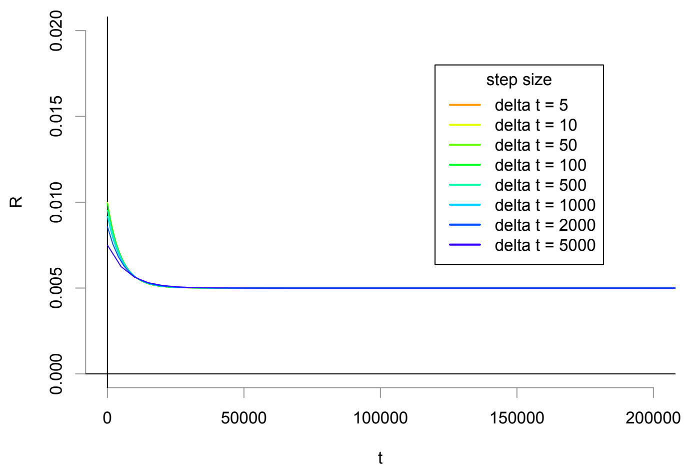
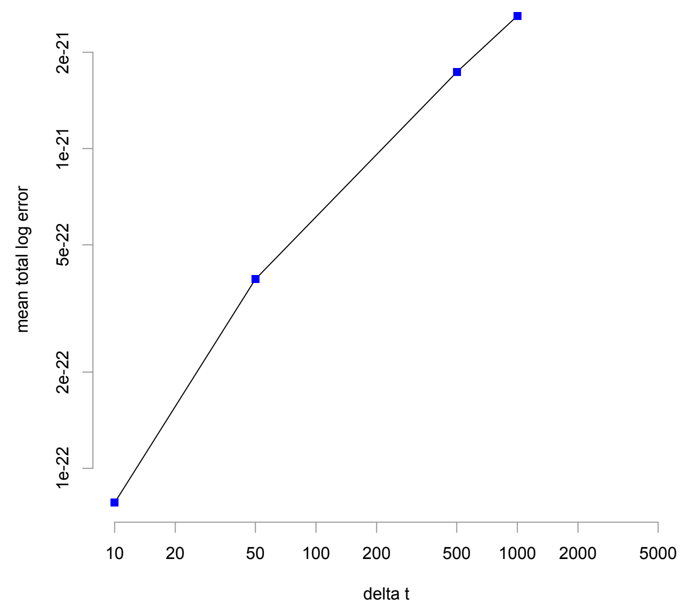
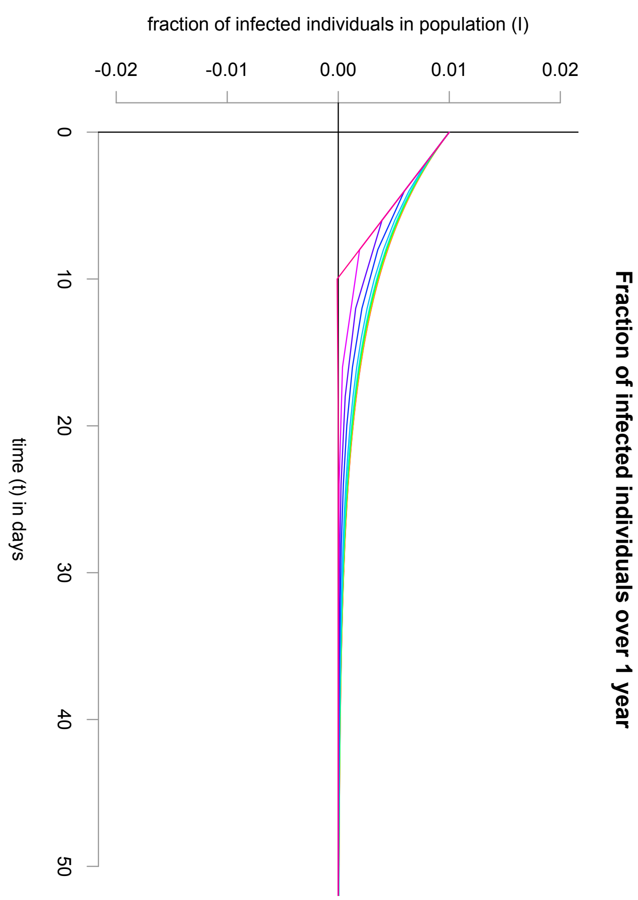


