Megan Liszewski
Assignment 3
Date: 11/23/09
question: 1
subquestion: a
other files:

Given the logistic equation:

$$N_{t+1} = 41\ N_t - 10\ \ N_t^2$$

$$= (41 - 10\ N_t)\ N_t$$

This can be written in the form: $N_{t+1} = (r - k\ N_t)\ N_t$.

Therfore, r = 41; k = 10.

To solve for the fixed/equilibrium points:

$$N^* = (41 - 10\ N^*)\ N^*$$

There are two solutions:

$N^* = 0$, which is stable for $r < 1$

$N^* = (r-1)/k = (41-1)/10 = 4$, which is stable for $1 < r < 3$.

Since in this case r = 41, neither of these equilibrium points is stable. Additionally, since there is no stable fixed point, there is no value for the population to approach in the long run.

Megan Liszewski
Assignment 3
Date: 11/23/09
question: 1
subquestion: b
other files:

Given the logistic equation:

$$N_{t+1} = 41\ N_t + 2\ N_t^2$$

$$= (41 + 2\ N_t)\ N_t$$

This can be written in the form: $N_{t+1} = (r - k\ N_t)\ N_t$.

Therfore, $r = 41$; $k = -2$.

To solve for the fixed/equilibrium points:

$$N^* = (41 + 2\ N^*)\ N^*$$

There are two solutions:

$N^* = 0$, which is stable for $r < 1$

$N^* = (r-1)/k = (41-1)/(-2) = -20$, which is stable for $1 < r < 3$.

Since in this case $r = 41$, neither of these equilibrium points is stable. Additionally, since there is no stable fixed point, there is no value for the population to approach in the long run.

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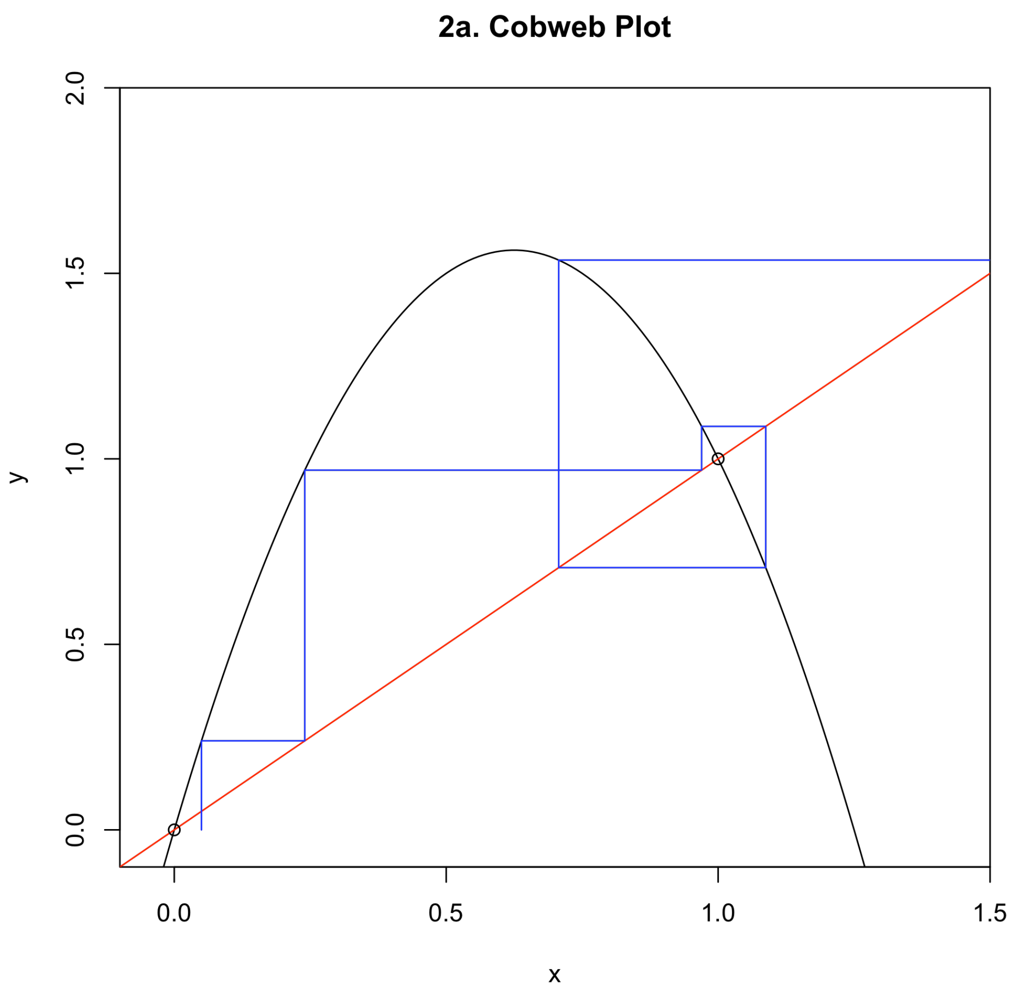
Assignment 3
Date: 11/23/09
question: 2
subquestion: a
other files: R-file

