Solving Linear Systems in C

Various Matrix Decompositions for $Ax = b$

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IAM851: High Performance Computing

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April 13, 2017

A Note on Notation

- Hermitian Conjugate: A^*, $A^\dagger$, or A^H
  - A^* is complex case of the transpose: A^T
  - Hermitian Matrix: $A = A^*$
- Unitary Matrix: Q
  - Q^* is complex case of an orthogonal matrix ($A^{-1} = A^T$)
  - I.e., $QQ^* = Q^*Q = I$ and $AA^T = A^TA = I$
- Lower Triangular Matrix: L
- Upper Triangular Matrix: U
- Diagonal Matrix: D

- QR decomposition: $A = QR$ (need not be square)
- LU decompositions:
  - No pivoting: $A = LU$
  - Partial pivoting: $PA = LU$
  - Full pivoting: $PAQ = LU$
  - Unit triangular and diagonal: $A = LDU$
  - LU reduction (runs LU in parallel)
- Cholesky decomposition: $A = L^*L$

Matrix Decompositions

Eigenvalues and related concepts: $Ax = \lambda x$

- Eigendecomposition: $A = PDP^{-1}$
- Jordan: $A = PJP^{-1}$
- Schur: $A = QUQ^{*}$
- Singular Value: $A = USV^{*}$ (need not be square)

QR

$$A = QR$$
$$Ax = b$$
$$QRx = b$$
$$Q^*QRx = Q^*b$$
$$Rx = Q^*b$$
$$Rx = y$$

LU

- Twice as fast as QR
- Pivoting adds stability

$$
\begin{aligned}
A &= LU \\
LUx &= b \\
Ly &= b \\
Ux &= y
\end{aligned}
$$

Cholesky

- Must be Hermitian and positive-definite
- I.e., positive eigenvalues
- Twice as fast as LU (where appropriate)

$$
\begin{aligned}
A &= L^* L \\
L^* L x &= b \\
L^* y &= b \\
L x &= y
\end{aligned}
$$