Probability and Confidence Intervals

- An important role of statistics: use information gathered from a sample to make statements about the population from which it was chosen.

- Using samples as an estimate of the population.

- How good of an estimate is that sample providing us with?

Estimators of a Population

- A Point estimate is a single value that best describes the population of interest

- Sample mean is the most common point estimate

- An Interval estimate provides a range of values that best describes the population

Point Estimate

- Single value that best describes the population of interest
- Sample mean is most common point estimate
- Easy to calculate and easy to understand
- Gives no indication of how accurate the estimation really is

Interval Estimate

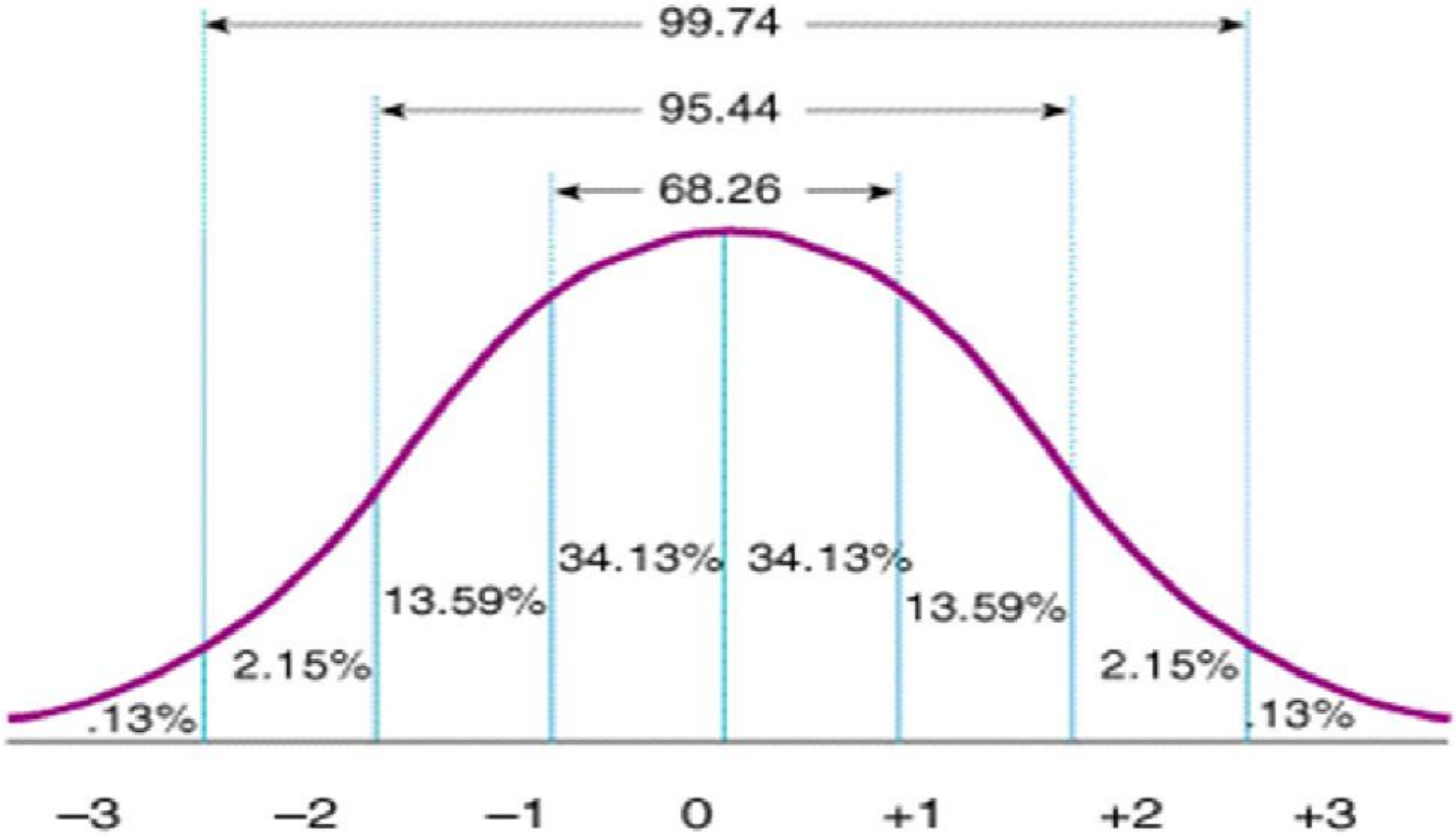
- To deal with uncertainty, we can use an interval estimate
- Provides a range of values that best describe the population
- To develop an interval estimate we need to learn about confidence levels