$f(x) = x(5 - 4x)$

The fixed points are 0 and 1. They are both unstable.



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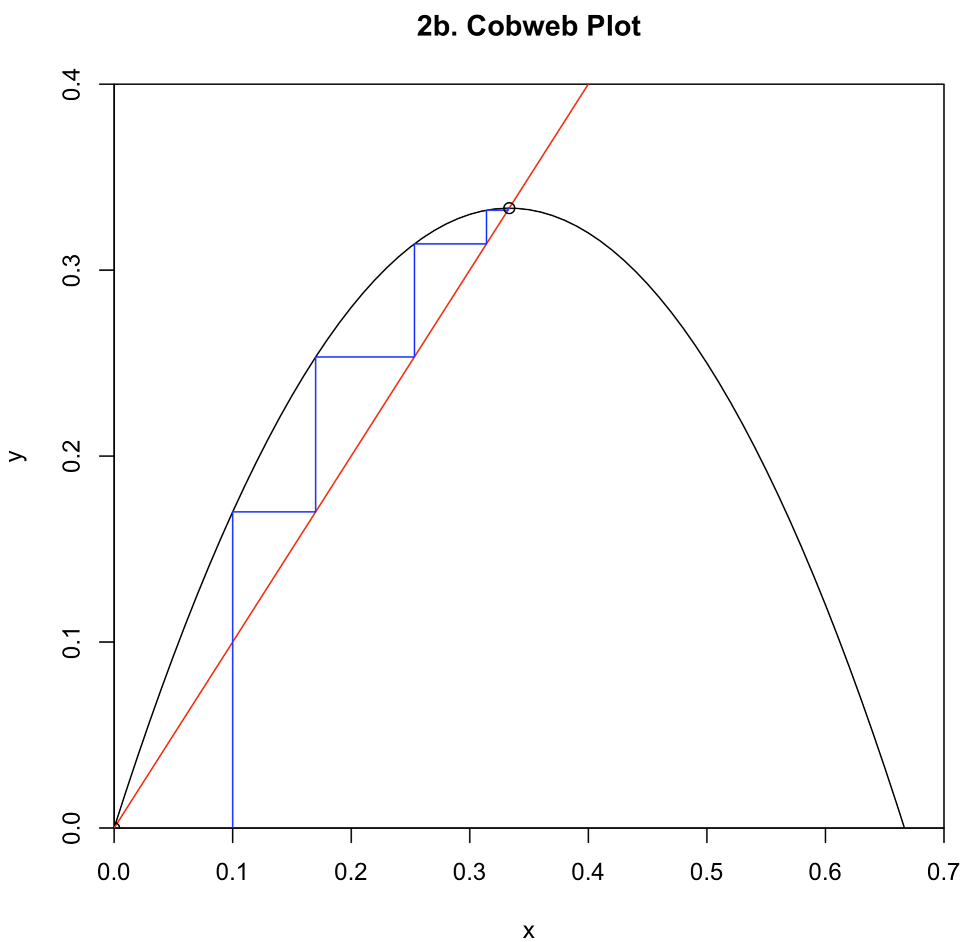
question: 2
subquestion: b
other files: R-file

$f(x) = 2x(1 - 3x/2) = x(2 - 3x)$

The fixed points are 0 and 1/3. The point 0 is unstable and 1/3 is stable.



