1 Problem

Consider all integer combinations of a^b for $2 \leqslant a \leqslant 5$ and $2 \leqslant b \leqslant 5$:

$2^2 = 4$ $2^3 = 8$ $2^4 = 16$ $2^5 = 32$
$3^2 = 9$ $3^3 = 27$ $3^4 = 81$ $3^5 = 243$
$4^2 = 16$ $4^3 = 64$ $4^4 = 256$ $4^5 = 1024$
$5^2 = 25$ $5^3 = 125$ $5^4 = 625$ $5^5 = 3125$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms:

$$4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125$$

How many distinct terms are in the sequence generated by a^b for $2 \leqslant a \leqslant 100$ and $2 \leqslant b \leqslant 100$?

2 Solution

```
import Data.List
import qualified Data.Map as Map
import Data.Maybe
import System.Environment
import Data.Numbers
import qualified Data.Set as Set

isPerfectPower :: Integer -> Bool
isPerfectPower x =
  let pf = group $ primeFactors x
      pl = max 2 (length $ maximumBy (\a b -> compare (length a) (length b)) pf)
      unitFactors = filter (\z -> length z /= pl) pf
  in unitFactors == []

getPerfectExponent :: Integer -> Int
getPerfectExponent x = (length . head . group . primeFactors) x

numUniqueEntries a b
  | not $ isPerfectPower a = b - 1
  | otherwise              = unEls
    where pexp = getPerfectExponent a
          fset      = Set.fromList [pexp * i | i <- [2..b]]
          nonUnique = Set.fromList [i * j | i <- [2..b], j <- [1..(pexp - 1)]]
          unEls     = Set.size $ Set.difference fset nonUnique

main = do
  let nUnique = sum $ map (\a -> numUniqueEntries a 100) [2..100]
  putStrLn $ "There are a total of " ++ show nUnique ++
    " distinct elements in the set {a^b | 2 <= a <= 100, 2 <= b <= 100}."
```

3 Result

```
runhaskell problem29.lhs
There are a total of 9183 distinct elements in the set {a^b | 2 <= a <= 100, 2 <= b <= 100}
```

There are a total of 9183 distinct elements in the set

$$\{a^b : 2 \leqslant a \leqslant 100, 2 \leqslant b \leqslant 100\}$$