Here we provide more detailed information about (A) the Infinity Interview and (B-D) report additional analyses exploring the robustness of our results to various cutoff criteria.

Our materials, data, and analysis code are available in a repository on the Open Science Framework: <https://osf.io/z6ky3/>

Table of Contents

[A: Infinity Interview 2](#_Toc33616246)

[A1: Detailed Coding scheme 2](#_Toc33616247)

[A2: Infinity transcript for a child classified as having no knowledge of infinity 3](#_Toc33616248)

[A3: Infinity transcript for a child classified as having only Successor knowledge of infinity 4](#_Toc33616249)

[A4: Infinity transcript for a child classified as having only Endless knowledge of infinity 5](#_Toc33616250)

[A5: Infinity transcript for a child classified as having Full knowledge of infinity 6](#_Toc33616251)

[B: Analysis of Productivity and Next Number accuracy with all participants 7](#_Toc33616252)

[C: Analysis of Infinity knowledge with all participants 8](#_Toc33616253)

[C1: Regression analyses predicting Successor Knowledge of Infinity 8](#_Toc33616254)

[C2: Regression analyses predicting Endless Knowledge of Infinity 8](#_Toc33616255)

[C3: Regression analyses predicting Full Knowledge of Infinity 9](#_Toc33616256)

[D: Reanalysis using a more conservative Productivity Classification 13](#_Toc33616257)

[D1: Relationship of Productivity to Highest Count and age 14](#_Toc33616258)

[D2: Relationship of Productivity to Next Number performance 15](#_Toc33616259)

[D3: Relationship of Productivity to Infinity knowledge 16](#_Toc33616260)

# 

# A: Infinity Interview

We describe a detailed coding scheme for classifying responses to the Infinity Interview. We first provide coding rules (A1), followed by an example transcript for each of the four possible Infinity knowledge classifications (A2-A5). The Infinity Interview was described in the main text (section 2.2.3) with results presented in section 3.2.

## A1: Detailed Coding scheme

The Infinity Interview consisted of 6 questions presented in the same order:

1. *“What is the biggest number you can think about?”* If the child did not answer, the experimenter probed them by asking how high they could count.
2. *“Is that the biggest number there could ever be?”*
   1. If yes, move on.
   2. If no, “*Can you think of a bigger number? Is that the biggest number there could ever be?”* The experimenter repeated this exchange up to 4 times or until the child affirmed that they had produced the biggest number.
3. “*If I keep counting, will I ever get to the end of numbers, or do numbers go on forever? Why?*”
4. “*If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn’t add any more? Why / Why not?*”
5. “*You said the biggest number you know is X. Tell me, is it possible to add one to X, or is X the biggest number possible? Why?*” For this question, X was the largest number the child had stated in the entire testing session.
6. “*Could I keep adding one? Why / Why not?*”
   1. If yes, “*What would happen if I kept adding one?*”

We coded successor and endless knowledge of infinity separately, as two binary variables.

To have endless knowledge of infinity, participants had to respond that there was no biggest number (Q1, Q2, Q5) or have provided four responses to the request to provide a bigger number (Q2). Participants who claimed to have provided the biggest number at some point in the interview (Q1, Q2, Q5) or said that numbers end (Q3) were classified as not exhibiting Endless knowledge of infinity.

To have successor knowledge of infinity, participants had to respond that it was possible to add 1 to some big number (Q4 and Q5) or claim that it was possible to keep adding one to that big number (Q6).

## A2: Infinity transcript for a child classified as having no knowledge of infinity

|  |  |
| --- | --- |
| E = Experimenter  C = Child (Age: 4 years 1 month) | |
| E: | What is the biggest number you can think about? |
| C: | One hundred. |
| E: | Is that the biggest number there could ever be? |
| C: | Yes. |
| E: | If I keep counting, will I ever get to the end of numbers, or do numbers go on forever? |
| C: | End. |
| E: | Why? |
| C: | Long and get to one hundred. |
| E: | If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn’t add any more? |
| C: | No. |
| E: | Why? |
| C: | Already so big. |
| E: | So you said that the biggest number you know is one hundred. Is it possible to add 1 to one hundred, or is one hundred the biggest number possible? |
| C: | Yes. |
| E: | Why? |
| C: | There’s also a hundred seventy. |
| E: | Could I keep adding 1? |
| C: | No. |
| E: | Why? |
| C: | It already has one. |