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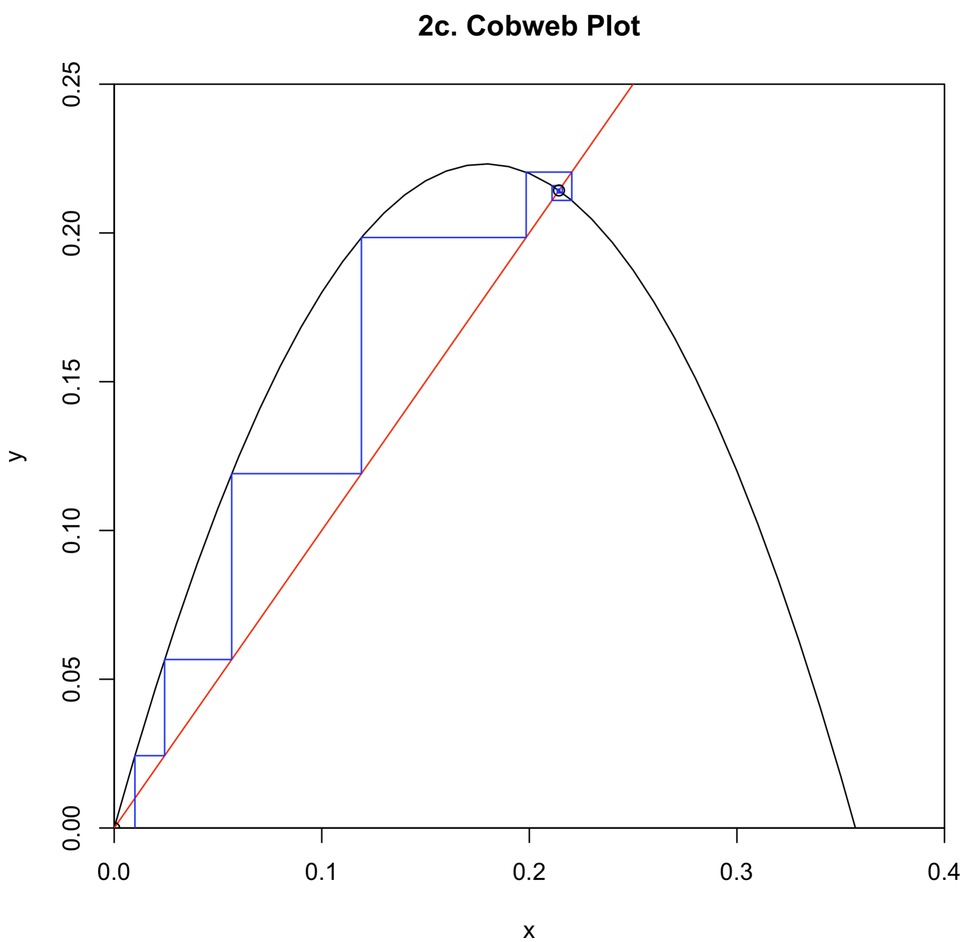
Date: 11/23/09
question: 2
subquestion: c
other files: R-file

f (x) = x (5/2 – 7 x)

The fixed points are 0 and 3/14. The point 0 is unstable and 3/14 is stable.



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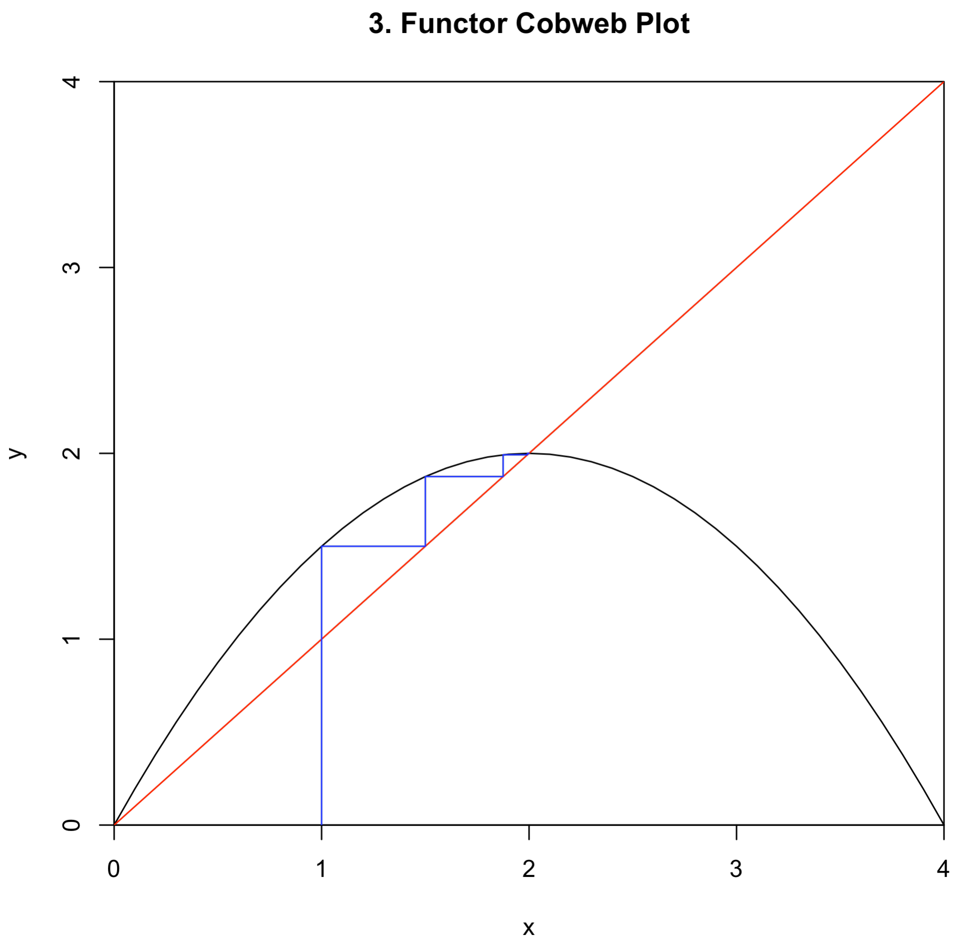
question: 3
subquestion:
other files: R-file



