated material and convection occurs from the surface of the insulated material to the air. This can be modelled by the circuit diagram below:

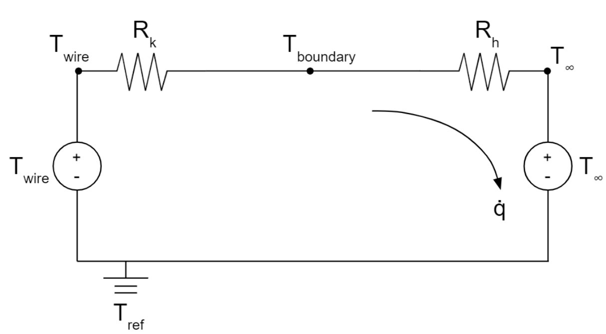

Figure 1: Circuit Model of Insulated Wire

In steady-state, the heat transfer rate due to convection and conduction are the same, so the resistances are connected in series and the equivalent resistance is the sum of the two. The heat transfer rate is inversely proportional to the equivalent resistance, so one can maximize the heat transfer rate with respect to the thickness of insulating material, Δx, by minimizing the equivalent resistance with respect to Δx.

$$R_{eq} = R_k + R_h = \frac{\Delta x}{kA} + \frac{1}{hA}$$

The area that conduction occurs over changes as the distance from the wire increases, so solving for R_k requires integration.

$$R_k(x) = \frac{\mathrm{d}x}{kA(x)} = \frac{\mathrm{d}x}{k(2\pi x L)}$$

$$R_k = \frac{1}{2\pi Lk} \int_{r_{wall}}^{r_{wall}+\Delta x} \frac{\mathrm{d}x}{x}$$

$$R_k = \frac{1}{2\pi Lk} \ln\left(\frac{r_{wall} + \Delta x}{r_{wall}}\right)$$

$$R_{eq} = R_k + R_h = \frac{1}{2\pi Lk} \ln\left(\frac{r_{wall} + \Delta x}{r_{wall}}\right) + \frac{1}{2\pi Lh(r_{wall} + \Delta x)}$$

$$\frac{\mathrm{d}R_{eq}}{\mathrm{d}x} = \frac{1}{2\pi Lk}\left(\frac{1}{r_{wall} + \Delta x}\right) - \frac{1}{2\pi Lh}\left(\frac{1}{(r_{wall} + \Delta x)^2}\right)$$

$$\frac{\mathrm{d}R_{eq}}{\mathrm{d}x} = \frac{1}{2\pi L}\left(\frac{h(r_{wall} + \Delta x) - k}{kh(r_{wall} + \Delta x)^2}\right)$$

$$\left(\frac{\mathrm{d}Req}{\mathrm{d}x} = 0\right) \iff \left(\Delta x = \frac{k}{h} - r_{wall}\right)$$

So the Δx which maximizes the heat transfer rate is $\frac{k}{h} - r_{wall} = 0.005\text{m}$. The corresponding R_{eq} is

$$R_{eq} = \frac{1}{2\pi kL}\left[\ln\left(\frac{1}{kh} + 1\right)\right] = 0.018\frac{K}{W},$$

Which makes the maximum heat transfer rate,

$$\dot{q} = \frac{T_\infty - T_{wire}}{R_k + R_h}$$

$$\dot{q}_{max} = \frac{90K}{0.018\frac{K}{W}} = 5009.78W$$

And the temperature at the boundary is,

$$T_{boundary} = T_\infty + \dot{q}_{max}R_{h,max} = \frac{5009.78W}{2\pi(100m)(0.15\frac{W}{mK})} = 53.16°C$$

(c) Why does the insulation increase the heat transfer rate instead of decreasing it?

The insulation adds a conductive resistance to the equivalent resistance, but it also decreases convective resistance by increasing the surface area over which convection occurs. When $\Delta x = 0.005$ m, the decrease in convective resistance is greater than the increase in conductive resistance, so the heat transfer rate still increases.

(d) What happens when the surrounding material is replaced with a better insulator?

When $\Delta x_{max} = \dfrac{k}{h} - r_{wall} < 0$, then there is no value of Δx that will make the decrease in convective resistance greater than the increase in conductive resistance. This happens when $k < 0.075\frac{W}{mK}$, so if the surrounding material is a better insulator, it will always reduce the heat transfer rate out of the wire.

(e) What happens when the surrounding fluid is replaced with a liquid?

The convection coefficient for a liquid is greater than that of a gas, so the convective resistance decreases and the heat transfer rate is greater.

Final Results

- The steady-state rate of heat loss for a bare wire is **4241.15 W**.
- The insulation thickness that maximizes the heat loss rate is **0.005 m**.
- The maximum steady-state heat transfer rate of an insulated wire is **5009.78 kW**.
- The corresponding outer temperature of the insulating material is **53.16 °C**.
- **525 K (251.85 °C)**.
- A poor insulating material (k > 0.075 W/mK) can **increase the heat loss rate by increasing the area over which convection occurs**.
- A good insulating material (k < 0.075 W/mK) will **always decrease the heat loss rate**.
- Replacing the surrounding fluid with a liquid will **increase the heat transfer rate**.