Confidence Levels

- A confidence level is the probability that the interval estimate will include the population parameter (such as the mean)

- A parameter is a numerical description of a characteristic of the population

Standard Normal Distribution



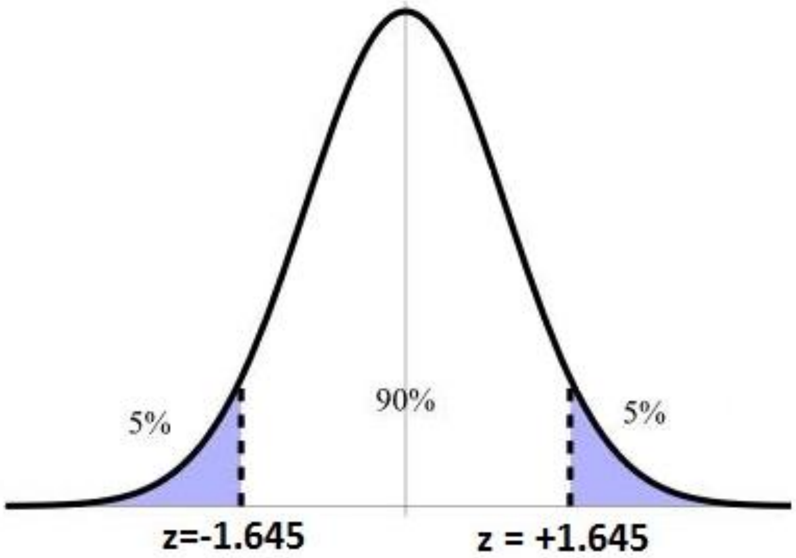
- Normal distribution with μ = 0 and SD = 1

Confidence Levels

- Sample means will follow the normal probability distribution for large sample sizes ($n \geq 30$)

- To construct an interval estimate with a 90 % confidence level

- Confidence level corresponds to a z-score from the standard normal table equal to 1.645



Confidence Intervals

- A confidence interval is a range of values used to estimate a population parameter and is associated with a specific confidence level

- Construct confidence interval around a sample mean using these equations:

$$\overline{x} \pm z\sigma_{\bar{X}}$$

Confidence Intervals

$$\overline{x} \pm z\sigma_{\bar{X}}$$

Where:

$\overline{x}$ = the sample mean

z = the z-score, which is the number of standard deviations based on the confidence level

$\sigma_{\bar{X}}$ = the standard error of the mean

Confidence Intervals

- A confidence interval is a range of values used to estimate a population parameter and is associated with a specific confidence level

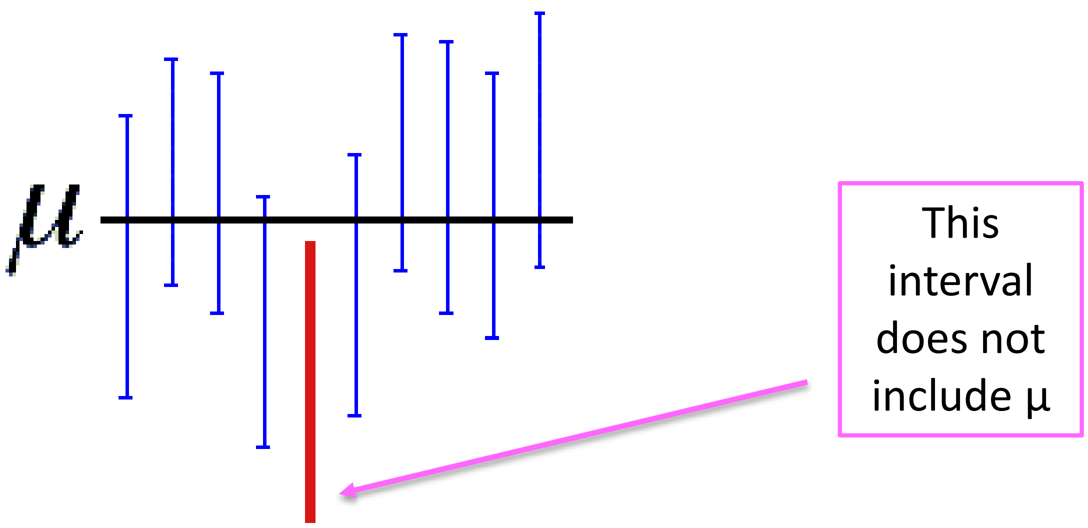
- Associated with specific confidence level