## A3: Infinity transcript for a child classified as having only Successor knowledge of infinity

|  |  |
| --- | --- |
| E = Experimenter  C = Child (Age: 5 years 6 months) | |
| E: | What is the biggest number you can think about? |
| C: | Sixty one. |
| E: | Is that the biggest number there could ever be? |
| C: | No, one hundred fifty nine sixty. |
| E: | Is that the biggest number there could ever be? |
| C: | I don’t know. |
| E: | Well, can you think of a bigger number? |
| C: | Eighteen ninety sixty one. |
| E: | Is that the biggest number there could ever be? |
| C: | No. |
| E: | Can you think of a bigger number? |
| C: | No. |
| E: | If I keep counting, will I ever get to the end of numbers, or do numbers go on forever? |
| C: | Forever. |
| E: | Why? |
| C: | End. |
| E: | Why? |
| C: | actually, you won't (get to the end of numbers) my friend can count to 100 and after and I learnt to count to 100 but not really. |
| E: | Do you think she could count forever? |
| C: | I don’t know. |
| E: | If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn’t add any more? |
| C: | Yes. |
| E: | Why? |
| C: | It makes it bigger. |
| E: | So you said that the biggest number you know is eighteen ninety sixty one. Is it possible to add 1 to eighteen ninety sixty one, or is eighteen ninety sixty one the biggest number possible? |
| C: | Yes. |
| E: | Why? |
| C: | Eighty nine. |
| E: | Could I keep adding 1? |
| C: | Yes |
| E: | Why? |
| C: | It makes it bigger and bigger and bigger. |
| E: | What would happen if I kept adding 1? |
| C: | It will get too big. |
| E: | Do we have to stop or could we keep adding 1? |
| C: | Stop. |

## A4: Infinity transcript for a child classified as having only Endless knowledge of infinity

|  |  |
| --- | --- |
| E = Experimenter  C = Child (Age: 4 years 9 months) | |
| E: | What is the biggest number you can think about? |
| C: | One hundred. |
| E: | Is that the biggest number there could ever be? |
| C: | No, three hundred. |
| E: | Is that the biggest number there could ever be? |
| C: | No, four hundred. |
| E: | Is that the biggest number there could ever be? |
| C: | Yes. |
| E: | If I keep counting, will I ever get to the end of numbers, or do numbers go on forever? |
| C: | Forever. |
| E: | Why? |
| C: | There’s a lot of numbers and they never end. |
| E: | If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn’t add any more? |
| C: | Couldn’t add more. |
| E: | Why? |
| C: | Because we couldn’t add nine hundred to it. |
| E: | So you said that the biggest number you know is four hundred. Is it possible to add 1 to four hundred, or is four hundred the biggest number possible? |
| C: | No. |
| E: | Why? |
| C: | Take a long time. |
| E: | Could I keep adding 1? |
| C: | I don’t know. |

## A5: Infinity transcript for a child classified as having Full knowledge of infinity

|  |  |
| --- | --- |
| E = Experimenter  C = Child (Age: 5 years 10 months) | |
| E: | What is the biggest number you can think about? |
| C: | A Jillion. |
| E: | Is that the biggest number there could ever be? |
| C: | No, infinity. |
| E: | If I keep counting, will I ever get to the end of numbers, or do numbers go on forever? |
| C: | Forever. |
| E: | Why? |
| C: | There’s lots of numbers in the world. |
| E: | If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn’t add any more? |
| C: | Yes. |
| E: | Why? |
| C: | Every number has a partner. Like 5 goes with 10s, twenties goes thirties, forties goes fifties, and sixties goes seventies, eighty goes ninety, one hundred goes one hundred and one. |
| E: | So you said that the biggest number you know is infinity. Is it possible to add 1 to infinity, or is infinity the biggest number possible? |
| C: | Yes. |
| E: | Why? |
| C: | I can if I want to. |
| E: | Could I keep adding 1? |
| C: | Yes. |
| E: | What would happen if I kept adding 1? |
| C: | I’d get to infinity, it’ll be a million and one. |

# B: Analysis of Productivity and Next Number accuracy with all participants

This analysis is referenced in Section 3.1.2 of the main text (Footnote 6 and 7). In the main text, we reported analyses excluding participants with an Initial Highest Count of 99. Here, we compare Non-Productive Counters and Productive Counters in terms of their accuracy on the Next Number task when all participants are included.

