Identifiability of the Causal Effect of $Infection$ on $Death$

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1 Task

The qualitative knowledge of causal relationships in the domain is represented by a causal model shown in Fig. 1. The treatment variable is $Infection$ and the outcome variable is $Death$. We show that the causal effect $do(Infection = infection)$ on $Death$, written as $P_{Infection}(Death)$, is identifiable from a distribution over the observed variables $P(AHD, COPD, Death, Diabetes, HT, ICU, Infection)$.

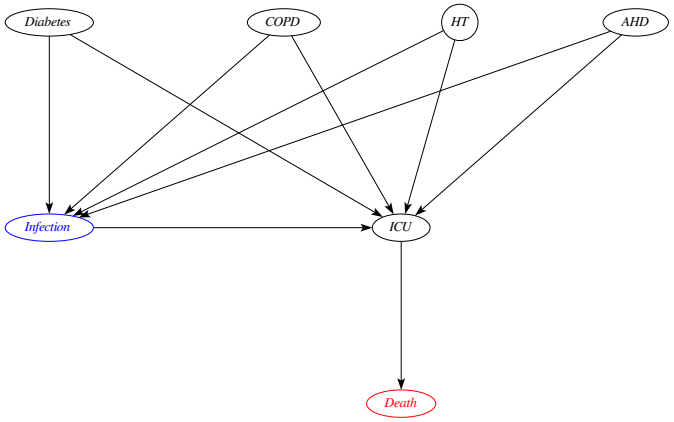

Figure 1: Causal Graph G. $Infection$ is the treatment variable, and $Death$ is the outcome variable.

2 Derivation

Theorem 1. *The causal effect of $Infection$ on $Death$ is identifiable from $P(AHD, COPD, Death, Diabetes, HT, ICU, In$ and is given by the formula*

$$P_{Infection}(Death) = \sum_{AHD,COPD,Diabetes,HT} P(Death|Infection, AHD, COPD, Diabetes, HT)\, P(AHD, COPD, Diabet$$

Proof.

$$P_{Infection}(Death) \tag{1}$$

$$= \sum_{AHD,COPD,Diabetes,HT,ICU} P_{Infection}(AHD, COPD, Death, Diabetes, HT, ICU) \tag{2}$$

$$= \sum_{AHD,COPD,Diabetes,HT,ICU} P_{COPD,Death,Diabetes,HT,ICU,Infection}(AHD)\, P_{AHD,Death,Diabetes,HT,ICU,Infection} \tag{3}$$

Eq. (2) follows from summing over $\{AHD, COPD, Diabetes, HT, ICU\}$ and Eq. (3) from C-component factorization.

Task 1: Compute $P_{COPD,Death,Diabetes,HT,ICU,Infection}\left(AHD\right)$

$$P_{COPD,Death,Diabetes,HT,ICU,Infection}\left(AHD\right) \tag{4}$$
$$= P(AHD) \tag{5}$$

Eq. (5) follows from the third rule of do-calculus with the independence ($COPD, Death, Diabetes, HT, ICU, Infection \perp$
(refer to Fig. 2).

Task 2: Compute $P_{AHD,Death,Diabetes,HT,ICU,Infection}\left(COPD\right)$

$$P_{AHD,Death,Diabetes,HT,ICU,Infection}\left(COPD\right) \tag{6}$$
$$= P(COPD) \tag{7}$$

Eq. (7) follows from the third rule of do-calculus with the independence ($AHD, Death, Diabetes, HT, ICU, Infection \perp C$
(refer to Fig. 2).

Task 3: Compute $P_{AHD,COPD,Death,HT,ICU,Infection}\left(Diabetes\right)$

$$P_{AHD,COPD,Death,HT,ICU,Infection}\left(Diabetes\right) \tag{8}$$
$$= P(Diabetes) \tag{9}$$

Eq. (9) follows from the third rule of do-calculus with the independence ($AHD, COPD, Death, HT, ICU, Infection \perp Dia$
(refer to Fig. 2).

Task 4: Compute $P_{AHD,COPD,Death,Diabetes,ICU,Infection}\left(HT\right)$

$$P_{AHD,COPD,Death,Diabetes,ICU,Infection}\left(HT\right) \tag{10}$$
$$= P(HT) \tag{11}$$

Eq. (11) follows from the third rule of do-calculus with the independence ($AHD, COPD, Death, Diabetes, ICU, Infection$
(refer to Fig. 2).

Task 5: Compute $P_{AHD,COPD,Death,Diabetes,HT,Infection}\left(ICU\right)$

$$P_{AHD,COPD,Death,Diabetes,HT,Infection}\left(ICU\right) \tag{12}$$
$$= P_{AHD,COPD,Diabetes,HT,Infection}\left(ICU\right) \tag{13}$$
$$= P(ICU|AHD, COPD, Diabetes, HT, Infection) \tag{14}$$

Eq. (13) follows from the third rule of do-calculus with the independence ($Death \perp ICU|AHD, COPD, Diabetes, HT, Inf$
(refer to Fig. 3). Eq. (14) follows from the second rule of do-calculus with the independence ($AHD, COPD, Diabetes, HT,$
(refer to Fig. 4).

