BIOS 26210: Lab Exercise 2

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October 13, 2009

Exercise 1

Code: **HW2-2-1_SiTang.R**

Additional Notes:
(1). A function named "sortExponents" is used to re-arrange the polynomial with descending order of x. See more detailed annotation of the function in the code.
(2). I have found that the function "parsePolynomial" is unable to deal with polynomials containing coefficients of non-integers. So I changed one line in the body of the function.(See my code for more details).
(3). I changed one line in function "deparsePolynomial", so that it is able to display polynomials with negative numbers at the very beginning of the polynomials, such as $-x^4 + x^3$ and $-x^2 + 1$. (See my code for more details.)
(4). In the function "*deparsePolynomial*", I think it is not appropriate to substitute all '1x' with 'x'. (For example: '11x' will be replaced by '1x', which is not correct). So I commented that line out, {'# out_string <- gsub('1x', 'x', out_string);'}, however, '1· x' will be displayed as 'x' as a result.

Exercise 2

2a.

Code: **HW2-2-1_SiTang.R**

For the logistic model $f(x) = 2x(2 - \frac{x}{2})$, the difference equation for this model is:

$$x_{n+1} - x_n = f(x_n) - x_n = 2x_n(2 - \frac{x_n}{2}) - x_n = 3x_n - x_n^2$$

At the fixed points, we have

$$x_{n+1} - x_n = 0 \;\; or \;\; f(x^*) = 2x^*(2 - \frac{x^*}{2}) = x^*$$

thus, the fixed points are: $x^* = 0$ and $x^* = 3$.

The following figure shows the curve of the logistic model and the fixed points.

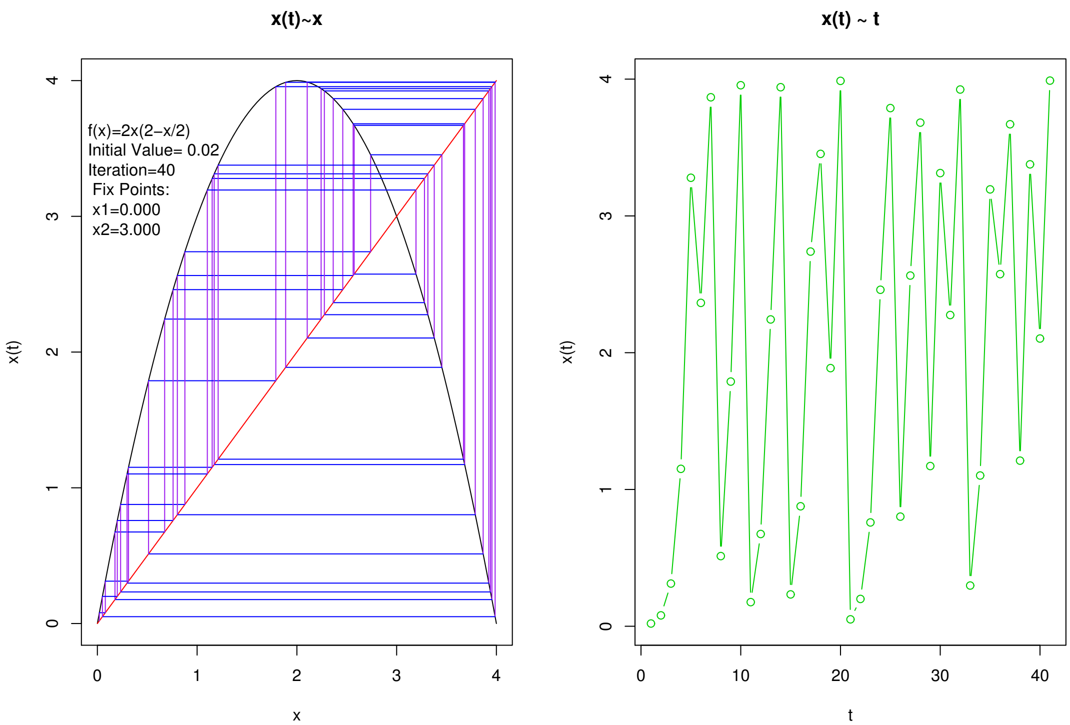

Fig.2-1 The iteration map (left figure, starting at initial value 0.5 with iteration times 20) and numeric solution (right figure) of logistic model $f(x) = 2x(2 - \frac{x}{2})$. Other iteration maps and numeric solutions, based on different initial values and iteration times,can be obtained by simply re-running my R code.