A mixed effects logistic regression predicting item-level accuracy from age, Productivity, Initial Highest Count and Productivity by IHC interaction (with random intercepts for subject and item magnitude), found no significant effect of Productivity (𝛘2(1) =2.43, *p* = 0.12) or Age (𝛘2(1) < 1, *p* = 0.93), or Productivity by IHC interaction (𝛘2(1) =2.45, *p* = .12). However, there was a significant effect of Initial Highest Count (*β* = 1.92, OR=6.83, 95% CI=[3.99, 11.7], 𝛘2(1) = 53.79, *p* < .001).

Next, we conducted a post-hoc analysis testing whether Productive counters may show a selective advantage on the Next Number task for mid-decade items but not decade transition items. This analysis found a significant main effect of Item Type (*β* = 2.10, OR=8.18, 95% CI=[3.46, 19.3], 𝛘2(1) = 22.96, *p* < .001), with greater accuracy on Mid-Decade items (*M* = 61%, *SD* = 49%) than on Decade Transition items (*M* = 34%, *SD* = 47%). Adding a Productivity by Item type (Decade transition/Mid-decade) interaction did not significantly improve model fit (𝛘2(1) < 1, *p* = .99), indicating that both Productive Counters and Non-Productive Counters found mid-decade items easier than decade transition items. This result suggests that both Productive and Non-Productive Counters have more difficulty generating decade terms than generating successive numbers within the same decade.

# C: Analysis of Infinity knowledge with all participants

These analyses are referenced in Section 3.3 of the main text (Footnote 11). In the main text, we exclude from analysis Productive Counters who had counted to 99 on their own without error. That allows for a more conservative test of the hypothesis that Productive knowledge of counting might relate to beliefs about the successor function and infinity. Here, we provide analysis results when all participants are included, and note any qualitative differences from the analysis in the main text.

We conduct separate analyses to predict children’s belief that every number has a successor (C1: Successor Knowledge), their belief that numbers never end (C2: Endless Knowledge), and children’s status as Full Infinity Knowers (C3: Full Knowledge). The analysis approach is reported in the main text; we constructed logistic regression models in a hierarchical fashion using Likelihood Ratio Tests to evaluate the contributions of additional variables, and selected final models based on a significant chi-squared statistic and reduced AIC value.

Note that, because Initial Highest Count was significantly correlated with both Productivity Group (χ2(1) = 80.45 *p* < .001) and Next Number accuracy (*r*(88) = .77, *p* < .001), whenever these latter two variables were entered in the final model we also included Initial Highest Count and its interaction in order to test for the role of decade+unit rule knowledge above and beyond rote counting ability.

## C1: Regression analyses predicting Successor Knowledge of Infinity

(See Table S1). Similar to analyses with the partial sample, initial models predicting Successor Knowledge of Infinity found that none of the three predictors explained a significant proportion of additional variance compared to the base model.

## C2: Regression analyses predicting Endless Knowledge of Infinity

(See Table S2). For models predicting children’s possession of Endless knowledge, all three counting measures explained significant additional variance relative to the base model. Controlling for age, children were more likely to have Endless knowledge if they were Productive counters (*β* = 1.70, OR = 5.46, 95% CI = [1.69, 21.58], 𝛘2(1) = 8.38, *p* = .004), had greater accuracy on the Next Number task (*β* = 0.69, OR = 2.00, 95% CI = [1.21, 3.46], 𝛘2(1) = 7.52, *p* = .006), or had greater Initial Highest Counts (*β* = 0.67, OR = 1.96, 95% CI = [1.22, 3.20], 𝛘2(1) = 7.81, *p* = .005).

