Stress and Applied Elasticity - Lab Report

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1 Introduction

The goal of the project was to evaluate failure of a infinite plate with a hole of radius $r = a$. The plate has a distributed load S applied to each end.

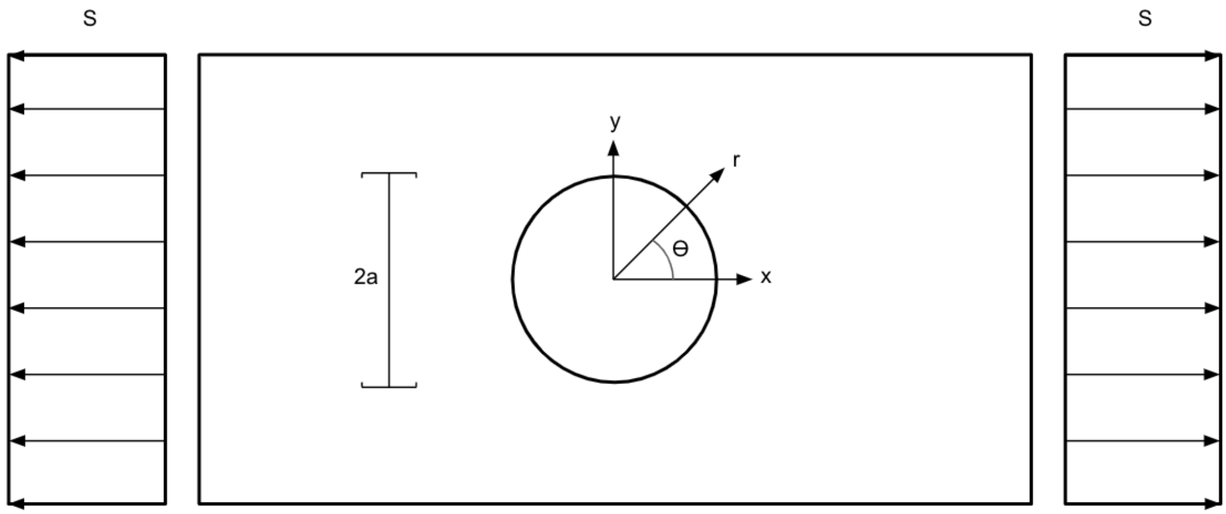

Figure 1: Infinite plate with hole of radius $r = a$

First, the equivalent stress, σ_e, normalized by S was plotted against $\frac{r}{a}$ for $1 \leq \frac{r}{a} \leq 4$ for $\theta = 0°, 30°, 60°, 90°$. Next, MATLAB was used to draw a contour plot of $\frac{\sigma_e}{S}$ for $-4 \leq \frac{x}{a} \leq 4$, $-4 \leq \frac{y}{a} \leq 4$, and $\frac{r}{a} \geq 1$. Finally, the maximum value of $\frac{\sigma_e}{S}$ was found, and this value was used to find the region of the plate where plastification will first occur.

2 Equations

This problem required the calculation of normal stresses and shear stresses in two-dimensional radial coordinates and the calculation of the von Mises yield criterion using these stresses. The equations were given in the textbook and in class.

Stress in the Radial Direction (Equation 3.55a, Edition 5):

$$\frac{\sigma_r}{S} = \frac{1}{2}\left(\left[1 - \left(\frac{r}{a}\right)^{-2}\right] + \left[1 + 3\left(\frac{r}{a}\right)^{-4} - 4\left(\frac{r}{a}\right)^{-2}\right]\cos 2\theta\right)$$

Stress in the Angular Direction (Equation 3.55b, Edition 5):

$$\frac{\sigma_\theta}{S} = \frac{1}{2}\left(\left[1 + \left(\frac{r}{a}\right)^{-2}\right] - \left[1 + 3\left(\frac{r}{a}\right)^{-4}\right]\cos 2\theta\right)$$

Shear Stress on the Plane (Equation 3.55c, Edition 5):

$$\frac{\tau_{r\theta}}{S} = -\frac{1}{2}\left[1 - 3\left(\frac{r}{a}\right)^{-4} + 2\left(\frac{r}{a}\right)^{-2}\right]\sin 2\theta$$

von Mises Yield Criterion in Two-Dimensional Cylindrical Coordinates:

$$|\sigma_e| < \sigma_{yield}$$

$$\sigma_e = \sqrt{\sigma_r^2 - \sigma_r\sigma_\theta + \sigma_\theta^2 + 3\tau_{r\theta}^2}$$

3 Results

3.1 σ_e versus $\frac{r}{a}$

The equivalent stress, σ_e, normalized by S was plotted against $\frac{r}{a}$ for $1 \leq \frac{r}{a} \leq 4$, $\theta = 0°, 30°, 60°, 90°$.