Concluding Statements This problem shows which factors impact the heat transfer rate for convective and conductive heat transfer. Factors that were examined specifically were how composing conduction and convection affect the heat loss rate, how changing the material properties and geometry of an object affects conductive heat transfer, and how changing fluid properties affects convective heat transfer.

2 Hovering Hot Air Balloon

Problem Statement: A spherical hot air balloon with diameter $d = 10m$ weighs $m = 130kg$, including the weight of the gondola and the passenger. How much fuel, in $\frac{kJ}{hr}$, must be consumed if the balloon is to hover at low altitude in still $27°C$ air? Assume that the convection coefficient for the inside of the balloon is $h_i = 126\frac{W}{m^2K}$ and outside of the balloon is $h_o = 215\frac{W}{m^2K}$.

Assumptions:

(1) Assume that the mass of the air inside of the balloon is not included in the given $m = 130kg$.

(2) Assume that the air acts as an ideal gas.

(3) Assume that conduction through the thickness of the balloon is negligible.

Solution: To begin solving the heat transfer problem, the temperature of the air inside of the balloon, T_i, must be found. To do this, the static equilibrium equation is setup and solved to find the density of the air inside of the balloon.

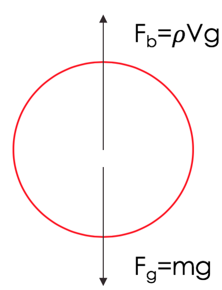

Figure 2: Free Body Diagram of the Hot Air Balloon

$$\sum F = 0 = F_b - mg$$

$$\rho Vg - (m_{balloon} + m_{air})g = 0$$

$$\rho V = m_{balloon} + m_{air}$$

$$m_{air} = \rho\frac{4}{3}\pi r_{balloon}^3 - m_{balloon}$$

$$m_{air} = (1.117\frac{kg}{m^3})(\frac{4}{3}\pi 5^3) - 130kg$$

$$m_{air} = 486.28kg$$

$$\rho_i = m_i/V_i = \frac{486.28kg}{\frac{4}{3}\pi 5^3} = 0.93\frac{kg}{m^3}$$

Then, the temperature inside of the balloon can be found using the ideal gas law.

$$T_i = \frac{\rho_o}{\rho_i}T_o$$

$$T_i = \frac{1.177\frac{kg}{m^3}}{0.93\frac{kg}{m^3}}(300.15K)$$

$$T_i = 380.40K = 107.25°C$$

The heat transfer from the inside of the balloon to the outside air is convective. Convection occurs within the inside of the balloon, and from the surface of the balloon to the outside air. This can be modelled by the circuit diagram below, where the red circle represents the boundary between the balloon and the air.

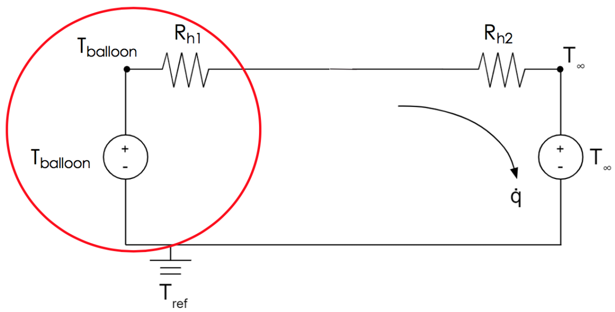

Figure 3: Circuit Model of the Hot Air Balloon

The two convective resistors are in series, and an equivalent resistance can be found by taking the sum of the two.

$$R_{eq} = R_h1 + R_h2 = \frac{1}{Ah_1} + \frac{1}{Ah_2}$$

$$R_{eq} = \frac{1}{A}(\frac{1}{h_1} + \frac{1}{h_2})$$

The heat transfer rate, $\dot{q}$, can then be found using the relationship

$$\dot{q} = \frac{T_o - T_i}{R_{eq}}$$

$$\dot{q} = \frac{A(T_o - T_i)}{\frac{1}{h_o} + \frac{1}{h_i}}$$

$$\dot{q} = \frac{4\pi(5m)^2(27°C - 107.25°C)}{\frac{1}{126\frac{W}{m^2K}} + \frac{1}{215\frac{W}{m^2K}}}$$

$$\dot{q} = 2002855.14W = 2002.86kW = 2002.86\frac{kJ}{s}$$

$$\dot{q} = 2002.86\frac{kJ}{s} \cdot \frac{60s}{min} \cdot \frac{60min}{hr} = 721029\frac{kJ}{hr}$$

Final Results

- The required fuel consumption to allow the balloon to hover at 25$°C$ is **721029 kJ/hr**

Concluding Statements This problem provides a real-life example of convective heat transfer. The problem also combines prior knowledge from statics and thermodynamics. It can be seen that, in colder environments, the required fuel consumption to continue hovering increases. It can also be seen that larger balloons require higher fuel consumption. These considerations are important when engineering, as the costs and benefits of designing a product a certain way have to be quantified.