

1.)

Sam plays baseball or Paul plays baseball

$x = \text{Sam}, y = \text{Paul}, x \vee y$

Sam plays baseball or Ryan doesn't play baseball

$x = \text{Sam}, z = \text{Ryan}, x \vee \neg z$

Knowledge Base = $(x \vee y) \wedge (x \vee \neg z)$

Boolean Expression

x	y	z	$(x \vee y) \wedge (x \vee \neg z)$	$(x \wedge z)$	$(x \vee y \vee z)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	F	F	T
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	F

A. It doesn't entail anything. Sam and Ryan both play baseball is false, but knowledge base is true when $x = F, z = F, y = T$.

B. Yes. At least one among Sam, Paul and Ryan play baseball is true. Knowledge base is also true when $x = T, z = T, y = F$

2.

a. If Ana eats, Bret eats
 x y

$$x \rightarrow y$$

b. Charles eats and Derek doesn't eat
 x y

$$x \rightarrow \neg y$$

c. Bret doesn't eat
 x

a $\neg x$

d. If Derek doesn't eat at least one among
Ana, Earl and Fred eats
 x y z

$$\neg a \rightarrow (x \vee y \vee z)$$

c. If at least one of Charles and Gary eats,
Earl doesn't eat
 x y z

$$x \vee y \rightarrow \neg z$$

3. Using the premises in Q2, show that Fred eats

Q2 Given:

Ana = A

Derek = D

Gary = G

Bret = B

Fred = F

Charles = C

Earl = E

- $A \rightarrow B$. If Ana eats, Bret eats
- $C \rightarrow \neg D$. Charles eats and Derek doesn't eat
- $\neg B$. Bret doesn't eat
- $\neg D \rightarrow (A \wedge E \wedge F)$. If D doesn't eat, Among A, E, F eats
- $C \vee G \rightarrow \neg E$
 - If at least one of Charles and G eats, E doesn't eat.

1. Show Fred eats = F

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|---|-----------------------------|
| 1. $C \rightarrow \neg D$ | Given |
| 2. $\neg D$ | 1, Simplification |
| 3. $\neg D \rightarrow (A \wedge E \wedge F)$ | Given |
| 4. $A \vee E \vee F$ | 2, 3, Modus Ponens |
| 5. A $A \rightarrow B$ | Given |
| 6. $\neg B$ | Given |
| 7. $\neg A$ | 5, 6, Modus Tollens |
| 8. $E \vee F$ | 4, 7 Disjunctive Syllogism |
| 9. $C \vee G \rightarrow \neg E$ | Given |
| 10. $C \rightarrow \neg E$ | 9, Simplification |
| 11. C | 1, simplification |
| 12. $\neg E$ | 10, 11 Modus Ponens |
| 13. F | 8, 12 Disjunctive syllogism |