To evaluate the relative contribution of each predictor, we constructed a full model which included all three predictors while controlling for age. This model explained significant additional variance compared to the base model with only age (𝛘2(3) = 11.31, *p* = 0.01), but did not improve model fit relative to any of the initial models. In addition, none of the predictor coefficients in this full model were significantly different from zero, suggesting that these measures of counting ability may explain overlapping variance in predicting Endless Knowledge among our sample.

## C3: Regression analyses predicting Full Knowledge of Infinity

(See Table S3). For initial models predicting Full Infinity knowledge, accuracy on the Next Number task explained significant additional variance relative to the base model (𝛘2(1) = 3.85, *p* = .0496). However, the model estimated coefficient for Next Number accuracy did not meet the threshold for significance (*β* = 0.57, *p* = 0.059, OR = 1.77, 95% CI = 1.00, 3.34]).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table S1 | | | | | | | | | | | | | | |
| *Regression models for predicting Successor knowledge on the Infinity Interview (All participants, N=122)* | | | | | | | | | | | | | | |
| Models | | | | Coefficient Estimates (ß) | | | | | | |  | Summary statistics | | |
| Age |  | IHC |  | NN |  | Productivity |  | Loglikelihood a | AIC | R2Nagelkerke |
| **Base Model** | | | |  |  |  |  |  |  |  |  |  |  |  |
| Age | | | | 0.342 |  |  |  |  |  |  |  | -81.54 | 167.08 | 0.037 |
| **Initial Models** | | | |  |  |  |  |  |  |  |  |  |  |  |
| Age + IHC | | | | 0.304 |  | 0.074 |  |  |  |  |  | -81.48 | 168.96 | 0.038 |
| Age + NN | | | | 0.286 |  |  |  | 0.126 |  |  |  | -81.36 | 168.72 | 0.041 |
| Age + Productivity | | | | 0.163 |  |  |  |  |  | 0.661 |  | -80.51 | 167.03 | 0.059 |
|  |  |  | *Notes.* IHC: Initial Highest Count; NN: Next Number accuracy; Coefficients were compared against 0 using *t*-tests. Model comparisons done using Likelihood Ratio Tests.  *a* Each initial model was compared against the base model. | | | | | | | | | | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| Table S2  *Regression models for predicting Endless knowledge on the Infinity Interview (All participants, N=122)* | | | | | | | | | | | | | | | | | | | | | | |
| Models | | | | Coefficient Estimates (ß) | | | | | | | | |  |  | |  |  |  | Summary statistics | | | |
| Age |  | IHC |  | NN |  | Productivity |  | IHC \* NN | | |  | IHC \* Productivity | |  | | Loglikelihood a | AIC | R2Nagelkerke |
| **Base Model** | | | |  |  |  |  |  |  |  |  |  | | |  |  | |  | |  |  |  |
| Age | | | | 0.707\*\* |  |  |  |  |  |  |  |  | | |  |  | |  | | -66.66 | 137.31 | 0.125 |
| **Initial Models** | | | |  |  |  |  |  |  |  |  |  | | |  |  | |  | |  |  |  |
| Age + IHC | | | | 0.403 |  | 0.675\*\* |  |  |  |  |  |  | | |  |  | |  | | -62.75\*\* | 131.50 | 0.207 |
| Age + NN | | | | 0.448 |  |  |  | 0.693\*\* |  |  |  |  | | |  |  | |  | | -62.89\*\* | 131.79 | 0.204 |
| Age + Productivity | | | | 0.329 |  |  |  |  |  | 1.698\*\* |  |  | | |  |  | |  | | -62.46\*\* | 130.93 | 0.212 |
| **Additional Models** | | | |  |  |  |  |  |  |  |  |  | | |  |  | |  | |  |  |  |
| Age + IHC \* NN | | | | 0.392 |  | 0.364 |  | 0.402 |  |  |  | .066 | | |  |  | |  | | -62.14 | 134.29 | 0.219 |
| Age + IHC \* Productivity | | | | 0.268 |  | 0.353 |  |  |  | 1.320 |  |  | | |  | 0.206 | |  | | -61.30 | 132.60 | 0.236 |
| **Full Model** | | | |  |  |  |  |  |  |  |  |  | | |  |  | |  | |  |  |  |
| Age + IHC + NN + Productivity | | | | 0.251 |  | 0.251 |  | 0.291 |  | 1.092 |  |  | | |  |  | |  | | -61.00\* | 131.99 | 0.242 |
|  |  |  | *Notes.*  IHC: Initial Highest Count; NN: Next Number accuracy. Coefficients were compared against 0 using *t*-tests. Model comparisons done using Likelihood Ratio Tests.  *a* Using Likelihood Ratio Tests, each initial model and the full model was compared against the base model. Each additional model was compared to the corresponding initial model without IHC. \* p < 0.05, \*\* p < 0.01 | | | | | | | | | | | | | | | | | | | |
|  |  |  |  | | | | | | | | | | | | | | | | | | | |