2b.

Code: **HW2-2-2_SiTang.R**
My code includes a section of trying to find the fixed points of the logistic models computationally in R. And the legends in the figures contains the fixed points solved by R. The code trunk is very preliminary attempt, since I do not have any previous experience in solving equations by computer. I truely welcome any advice from you.

Exercise 3. Function qsort()

Code: **HW2-3_SiTang.R**

Exercise 4

To sort n numbers, there should be no more than $(n-1)+(n-2)+\ldots+2+1 = \frac{1}{2}n(n-1)$ times of pairwise comparison, which is resulted from comparing the first number with the other $(n-1)$ numbers, comparing the second number with the rest $(n-2)$ numbers, and so on until the comparison between the last two numbers. The maximum times of pairwise comparison guaranteed any two numbers from the n numbers are compared one time, thus the order of the n numbers is then determined.

The worst conditions of such 'qsort' algorithm are those when the n numbers to be sorted are **already sorted** before applying the algorithm. Under these circumstances, each time of calling the function 'qsort' can only determine the relationship of the first number with other numbers, and only the first number in the array are assigned to 'pivot' and the other numbers, being passed to the function during this round of function calling, are ALL categorized as either 'head' or 'tail', waiting to be sorted during the next time of function calling. Then the function is called n times in total under the worst conditions, and generally at the kth time, $(n-k)$ times of comparison are performed among $n-k+1$ numbers, determine only one number's position, leaving all other $(n-k)$ numbers for the next round of comparison.

So the total comparison times are $\displaystyle\sum_{k=1}^{n}(n-k) = (n-1)+(n-1)+\ldots+2+1 = \frac{1}{2}n(n-1)$, which is the same as the maximum times of pairwise comparison. Thus, the computational cost under these conditions is $O(n^2)$

Exercise 3 in 'Chapter 1 Exercise REVISED'

Code: **HW2-3_SiTang-reviesed.R**

a.

(1). Calculate the fixed points.
From the logistic model, $f(x) = 2x(1-x)$, we can obtain the fixed point by solving:

$$\begin{aligned} f(x^*) = 2x^*(1-x^*) &= x^* \\ x^* - 2x^{*2} &= 0 \\ x^* = 0 \quad or \quad x^* &= \frac{1}{2} \end{aligned}$$

(2). Plot the logistic model.