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subquestion: a
other files: R-file

$$\frac{dG}{dt} = -k\,G(t)$$

This is first order, linear, homogeneous and autonomous.

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question: 4
subquestion: b
other files: R-file

$$\frac{dG}{dt} = -k\,G(t)$$

$$\frac{dG}{G(t)} = -k\,dt$$

$$\int \frac{dG}{G(t)} = -\int k\,dt$$

$$\ln\big(G(t)\big) = -k\,t + C$$

$$G(t) = e^{-k\,t+C} = A\,e^{-k\,t}$$

Plugging in the initial condition: $G(0) = G_O = A\,e^{-k\cdot 0} = A$

Thus, the solution is: $G(t) = G_O\,e^{-k\,t}$

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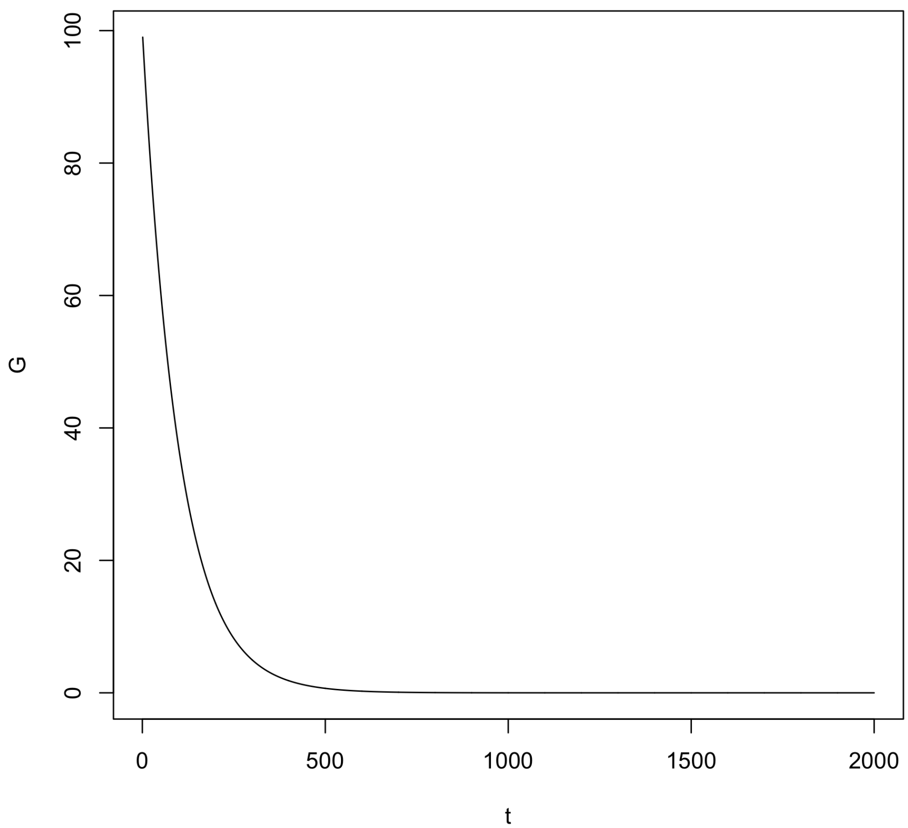
subquestion: c
<u>other files: R-file</u>



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subquestion: d
<u>other files: R-file</u>

The equilibrium of blood sugar in this model is 0 mg/dl, and it is stable.

