DM - Book Activities - Week 1

Ma. Rosario Pajaroja-Villanueva

October 24, 2023

Rewrite each set using the listing method:

1. The set of months that begins with the letter A.
   Answer: A = {April, August}
2. The set of letters of the word GOOGOL.
   Answer: A = {G,O,L}
3. The set of months with exactly 31 days.
   Answer: A = {January, March, May, July, August, October, December}
4. The set of solutions of the equation:
   $x^2 - 5x + 6 = 0$
   =(x-3)(x-2)
   x-3=0
   ~~x-3+3~~=∅+3
   x=3

   x-2=0
   ~~x-2+2~~=∅+2
   x=2

   Answer: A = {3,2}

   Rewrite each set using the set-builder notation.
5. The set of integers between 0 and 5.
   Answer: $A = \{x | x \in Z \, and \, 0 < x < 5\}$
6. The set of January, February, May, and July.
   Answer: A = {x|x is a month name ending in "y"}
7. The set of all members of the United Nations.
   Answer: A = {x|x is all members of the United Nations}
8. {Asia, Australia, Antarctica}
   Answer: A = {x|x is Asia, Australia, Antarctica}

Mark each as true or false.

13. $a \in \{alfa]$
Answer: FALSE

14. $b \subseteq \{a, b, c\}$
Answer: FALSE

b $\in \{a, b, c\}$ *but* $b \nsubseteq \{a, b, c\}$ *because* $a, b, c \subseteq \{a, b, c\}$

15. $\{x\} \subseteq \{x, y, z\}$
Answer: TRUE

16. $\{0\} = \emptyset$
Answer: FALSE

Zero is not equal to an empty set.

17. $0 \in \emptyset$
Answer: FALSE

Zero is not an element of an empty set.

18. $\{\emptyset\} = 0$
Answer: FALSE

An empty set is not equal to 0.

19. $\{\emptyset\} = \emptyset$
Answer: FALSE

An empty set is not equal to empty set.

20. $\emptyset \subseteq \emptyset$
Answer: TRUE

An empty set is a subset of any set.

21. $\emptyset \in \{\emptyset\}$
Answer: TRUE

An empty set is an element of an empty set.

22. $\{x | x \neq x\} = \emptyset$
Answer: TRUE

$\{x \neq x\} = \{\,\} = \emptyset$

23. $\{x, y\} = \{y, x\}$
Answer: TRUE

24. $\{x\} \in \{\{x\}, y, z\}$
Answer: TRUE

To avoid confusion, I am noting: $\{x\}$ is an element of $\{\{x\}, y, z\}$ but not an element of $\{x, y, z\}$ *and* $\{x\}$ is a subset of $\{x, y, z\}$

25. $\emptyset$ is a subset of every set.
Answer: TRUE

26. Every set is a subset of itself.
Answer: TRUE

27. Every nonempty set has at least two subsets.
Answer: TRUE

The $\emptyset$ and another element.

Find the power set of each set.

30. $\emptyset$
Answer: $P(\emptyset) = \{\emptyset\}$

31. $\{a\}$
Answer: $P(\{a\}) = \{\{\}, \{a\}\}$

32. $\{a, b, c\}$
Answer: $P(\{a, b, c\}) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{\}\}$

$\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{\}$

In Exercises 41-44, a language L over $\sum = \{a, b\}$ is given. Find five words in each language.

41. $L = \{x \in \Sigma^* | x \textit{ begins with and ends in } b.\}$
Answer: $L = \{bab, bbabb, baab, bbaab, baaaab\}$

42. $L = \{x \in \Sigma^* | x \textit{ contains exactly one } b.\}$
Answer: $L = \{ba, aba, ab, aaab, baa\}$

43. $L = \{x \in \Sigma^* | x \textit{ contains an even number of } a's.\}$
Answer: $L = \{baa, baaaab, aaaaaabb, aaaaaaaabb, bbaabbaabbaabbaaaa\}$

44. $L = \{x \in \Sigma^* | x \textit{ contains an even number of } a's \textit{ followed by an odd number of } b's.\}$

Answer: $L = \{aab, aaaabbb, aaaaaabbbbb, aaaaaaaabbbbbbb, aaaaaaaaaabbbbbbbbb\}$