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| Table S3  *Regression models for predicting Full Infinity knowledge on the Infinity Interview (All participants, N = 122)* | | | | | | | | | | | | | | | | | | |
| Models |  |  | Coefficient Estimates (ß) | | | | | | | | | |  | Summary statistics | | | |
| Age | | |  | IHC |  | NN |  | Productivity |  | IHC \* NN |  | | | Loglikelihood a | AIC | R2Nagelkerke | | |
| **Base Model** |  | | |  |  |  |  |  |  |  |  |  | | |  |  |  | | |
| Age | 0.752\*\* | | |  |  |  |  |  |  |  |  |  | | | -54.36 | 112.71 | 0.120 | | |
| **Initial Models** |  | | |  |  |  |  |  |  |  |  |  | | |  |  |  | | |
| Age + IHC | 0.560 | | |  | 0.400 |  |  |  |  |  |  |  | | | -53.31 | 112.62 | 0.145 | | |
| Age + NN | 0.532 | | |  |  |  | 0.573 |  |  |  |  |  | | | -52.43\* | 110.86 | 0.166 | | |
| Age + Productivity | 0.480 | | |  |  |  |  |  | 1.219 |  |  |  | | | -52.78 | 111.57 | 0.158 | | |
| **Additional Models** |  | | |  |  |  |  |  |  |  |  |  | | |  |  |  | | |
| Age + IHC \* NN | 0.600\* | | |  | -0.251 |  | 0.644 |  |  |  | 0.365 |  | | | -51.93 | 113.87 | 0.178 | | |
| *Notes.*  IHC: Initial Highest Count; NN: Next Number accuracy. Coefficients were compared against 0 using *t*-tests. Model comparisons done using Likelihood Ratio Tests.  *a* Each initial model was compared against the base model. | | | | | | | | | | | | | | | | | | |

# D: Reanalysis using a more conservative Productivity Classification

In this section we describe a stricter coding scheme for classifying participants’ Productivity status based on the Highest Count Task (originally described in section 2.2.1 of manuscript).

These re-analyses led to the same conclusions as reported in the main text regarding how Productivity influences Highest Count performance (main text, section 3.1), Next Number performance (main text, section 3.2), and Successor Knowledge of Infinity (main text, section 3.2). However, there was a different result for regression analysis predicting Endless knowledge and Full knowledge of infinity. First, the original findings reported that Productivity group significantly predicted Endless knowledge; this effect did not replicate using the stricter definition. Second, the original findings did not yield any significant predictors of Full infinity knowledge when controlling for age; however, the stricter Productivity definition significantly predicts Full infinity knowledge, even when controlling for age or Initial Highest Count. Given that Full infinity knowledge requires Endless knowledge, these results generally converge.

In summary, these analyses indicate three robust findings: (1) there exists significant individual differences in knowledge of the decade+unit rule, which affects performance on counting and Next Number tasks; (2) knowledge of this productive rule does not predict successor function knowledge (i.e. the belief that we can always add one); (3) Productivity does predict infinity understanding (i.e. the belief that numbers never end), although particular regression results may depend on how strictly we define productivity and infinity knowledge (i.e. Endless knowledge vs. Endless+Successor knowledge).

## D1: Relationship of Productivity to Highest Count and age

In this stricter definition, we allow for only one error when classifying participants’ Productivity Status. Thus, to be Productive, participants have to count to 99 on their own with at most one error (“Productive Counters, IHC ≥ 99”) or count past a Decade-Change Error by at least 2 decades, with at most one error in those 2 decades (“Productive Counters, IHC < 99”). This stricter definition reclassifies 6 participants as Non-Productive counters instead of Productive Counters. Their subject IDs (in main manuscript, Fig. 3) are: 34, 78, 79, 83, 85, 89. Below, we replicate analyses reported in the main manuscript (Section 3.1).

