Hint for exercise 3 on the homework.

$$\begin{aligned}
(a_k) &= a_1, a_2, \ldots \\
\text{and } a_k &= b_k + c_k\, i, \quad b_k, c_k \in \mathbb{Q} \\
&= \sqrt{b_k^2 + c_k^2 e}^{\,i\left(\arctan\left(\frac{b_k}{c_k}\right)\right)} \\
\text{Let } r_k &= \sqrt{b_k^2 + c_k^2} \text{ and } \psi_k = \arctan\left(\frac{b_k}{c_k}\right) \\
\text{Then } a_k &= r_k e^{\psi_k i} \\
a_k^n &= \left(r_k e^{\psi_k i}\right)^n \\
&= {r_k}^n e^{ni\psi_k}
\end{aligned}$$

Since $x^n - 1, {r_k}^n = 1$ and $e^{ni\psi_k} = 0 \pm 2c\pi$, where $c \in \mathbb{Z}$, we know that ${r_k}^n = 1$ and further, $r_k = 1$. We also know that $0 \leq \psi_k \leq 2\pi$. The first two solutions, then, are given by

$$\begin{aligned}
n\psi_k &= 2\pi & n\psi_k &= 0 \\
\psi_k &= \frac{2\pi}{n} & \psi_k &= 0
\end{aligned}$$

Furthermore, the set of all solutions is the set of linear combinations of these two solutions, modulo 2π.

$$\begin{aligned}
\psi_0 &= 0 \\
\psi_1 &= \frac{2\pi}{n} \\
\psi_2 &= 2\left(\frac{2\pi}{n}\right) \\
&\vdots \\
\psi_n &= n\left(\frac{2\pi}{n}\right) = 2\pi \equiv 0
\end{aligned}$$

Therefore $\forall k \in \mathbb{Z} : 0 \leq k < n$, the complex number $e^{\frac{2ki\pi}{n}}$ is a solution for $x^k - 1 = 0$.