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subquestion: a
other files: R-file

$$\frac{dG}{dt} = a - k\,G(t)$$

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$$\frac{dG}{dt} = -k\,G(t)$$

Use the integration factor: $e^{\int k\,dt} = e^{k\,t}$ (ignoring the constant, which would just be factored out later).

$$\frac{dG}{dt}e^{k\,t} + k\,G(t)\,e^{k\,t} = a\,e^{k\,t}$$

Then, by the product rule: $\frac{d}{dt}\left(G(t)\,e^{k\,t}\right) = a\,e^{k\,t}$

Integrate both sides to get:

$$G(t)\,e^{k\,t} = \frac{a}{k}e^{k\,t} + C \quad ; \quad G(t) = \frac{a}{k} + C\,e^{-k\,t}$$

Then, using the initial condition:

$$G(0) = \frac{a}{k} + C\,e^{-k\cdot 0} = \frac{a}{k} + C = G_O \quad ; \quad C = G_O - \frac{a}{k}$$

Therefore, the solution is:

$$G(t) = \frac{a}{k} + \left(G_O - \frac{a}{k}\right)e^{-k\,t}$$

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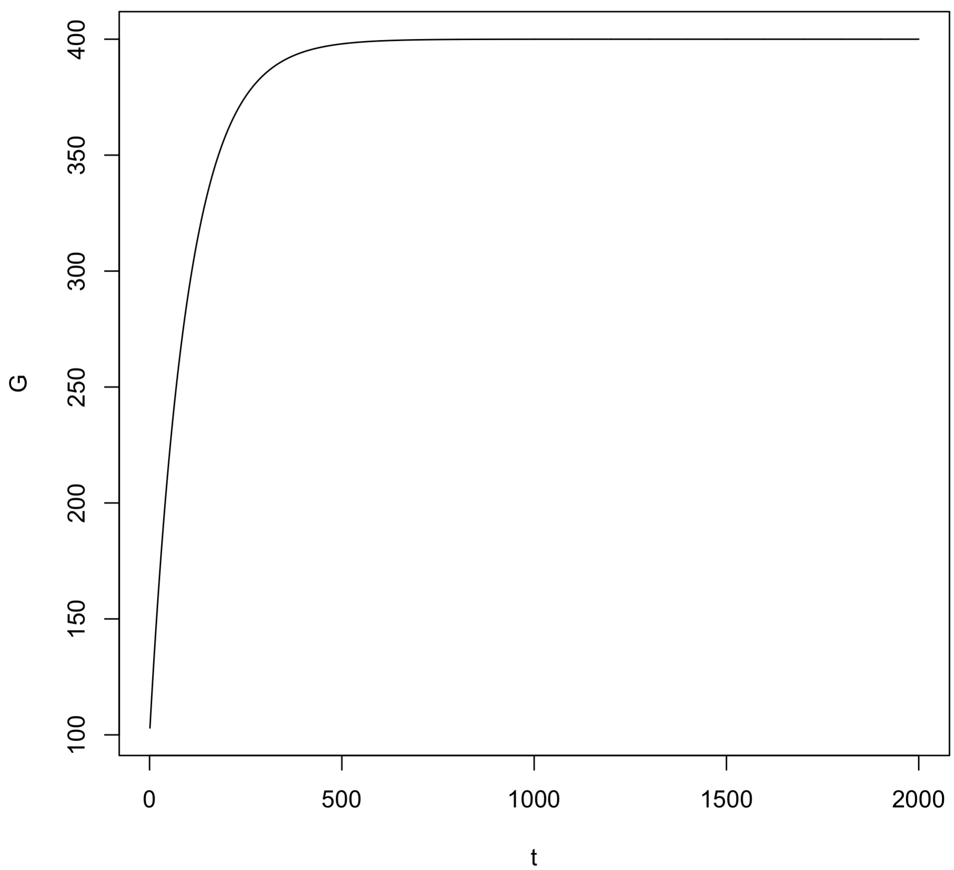

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The equilibrium concentration is stable and it is a/k.