Productive Counters had, on average, a higher Initial Highest Count (*Mean* = 70.4, *SD* = 29.4, *Median* = 76) than Non-Productive Counters (*Mean* = 26.3, *SD* = 18.4, *Median* = 18). This difference remained if we considered only Productive Counters with Initial Highest Count below 99 (*Mean* = 44.3, *SD* = 14.3, *Median* = 49).

Productive Counters also made greater improvements past their Initial Highest Counts. Non-Productive Counters had a median Final Highest Count of 29 (*M* = 39.3, *SD* = 26.8, *Range* = 5 to 99), which was only 11 numbers past their median Initial Highest Count of 18. In contrast, Productive Counters had a median Final Highest Count of 99 (*M* = 96.1, *SD* = 9.9, *Range* = 49 to 99), which was 23 numbers higher than their median Initial Highest Count of 76. When considering only Productive Counters (IHC<99), the improvement between initial and final highest counts is even greater: their median Final Highest Count was 99, which was 50 numbers higher than their median Initial Highest Count of 49.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Table S4: Distribution of Age and Highest Count measures by Productivity | | | | | | | |
| **Classification** | **Variable** | *n* | *M* | *SD* | *Median* | *Min* | *Max* |
| Original definition |  |  |  |  |  |  |  |
| Nonproductive | Age (years) | 49 | 4.60 | 0.42 | 4.49 | 4.00 | 5.61 |
|  | IHC | 49 | 22.63 | 14.86 | 15 | 5 | 77 |
|  | FHC | 49 | 32.00 | 17.62 | 29 | 5 | 99 |
|  | DCE | 19 | 30.58 | 8.34 | 29 | 19 | 49 |
| Productive  (IHC < 99) | Age (years) | 41 | 5.22 | 0.48 | 5.24 | 4.25 | 5.99 |
| IHC | 41 | 46.02 | 15.34 | 49 | 14 | 79 |
| FHC | 41 | 94.32 | 12.33 | 99 | 49 | 99 |
| DCE | 34 | 47.82 | 15.33 | 49 | 29 | 89 |
| Stricter definition |  |  |  |  |  |  |  |
| Nonproductive | Age (years) | 55 | 4.65 | 0.43 | 4.56 | 4.00 | 5.61 |
|  | IHC | 55 | 26.25 | 18.37 | 18 | 5 | 77 |
|  | FHC | 55 | 39.31 | 26.84 | 29 | 5 | 99 |
|  | DCE | 20 | 33.50 | 15.38 | 29 | 19 | 89 |
| Productive (IHC < 99) | Age (years) | 35 | 5.26 | 0.49 | 5.35 | 4.25 | 5.99 |
| IHC | 35 | 44.34 | 14.35 | 49 | 14 | 79 |
| FHC | 35 | 93.51 | 13.21 | 99 | 49 | 99 |
| DCE | 33 | 46.58 | 13.70 | 49 | 29 | 79 |

## D2: Relationship of Productivity to Next Number performance

We replicate the effect of Productivity on Next Number performance: Productive Counters (72% correct; *SD* = 27%) significantly outperformed Non-Productive Counters (31%; *SD* = 27%) on the Next Number task (*t*(120) = -8.13, *p* < .001). Below, we conduct more conservative analyses considering just Productive Counters (IHC<99) and Non-Productive Counters. We use mixed effects logistic regression to predict trial-level accuracy from predictors of interest, with random effects for subject and item magnitude.