Task 6: Compute $P_{AHD,COPD,Diabetes,HT,ICU,Infection}\left(Death\right)$

$$P_{AHD,COPD,Diabetes,HT,ICU,Infection}\left(Death\right) \tag{15}$$
$$= P(Death|AHD, COPD, Diabetes, HT, ICU, Infection) \tag{16}$$

Eq. (16) follows from the second rule of do-calculus with the independence ($AHD, COPD, Diabetes, HT, ICU, Infection$
(refer to Fig. 2).

Substituting Eq. (5), Eq. (7), Eq. (9), Eq. (11), Eq. (14), and Eq. (16) back into Eq. (3), we get

$$P_{Infection}\left(Death\right) = \sum_{AHD,COPD,Diabetes,HT} P\left(Death|Infection, AHD, COPD, Diabetes, HT\right) P\left(AHD, COPD, Diabet \tag{17}$$

□

3 Figures

The subgraphs used in the derivation of the causal effect of $Infection$ on $Death$ are as follows:

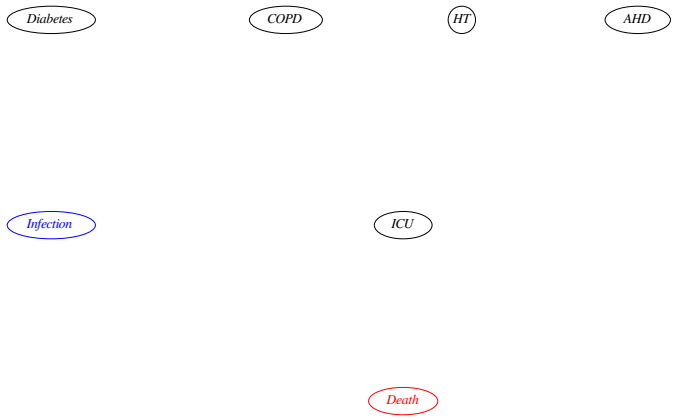

Figure 2: Causal Graph $G_{\overline{COPD,Death,Diabetes,HT,ICU,Infection}}$, $G_{\overline{AHD,Death,Diabetes,HT,ICU,Infection}}$, $G_{\overline{AHD,COPD,Death,H}}$

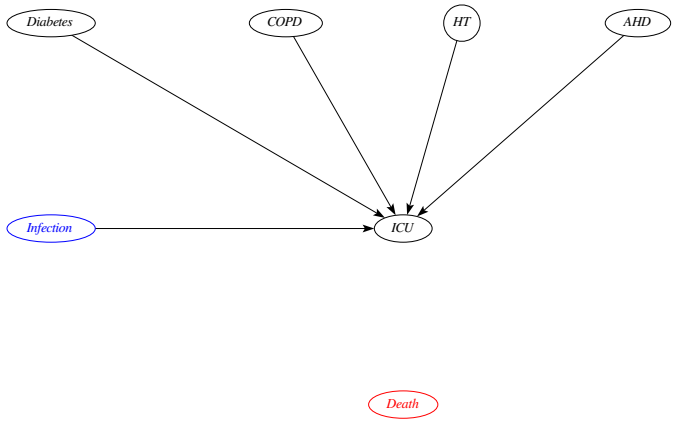

Figure 3: Causal Graph $G_{\overline{AHD,COPD,Death,Diabetes,HT,Infection}}$.

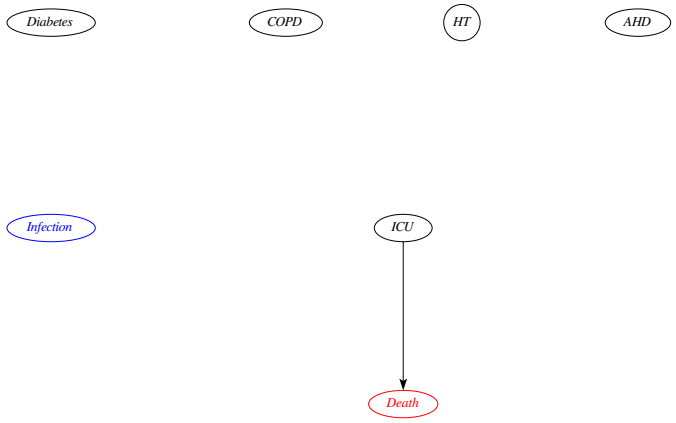

Figure 4: Causal Graph $G_{\underline{AHD,COPD,Diabetes,HT,Infection}}$.