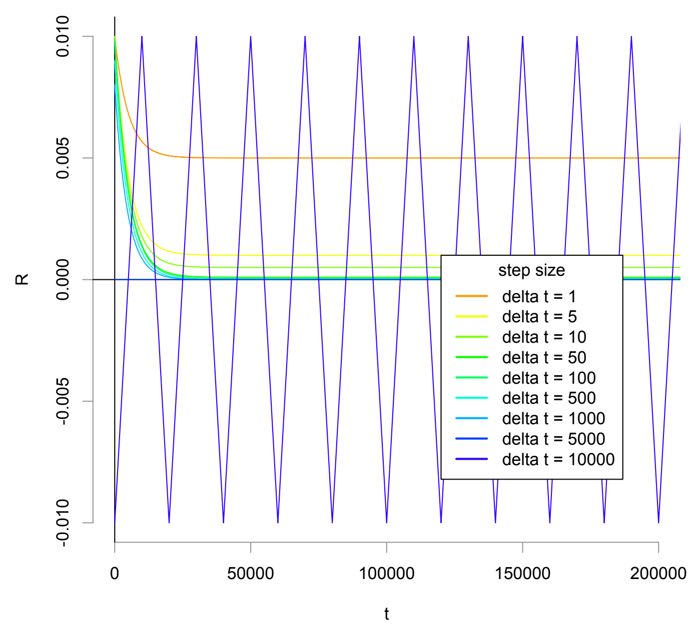
R
0.000
0.005
0.010
0.015
0.020
0
50000
100000
150000
200000
t
step size
delta t = 5
delta t = 10
delta t = 50
delta t = 100
delta t = 500
delta t = 1000
delta t = 2000
delta t = 5000



mean total log error
1e-22
2e-22
5e-22
1e-21
2e-21
10
20
50
100
200
500
1000
2000
5000
delta t



Fraction of infected individuals over 1 year
fraction of infected individuals in population (I)
-0.02
-0.01
0.00
0.01
0.02
time (t) in days
0
10
20
30
40
50



R
-0.010
-0.005
0.000
0.005
0.010
0
50000
100000
150000
200000
t
step size
delta t = 1
delta t = 5
delta t = 10
delta t = 50
delta t = 100
delta t = 500
delta t = 1000
delta t = 5000
delta t = 10000

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$$R\_dot = -(k_+ * S * R) + k_- * (R_0 - R)$$

$$= -(k_+ * S * R) + (k_- * R_0) - (k_- * R)$$

$$= -R * (k_+ * S + k_-) + (k_- * R_0)$$

$$R\_dot + R * \underset{a(t)}{\underset{\Uparrow}{(k_+ * S + k_-)}} = \underset{b(t)}{\underset{\Uparrow}{(k_- * R_0)}}$$

Integrating factor $= \mu = e^{\int (k+ \, * \, S + k-)dt} = e^{(k+ \, * \, S + k-)t}$

$$\mu' = (k_+ * S + k_-) * e^{(k+ \, * \, S + k-)t}$$

$$(R * \mu)' = \mu * k_- * R_0$$

$$\int (R * \mu)'dt = \int (\mu * k_- * R_0)dt = \int (e^{(k+ \, * \, S + k-)t} * k_- * R_0)dt$$

$$R * \mu + C = 1/(k_+ * S + k_-) * e^{(k+ \, * \, S + k-)t} * k_- * R_0$$

$$R * \mu = 1/(k_+ * S + k_-) * e^{(k+ \, * \, S + k-)t} * k_- * R_0 - C$$

$$R = 1/\mu * [1/(k_+ * S + k_-) * e^{(k+ \, * \, S + k-)t} * k_- * R_0 - C]$$

$$= e^{-(k+ \, * \, S + k-)t} * [1/(k_+ * S + k_-) * e^{(k+ \, * \, S + k-)t} * k_- * R_0 - C]$$

$$= (k_- * R_0) / (k_+ * S + k_-) - C * e^{-(k+ \, * \, S + k-)t}$$

Use initial condition to determine C:

$$R_0 = (k_- * R_0) / (k_+ * S + k_-) - C * e^0$$

$$= (k_- * R_0) / (k_+ * S + k_-) - C$$

$$\Rightarrow \quad C = (k_- * R_0) / (k_+ * S + k_-) - R_0$$

$$= (k_- * R_0) / (k_+ * S + k_-) - R_0 * (k_+ * S + k_-) / (k_+ * S + k_-)$$

$$= (k_- * R_0 - R_0 * k_+ * S - R_0 * k_-) / (k_+ * S + k_-)$$

$$= -(R_0 * k_+ * S) / (k_+ * S + k_-)$$

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Chapter 4 Lab Exercise, Question 2c

Hence, the complete solution is:

$$\boldsymbol{R} = (k_- * R_0) / (k_+ * S + k_-) - C * e^{-(k+ \, * \, S + k-)t}$$

$$= \boldsymbol{(k_- * R_0) / (k_+ * S + k_-) + [(R_0 * k_+ * S) / (k_+ * S + k_-)] * e^{-(k+ \, * \, S + k-)t}}$$

According to the derived equation, as t approaches ∞, the solution will approach stable equilibrium at $\boldsymbol{(k_- * R_0) / (k_+ * S + k_-)}$ since $e^{-\infty}$ approaches 0.