1 Problem

By replacing the 1^{st} digit of $\cdot 3$, it turns out that six of the nine possible values: 13, 23, 43, 53, 73, and 83, are all prime.

By replacing the $3^{\rm rd}$ and $4^{\rm th}$ digits of $56 \cdot \cdot 3$ with the same digit, this 5-digit number is the first example having seven primes among the ten generated numbers, yielding the family: 56003, 56113, 56333, 56443, 56663, 56773, and 56993. Consequently 56003, being the first member of this family, is the smallest prime with this property.

Find the smallest prime which, by replacing part of the number (not necessarily adjacent digits) with the same digit, is part of an eight prime value family.

2 Solution

```
import Data.List
import qualified Data.Map as Map
import Data. Maybe
import System. Environment
import Data.Numbers
{\bf import}\ Data. Numbers. Primes
import qualified Data. Set as Set
primesByReplacement :: (Integral \ a) \Rightarrow [a] \rightarrow [Integer]
primesByReplacement \ x = filter \ isPrime \ potPrimes
  where potPrimes = map (\lambda z \rightarrow replaceStars \ x \ z) \ [1...9]
sPrimes = takeWhile (<20000) primes
buildFamilies :: (Integral \ a) \Rightarrow a \rightarrow [([Int], [Integer])]
buildFamilies x =
  let digs = map \ (\lambda z \rightarrow (read \ (z : "") :: Int)) \ (Set.toList \$ \ (Set.fromList \circ show) \ x)
     fams = map \ (\lambda z \rightarrow replace With Star \ x \ z) \ digs
  in map (\lambda z \rightarrow (z, primesByReplacement z)) fams
replace\ WithStar :: (Integral\ a, Show\ a) \Rightarrow a \rightarrow Int \rightarrow [Int]
replace With Star \ x \ d = ds'
  where ds = map \ (\lambda z \rightarrow read \ (z : "") :: Int) \ (show \ x)
     ds' = map \ (\lambda k \to \mathbf{if} \ k \equiv d \ \mathbf{then} - 1 \ \mathbf{else} \ k) \ ds
replaceStars :: (Integral \ a) \Rightarrow [a] \rightarrow a \rightarrow Integer
replaceStars \ ds \ v = read \ (concat \$ \ map \ show \ (replaceStars' \ ds \ [\ ] \ v)) :: Integer
  where replaceStars' [] ds' v = ds'
     replaceStars' (d:ds) ds' v =
        let d' = if d < 0 then v else d
           ds'' = concat [ds', [d']]
        in replaceStars' ds ds" v
main = do
  let soln = head \$
        filter (\lambda x \to length \ (snd \ x) \ge 8) $
        concat (map buildFamilies primes)
     soln' = minimum \$ snd soln
  putStrLn \$ "The smallest prime that is a member of an eight-prime n" +
     "family is " + show soln' ++ "."
```

3 Result

```
runhaskell problem51.lhs
The smallest prime that is a member of an eight-prime family is 121313.
```