Chapter 6 lab lecture

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Solving the implicit

Eigenvalues/eigenvectors

Predicting long-term behavior

Spectral decomposition

Systematic calculation

Complex functions

Argument branching

Chapter 6 lab lecture

David M. Rosenberg

Committee on Neurobiology University of Chicago

November 18, 2009

Outline

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Homework

- Office hours
- LATEX and TEX interest

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Remember

- This differs from the *forward* Euler method.
- Typically solved analytically.
- The example in the text *only works for that* model ($\dot{x} = at$)
- The computational / cerebral costs of solving the implicit equation first should be considered when choosing a numerical method.

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Example

Consider the model $\dot{x} = 5x - 3$

$$f(x,t) = \dot{x} = 5x - 3$$

$$x_{n+1} = x_n + \Delta t f(x_{n+1})$$

$$x_{n+1} = x_n + \Delta t (5x_{n+1} - 3)$$

$$x_{n+1} (1 - 5\Delta t) = x_n - 3\Delta t$$

$$x_{n+1} = \left(\frac{x_n - 3\Delta t}{1 - 5\Delta t}\right)$$

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Example

Consider the model $\dot{x} = 5x - 3$

```
>data.frame(t=t, x=x);
t x
1 0 2.00000000
2 5 0.5416667
3 10 0.6024306
4 15 0.5999887
5 20 0.60000042
6 25 0.599998
7 30 0.6000000
8 35 0.6000000
9 40 0.6000000
```

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behavior

Stability:	$Re(\lambda_1, \lambda_2) > 0$	$Re(\lambda_1, \lambda_2) < 0$
real:	unstable node	stable node
complex:	unstable spiral	stable spiral

Stability:	$Re(\lambda_1) < 0, Re(\lambda_2) > 0$	$Re(\lambda_1 \ or \ \lambda_2) = 0$
real: complex:	saddle point N/A	degenerate node center point

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$$A = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^{-1}$$

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Spectral decomposition

$$A^2 = A \cdot A$$

$$= Q \Lambda Q^{-1} \cdot Q \Lambda Q^{-1}$$

$$= Q \Lambda^2 Q^{-1}$$

- Exponential matrix operations
- Functional calculus

By similar argument:

$$A^k = Q\Lambda^k Q^{-1}$$

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Argument branching Consider $\hat{A}=Q\Lambda^{-1}Q^{-1}$ $A\cdot\hat{A}=Q\Lambda Q^{-1}\cdot Q\Lambda^{-1}Q^{-1}$ =I

- Exponential matrix operations
- Matrix inversion
- Functional calculus

Thus we have:

$$A^{-1} = Q\Lambda^{-1}Q^{-1} \tag{1}$$

Furthermore if
$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$
, then $\Lambda^{-1} = \begin{bmatrix} \frac{1}{\lambda_1} & 0 \\ 0 & \frac{1}{\lambda} \end{bmatrix}$.

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Sanity checks Exponential matrix operations

- Matrix inversion
- Functional calculus

$$f(x) = a + bx + cx^{2} + \dots$$

$$f(A) = f(Q\Lambda Q^{-1})$$

$$= Qf(\Lambda)Q^{-1}$$

$$\Lambda = \begin{bmatrix} \lambda_{1} & 0\\ 0 & \lambda_{2} \end{bmatrix}$$

$$f(\Lambda) = \begin{bmatrix} f(\lambda_{1}) & 0\\ 0 & f(\lambda_{2}) \end{bmatrix}$$

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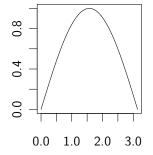
```
>pointCurve <- function(expr, from=0, to=1, xList=NULL, ...) {
   sexpr <- substitute(expr);</pre>
                                          ## Don't worry about this
   curve (expr, from=from, to=to, ...)
   if(!is.null(xList)) {
                                          ## This is KEY -
         Identifying whether
                                               or not to do the
                                                 optional
                                               'thina'
     v <- numeric(length=length(xList));
     for (ii in 1:length(xList)) {
       y[ii] <- eval(call(eval(as.character(sexpr)), xList[ii]));
     points(xList, v. col=rainbow(n=length(xList)), pch=19);
>layout(matrix(c(1, 2), nrow=1));
>par (mar=c(2,2,2,2));
>pointCurve(sin, 0, pi);
                                           ## One hehavior without
      the ara.
>pointCurve(sin. 0. pi. xList=runif(n=10. ## And one with it.
                         min=0. max=pi)):
```

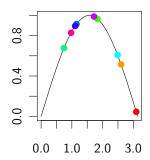
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■ TODO: Example needed