The finding that Initial Highest Count predicts Next Number accuracy for Non-Productive Counters, but not Productive Counters, is robust. We constructed a model predicting accuracy from Productivity, Initial Highest Count, age, and a Productivity by Initial Highest Count interaction and age. There was no significant main effect of Productivity (*β* = 0.90, 𝛘2(1) = 2.53, *p* = 0.11) or Age (𝛘2(1) = 0.026, *p* = .87), but there was a significant main effect of Initial Highest Count (β = 0.85, OR=2.34, 95% CI=[1.56, 3.51]; 𝛘2(1) = 20.9, *p* < .001). However, these effects are qualified by a significant Productivity by Initial Highest Count interaction (*β* = -0.91, OR=.40, 95% CI=[.19, .88], 𝛘2(1) = 5.20, *p* = 0.023). Inspection of simple slopes within Productivity group found that while Initial Highest Count positively predicted accuracy among Non-Productive Counters (*β* = 1.20, OR=3.32, 95% CI=[2.09, 5.27]), there was no effect for Productive Counters (*β* = 0.29, OR=1.34, 95% CI=[0.68, 2.64]).

Next, we evaluated how productivity classification might interact with various item-level covariates. In the main text, a *post-hoc* analysis testing whether Productive Counters did not find a selective advantage on the Next Number task for Mid-Decade items compared to decade transitions. This model predicted accuracy from Productivity, Item Type, Productivity by Item Type interaction, and Initial Highest Count. Using a stricter definition of Productivity, we replicated this finding: there was no significant interaction of Productivity and Item Type (𝛘2(1) = .67, *p* = .41). Instead, there was a main effect of Item type (*β* = 2.27, OR=9.70, 95% CI=[4.04, 23.30]; 𝛘2(1) = 12.43, *p* < .001), such that participants were overall more accurate on Mid-Decade items (M=49%, SD=34%) than Decade Transition items (M=17%, SD=30%).

Finally, we test the prediction that Productive Counters can generate successors for numbers both within and beyond their Initial Highest Count, while Non-Productive Counters are only able to generate successors within their Initial Highest Count. This regression model predicted trial-level accuracy from Productivity, Item Range (Within/Beyond IHC), a Productivity by Item Range interaction, as well as Initial Highest Count and age. Here, we found a significant main effect of Productivity (β = .71, OR=2.04, 95% CI = [1.00, 4.16]; 𝛘2(1) = 3.19, *p* = .0495) and Initial Highest Count (β = .88, OR=2.40, 95% CI = [1.59, 3.61]; 𝛘2(1) = 17.56, *p* < .001), but no significant effect of Item Range (𝛘2(1) < 1, *p* = .99). Planned contrasts indicate that performance on numbers within children’s Initial Highest Count was similar for Productive Counters (IHC<99) (*M* = 60%) and Non-Productive Counters (*M* = 62%; *p* = .86 by *t*-test) whereas accuracy for numbers beyond their Initial Highest count was significantly greater among Productive Counters (*M* = 55%, *SD* = 30%) than among Non-Productive Counters (*M* = 30%, *SD* = 29%; *t*(88) = -3.96, *p* < .001). However, this Item Range by Productivity interaction did not meet the threshold for significance (β = .94, OR=2.57, 95% CI = [.96, 6.87], 𝛘2(1) = 3.54, *p* = .06).

## D3: Relationship of Productivity to Infinity knowledge

Here we take the same regression approach as described in the main text, replacing the variable of Productivity (Non-Productive / Productive (IHC < 99)) with the stricter version. In predicting children’s Successor Knowledge (Table S5) and Endless Knowledge (Table S6), none of the three predictors explained a significant proportion of additional variance compared to the base model.

Finally, we constructed models predicting children’s status as Full Infinity Knowers (Table S7). Productivity Group explained significant additional variance relative to the base model (𝛘2(1) = 4.67, *p* = .031), though other measures of counting ability (Initial Highest Count, Next Number accuracy) did not explain additional variance when controlling for age. This final model thus included only Productivity Group and Age as predictors, and estimated that Productive counters were more likely than Non-Productive Counters to have Endless Knowledge (*β* = 1.70, *p* = .04, OR = 5.47, 95% CI: 1.17–31.80), while Age was not a significant predictor (*p* = 0.83). The effect of Productivity Group remained when controlling for Initial Highest Count, although these more complex models did not explain any additional variance.