- Needs to be described in the context of several samples

Confidence Intervals

- Select 10 samples and construct 90 % confidence intervals around each of the sample means

- Theoretically, 9 of the 10 intervals will contain the true population mean, which remains unknown



Confidence Intervals

- Careful not to misinterpret the definition of a confidence interval

- NOT Correct – "there is a 90 % probability that the true population mean is within the interval"

- CORRECT – "there is a 90 % probability that any given confidence interval from a random sample will contain the true population mean

Level of Significance

- As there is a 90 % probability that any given confidence interval will contain the true population mean, there is a 10 % chance that it won't
- This 10 % is known as the level of significance (α) and is represented by the purple shaded area

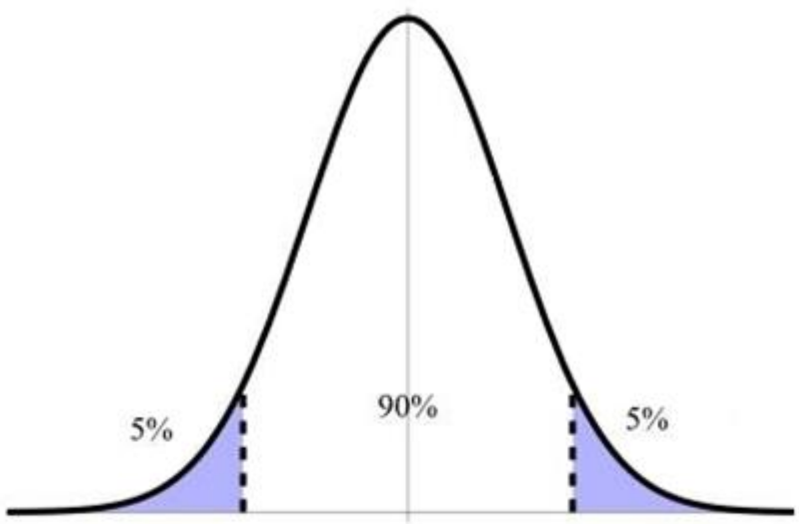


Image accessed: http://www.mathandstatistics.com/learn-stats/finding-z-critical-values

Level of Significance

- Level of significance (α) is the probability of making a type-I error.

- The probability for the confidence interval is a complement to the significance level.

- An $(1 - \alpha)$ confidence interval has a significance level equal to α.

When σ is Unknown

- So far our examples have assumed we know σ - the population standard deviation
- If σ is unknown we can substitute *s* (sample standard deviation) for σ
- $n \geq 30$
- We use $\hat{\sigma}_{\bar{X}}$ to show we have approximated the standard

error of the mean by using *s* instead of σ

Confidence Intervals for the Mean with Small Samples

- So far we have discussed confidence intervals for the mean where $n \geq 30$

- When σ is known, we are assuming the population is normally distributed and so we can follow the procedure for large sample sizes

- When σ is unknown (more often the case!) we make adjustments

When σ is Unknown – Small Samples

- Substitute *s*, sample standard deviation, for σ
- Because of the small sample size, this substitution forces us to use the t-distribution probability distribution
- Continuous probability distribution
- Bell-shaped and symmetrical around the mean
- Shape of curve depends on degrees of freedom (d.f) which equals n - 1