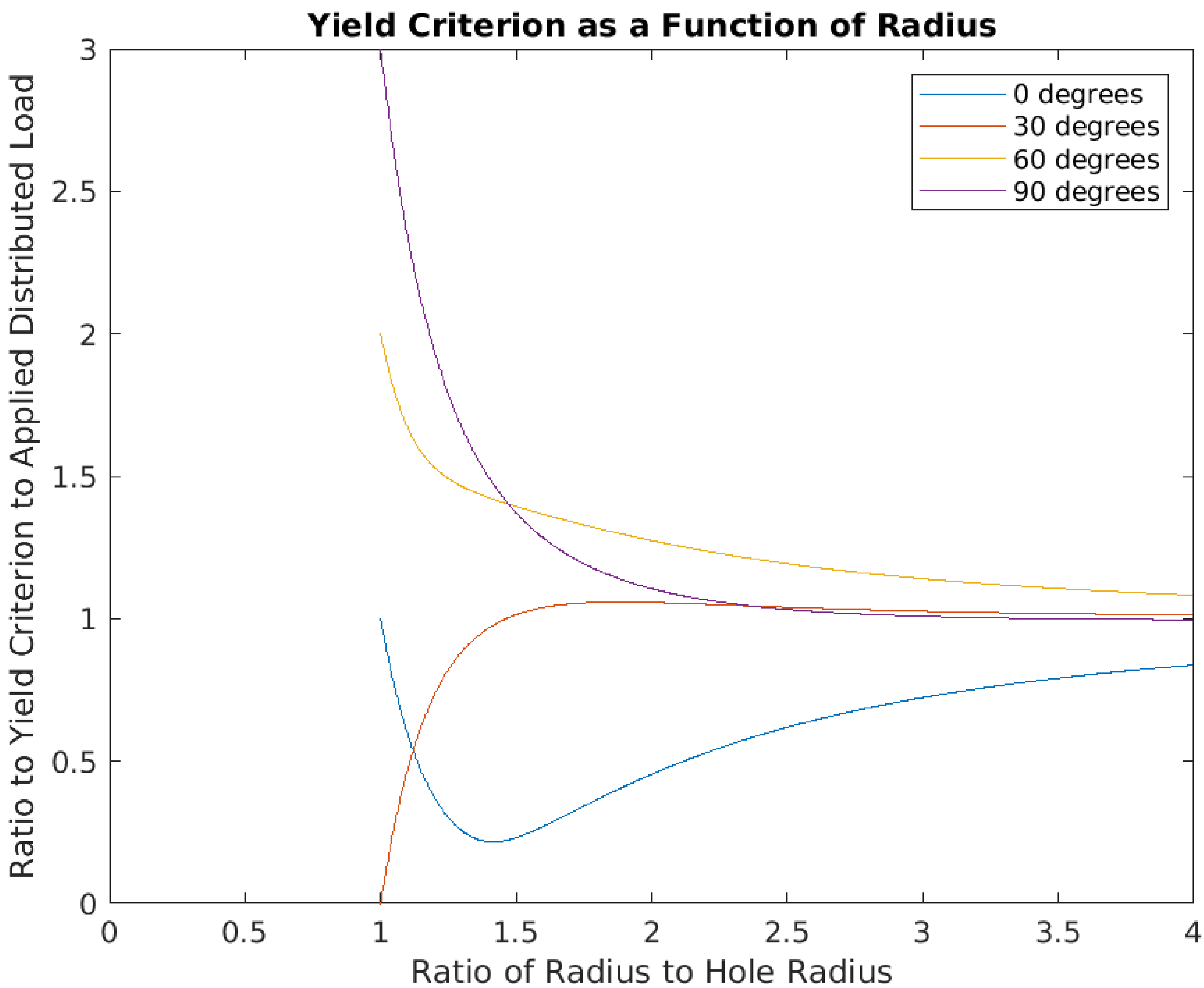

Figure 2: Yield Criterion as a Function of Radius

3.2 Contour plot of equivalent stress

A contour plot of the yield criterion, $\frac{\sigma_e}{S}$, was produced for $-4 \leq \frac{x}{a} \leq 4$, $-4 \leq \frac{y}{a} \leq 4$, and $\frac{r}{a} \geq 1$.

