1 Problem

It is possible to show that the square root of two can be expressed as an infinite continued fraction.

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \dots}}} = 1.414213\dots \tag{1}$$

By expanding this for the first four iterations, we get:

$$\begin{aligned}
1 + 1/2 = 3/2 &= 1.5 \\
1 + 1/(2 + 1/2) = 7/5 &= 1.4 \\
1 + 1/(2 + 1/(2 + 1/2)) = 17/12 &= 1.41666\dots \\
1 + 1/(2 + 1/(2 + 1/(2 + 1/2))) = 41/29 &= 1.41379\dots
\end{aligned}$$

The next three expansions are $\frac{99}{70}$, $\frac{239}{169}$, and $\frac{577}{408}$, but the eighth expansion, $\frac{1393}{985}$, is the first example where the number of digits in the numerator exceeds the number of digits in the denominator.

In the first one-thousand expansions, how many fractions contain a numerator with more digits than denominator?

2 Solution

```haskell
import Data.List
import qualified Data.Map as Map
import Data.Maybe
import System.Environment
import Data.Ratio

odds = filter odd [1..]

impFraction 1 = 3 / 2 :: Rational
impFraction n = 1 + 1 / (1 + impFraction (n - 1))

improperFraction 1 = (3, 2)
improperFraction n =
  let (a, b) = improperFraction (n - 1)
      b' = a + b
      a' = 2 * a + (odds !! (n - 2))
  in (a', b')

main = do
  let imps = take 998 $ iterate (\z -> 1 + 1 / (1 + z) :: Rational) (3 / 2 :: Rational)
      soln = length $ filter (\z -> length (show $ numerator z) > length (show $ denominator z)) imps
  putStrLn $ show soln
```

3 Result

```
runhaskell problem57.lhs
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```