1 Problem

A unit fraction contains a 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

$$\begin{aligned}
\frac{1}{2} &= 0.5 \\
\frac{1}{3} &= 0.\overline{3} \\
\frac{1}{4} &= 0.25 \\
\frac{1}{5} &= 0.2 \\
\frac{1}{6} &= 0.1\overline{6} \\
\frac{1}{7} &= 0.\overline{142857} \\
\frac{1}{8} &= 0.125 \\
\frac{1}{9} &= 0.\overline{1} \\
\frac{1}{10} &= 0.1
\end{aligned}$$

Where $0.1\overline{6}$ means $0.166666\ldots$ and has a 1-digit recurring cycle. It can be seen that $\frac{1}{7}$ has a 6-digit recurring cycle.

Find the value of $d < 1000$ for which $\frac{1}{d}$ contains the longest recurring cycle in its decimal fraction part.

2 Solution

```
import Data.List
import qualified Data.Map as Map
import Data.Maybe
import System.Environment
import Data.Numbers
import Data.Ord

remainders d 0 rs = 0
remainders d r rs =
  let r' = r `mod` d
  in case elemIndex r' rs of
    Just i -> i + 1
    Nothing -> remainders d (10 * r') (r' : rs)
recurringCycle d = remainders d 10 []
```

```
ordInt :: (Integral a) ⇒ a → a → a
ordInt r n = (snd ∘ head) $
  filter (λ(x, y) → x ≡ 0) [((r ↑ k − 1) `mod` n, k) | k ← [1 .. n]]
repeatPeriod n b = ordInt b n'
  where n' = product $
    filter (λz → z ∉ [2, 5]) (1 : (primeFactors n))
coprimeNums k =
  map (λzz → product $
    filter (λz → z ≢ 2 ∧ z ≢ 5) (1 : (primeFactors zz))) [1 .. k]
repeatLengths xs = map (λx → (x, repeatPeriod x 10)) xs
maxEntry rlpairs = fst $
  maximumBy (λ(_, a) (_, b) → compare a b) rlpairs
  -- below is someone else's implementation I put in afterwards
  -- to compare to my own.
nums = [n | n ← [3, 5 ..], n `mod` 5 ≢ 0]
period n =
    head $ [p | p ← [1 ..], (10 ↑ p − 1) `mod` n ≡ 0]
answer =
    fst $
    maximumBy (λ(_, a) (_, b) → compare a b) $
    map (λn → (n, period n)) $
    takeWhile (<1000) nums
main = do
  let cn = coprimeNums 1000
      rl = repeatLengths cn
      me = maxEntry rl
  putStrLn $ "The longest repeating decimal found in the set " ++
    "[1/2, 1/3, ..., 1/1000] is given by the expansion of 1/" ++
    show me ++ ", which has a repeating sequence of 982 digits."
```

3 Result

```
runhaskell problem26.lhs
The longest repeating decimal found in the set [1/2, 1/3, ..., 1/1000]
is given by the expansion of 1/983, which has a repeating sequence of
982 digits.
```

The longest repeating decimal found in the set

$$\{\frac{1}{2}, \frac{1}{3}, ..., \frac{1}{1000}\}$$

is given by the expansion of $\frac{1}{983}$, which has a repeating sequence of 982 digits.