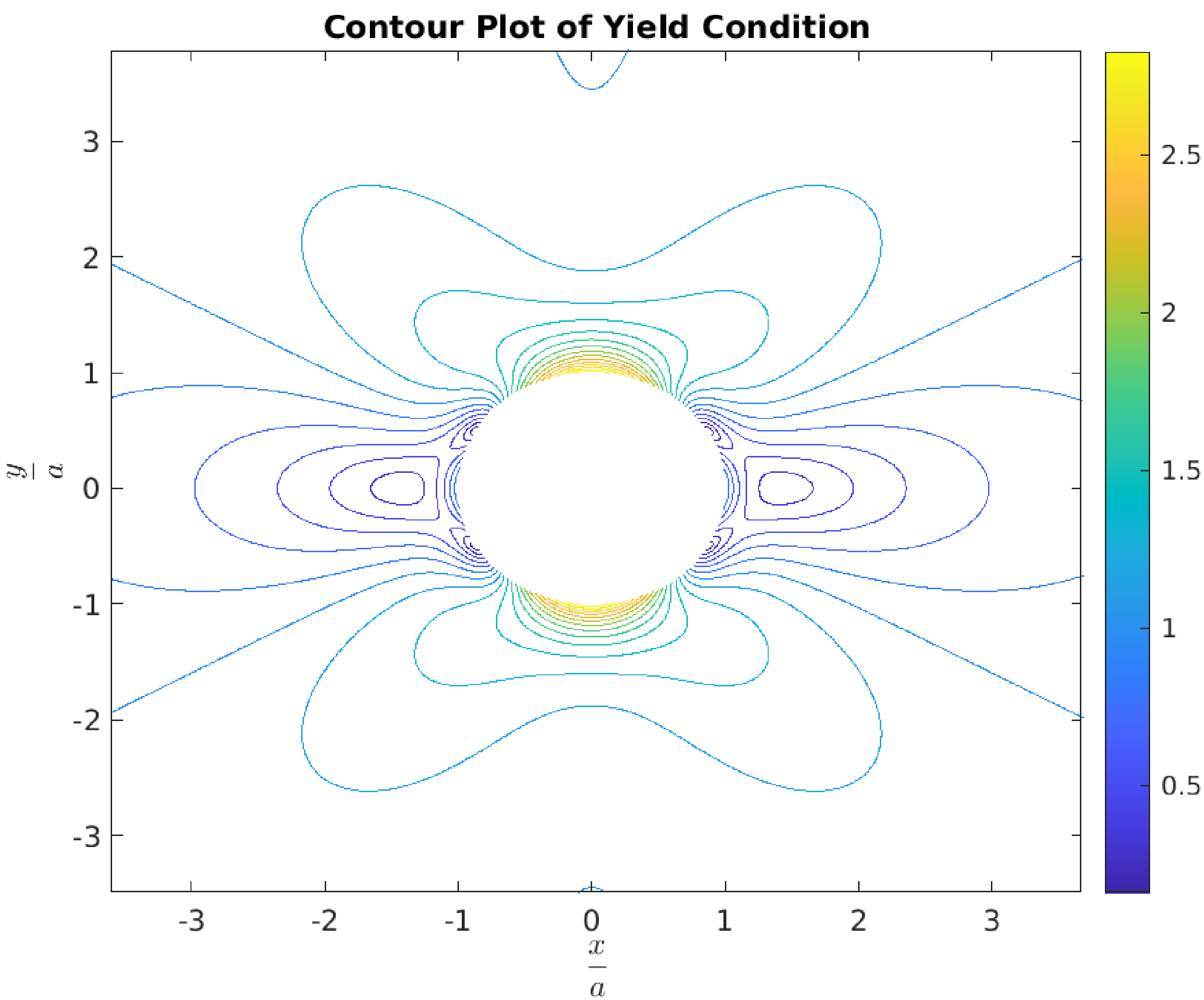

Figure 3: Contour plot of yield criterion

3.3 Critical region of the infinite plate

The yield condition was plotted against $\frac{x}{a}$ and $\frac{y}{a}$ for $-4 \le \frac{x}{a} \le 4$, $-4 \le \frac{y}{a} \le 4$, and $\frac{r}{a} \ge 1$.

