1 Problem

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square.

$$
\begin{aligned}
9 &= 7 + 2 \times 1^2 \\
15 &= 7 + 2 \times 2^2 \\
21 &= 3 + 2 \times 3^2 \\
25 &= 7 + 2 \times 3^2 \\
27 &= 19 + 2 \times 2^2 \\
33 &= 31 + 2 \times 1^2
\end{aligned}
$$

It turns out that the conjecture was false.

What is the smallest odd composite that cannot be written as the sum of a prime and twice a square?

2 Solution

```
import Data.List
import qualified Data.Map as Map
import Data.Maybe
import System.Environment
import Data.Numbers
import Data.Numbers.Primes

oddComposites n = filter (λz → (¬ $ isPrime z) ∧ (odd z)) [1 .. n]

compB n = filter (<n) [2 * k ↑ 2 | k ← [1 .. (round $ sqrt $ fromIntegral n)]]

checkOddComposite :: Integer → Bool
checkOddComposite x =
  let compAs = map ((−) x) (compB x)
      solns = filter isPrime compAs
  in length solns ≡ 0

main = do
  let ocs = oddComposites 10000
      soln = minimum $ filter checkOddComposite ocs
  putStrLn $ "The smallest odd composite that cannot be written as the " ++
        "sum of a prime and twice a squre is " ++ show soln ++ "."
```

3 Result

```
runhaskell problem46.lhs
The smallest odd composite that cannot be written as the sum
of a prime and twice a squre is 5777.
```