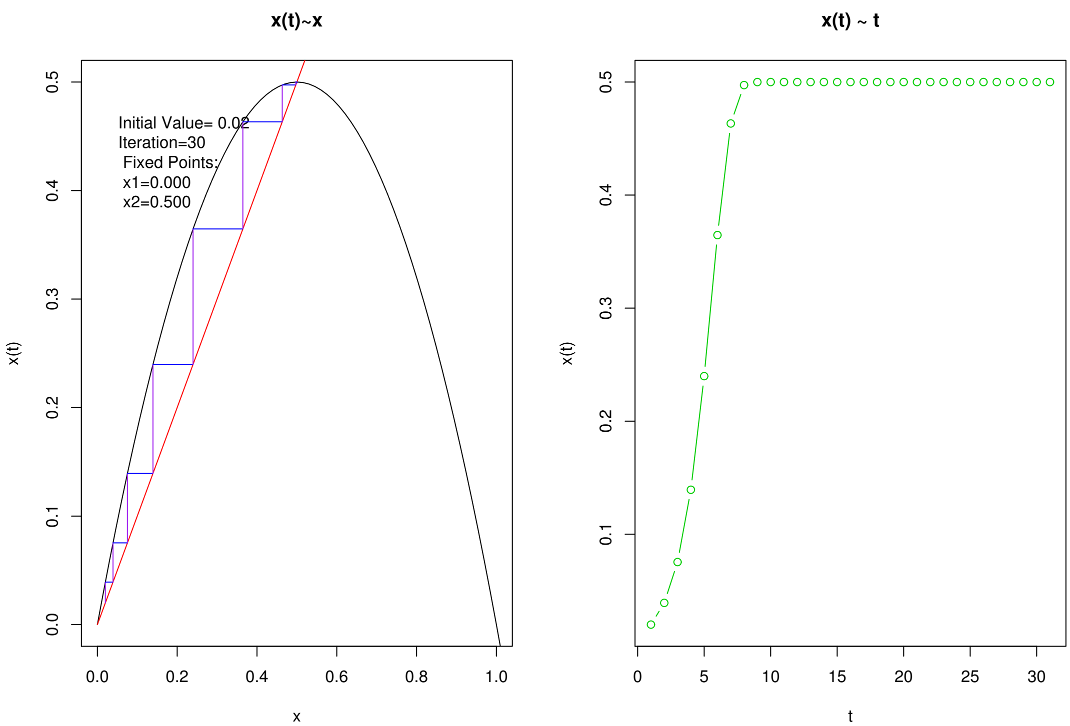

Fig.2-2 The iteration map (left figure, starting at initial value 0.02 with iteration times 30) and numeric solution (right figure) of logistic model $f(x) = 2x(1-x)$. Other iteration maps and numeric solutions, based on different initial values and iteration times,can be obtained by simply re-running my R code.

b.

(1). Calculate the fixed points.
From the logistic model, $f(x) = 4x(1-x)$, we can obtain the fixed point by solving:

$$\begin{aligned} f(x^*) = 4x^*(1-x^*) &= x^* \\ 3x^* - 4x^{*2} &= 0 \\ x^* = 0 \quad or \quad x^* &= \frac{3}{4} \end{aligned}$$

(2). Plot the logistic model.

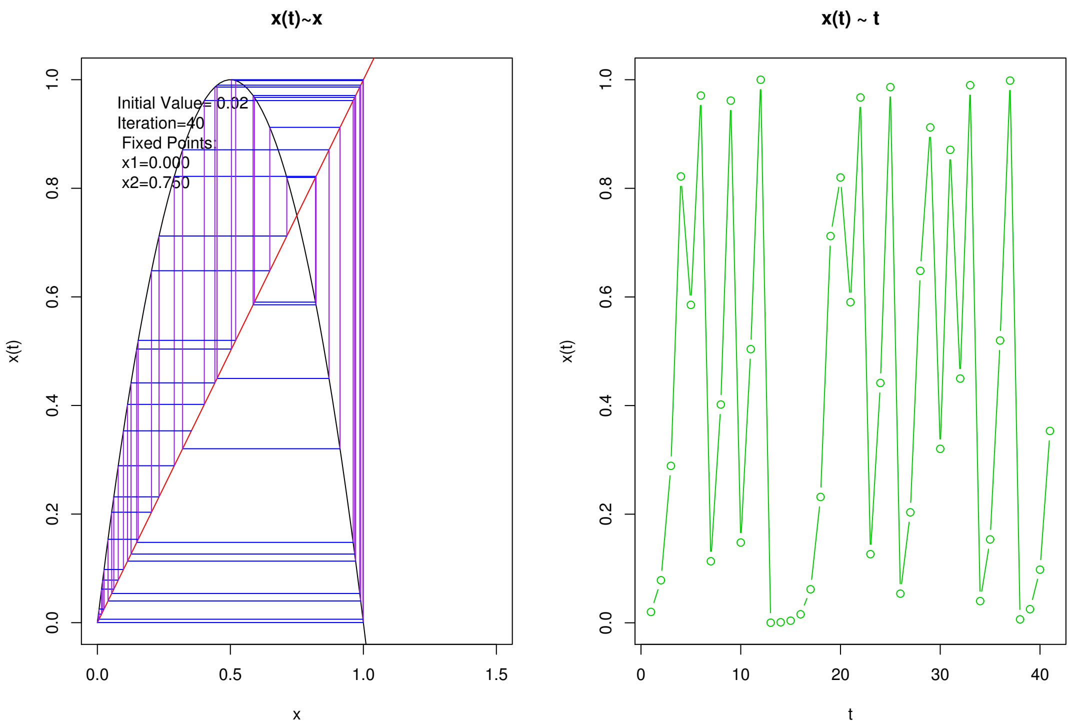

Fig.2-3 The iteration map (left figure, starting at initial value 0.02 with iteration times 40) and numeric solution (right figure) of logistic model $f(x) = 4x(1-x)$. Other iteration maps and numeric solutions, based on different initial values and iteration times,can be obtained by simply re-running my R code.