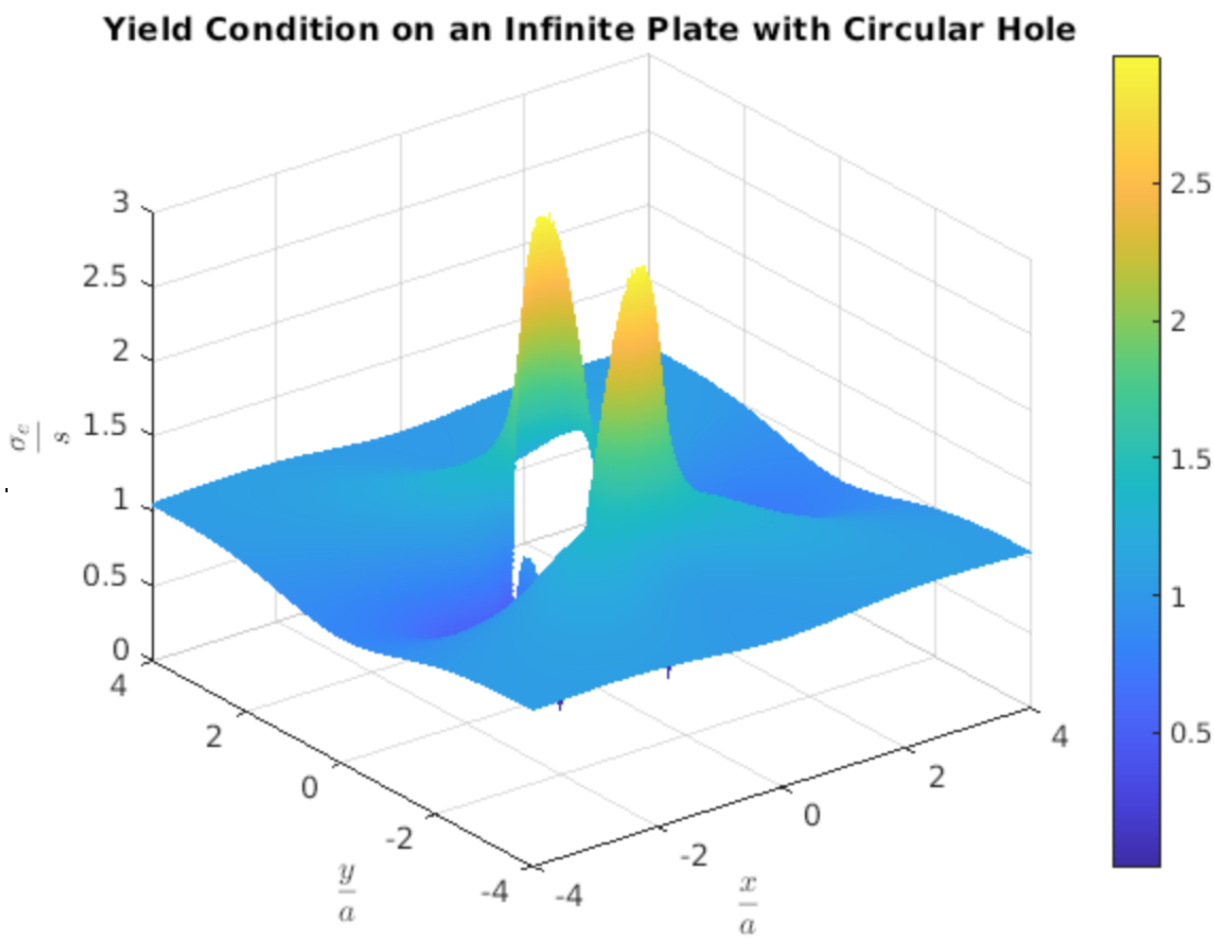

Figure 4: Yield condition vs $\frac{x}{a}$ and $\frac{y}{a}$

This gives a visual representation of the critical region, which is the region where plastification will first occur. Using MATLAB to obtain the maximum equivalent stress value in the given region, and the location of the maximum equivalent stress, it was found that the maximum equivalent stress is $\frac{\sigma_e}{s} = 2.9672$ at $\frac{x}{a} = -0.0841$, $\frac{y}{a} = -0.977$.

3.4 Discussion

It should be noted that the values for maximum equivalent stress and location were obtained by doing a coarse grid search over the region of interest, and that MATLAB returns only one maximum value. However, due to symmetry, there are actually two points of maximum equivalent stress. In reality, the maximum equivalent stress is $\frac{\sigma_e}{s} = 3$ at $\frac{x}{a} = 0$, $\frac{y}{a} = \pm 1$. These values can be obtained numerically by grid searching in the two symmetric regions and performing a fine grid search in the areas of interest, or they can be obtained analytically.