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| Table S5 | | | | | | | | | | | |
| *Regression models for predicting Successor knowledge on the Infinity Interview (Participants IHC <99, N=90)* | | | | | | | | | | | |
| Models | Coefficient Estimates (ß) | | | | | | |  | Summary statistics | | |
| Age |  | IHC |  | Next Number accuracy |  | Productivity Group |  | Loglikelihood a | AIC | R2Nagelkerke |
| **Base Model** |  |  |  |  |  |  |  |  |  |  |  |
| Age | 0.296 |  |  |  |  |  |  |  | -58.75 | 121.51 | 0.027 |
| **Initial Models** |  |  |  |  |  |  |  |  |  |  |  |
| Age + IHC | 0.598 \* |  | -0.520 |  |  |  |  |  | -57.05 | 120.10 | 0.077 |
| Age + Next Number accuracy | 0.358 |  |  |  | -0.155 |  |  |  | -58.55 | 123.10 | 0.033 |
| Age + Productivity Group | 0.056 |  |  |  |  |  | 0.885 |  | -57.39 | 120.77 | 0.067 |
| *Notes.* Coefficients were compared against 0 using *t*-tests. Model comparisons done using Likelihood Ratio Tests.  *a* Each initial model was compared against the base model. \* p < 0.05 | | | | | | | | | | | |

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| Table S6  *Regression models for predicting Endless Infinity knowledge on the Infinity Interview (Participants IHC <99, N=90)* | | | | | | | | | | | |
| Models | Coefficient Estimates (ß) | | | | | | |  | Summary statistics | | |
| Age |  | IHC |  | Next Number accuracy |  | Productivity Group |  | Loglikelihood a | AIC | R2Nagelkerke |
| **Base Model** |  |  |  |  |  |  |  |  |  |  |  |
| Age | 0.451 |  |  |  |  |  |  |  | -42.22 | 88.44 | 0.049 |
| **Initial Models** |  |  |  |  |  |  |  |  |  |  |  |
| Age + IHC | 0.261 |  | 0.349 |  |  |  |  |  | -41.63 | 89.26 | 0.070 |
| Age + Next Number accuracy | 0.287 |  |  |  | 0.466 |  |  |  | -41.04 | 88.07 | 0.090 |
| Age + Productivity Group | 0.133 |  |  |  |  |  | 1.171 |  | -40.66 | 87.32 | 0.103 |
| *Notes.* Coefficients were compared against 0 using *t*-tests. Model comparisons done using Likelihood Ratio Tests.  *a* Each initial model was compared against the base model. | | | | | | | | | | | |

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| Table S7 | | | | | | | | | | | | | | |
| *Regression models for predicting Full knowledge on the Infinity Interview (Participants IHC <99, N=90)* | | | | | | | | | | | | | | |
| Models | | | | Coefficient Estimates (ß) | | | | | | |  | Summary statistics | | |
| Age |  | IHC |  | Next Number accuracy |  | Productivity Group |  | Loglikelihood a | AIC | R2Nagelkerke |
| **Base Model** | | | |  |  |  |  |  |  |  |  |  |  |  |
| Age | | | | 0.527 |  |  |  |  |  |  |  | -33.91 | 71.81 | 0.058 |
| **Initial Models** | | | |  |  |  |  |  |  |  |  |  |  |  |
| Age + IHC | | | | 0.481 |  | 0.084 |  |  |  |  |  | -33.88 | 73.76 | 0.059 |
| Age + Next Number accuracy | | | | 0.417 |  |  |  | 0.307 |  |  |  | -33.51 | 73.03 | 0.073 |
| Age + Productivity Group | | | | 0.082 |  |  |  |  |  | 1.700 \* |  | -31.57\* | 69.14 | 0.148 |
| **Final Models** | | | |  |  |  |  |  |  |  |  |  |  |  |
| Age + Productivity Group + IHC | | | | 0.132 |  | -0.133 |  |  |  | 1.772 \* |  | -31.53 | 71.05 | 0.149 |
| Age + Productivity Group \* IHC | | | | 0.143 |  | -0.066 |  |  |  | 1.777 \* |  | -31.41 | 72.82 | 0.154 |
|  |  |  | *Notes.* Coefficients were compared against 0 using *t*-tests. Model comparisons done using Likelihood Ratio Tests.  *a* Each initial model was compared against the base model. \* p < 0.05, \*\* p < 0.01 | | | | | | | | | | | |