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Low-Fidelity Aerodynamic Analysis of a NACA 0012 Airfoil

The full implementation code is available at:
https://github.com/rosh03a/naca0012-aerodynamic-analysis.git

1. Introduction

The aerodynamic performance of an aircraft is strongly influenced by the pressure distribution and lift characteristics of its wing sections.

Early-stage aircraft design therefore relies heavily on low-order aerodynamic models to provide physical insight and to support preliminary design decisions before higher-fidelity numerical or experimental studies are undertaken.

This project investigates the aerodynamic behaviour of a symmetric NACA 0012 airfoil using low-fidelity computational modelling. The objectives of the study are to:

- generate the airfoil geometry computationally,
- investigate the relationship between angle of attack and lift coefficient,
- model chordwise pressure coefficient distributions,
- and validate lift trends against classical thin airfoil theory.

The project was undertaken to strengthen practical understanding of aerodynamics and fluid mechanics and to develop experience in implementing and interpreting mathematical models relevant to aeronautical engineering.

2. Airfoil Geometry

The NACA 0012 airfoil was selected as a standard symmetric reference profile. The airfoil has zero camber and a maximum thickness of 12% of the chord, making it a commonly used benchmark profile in aerodynamic studies.

The airfoil geometry was generated using the standard NACA four-digit thickness distribution, which defines the half-thickness of the airfoil as a function of chordwise position:

$$y_t(x) = 5t\,(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4)$$

Note: the coefficient −0.1015 is a commonly used standard form; some references use −0.1036 to enforce a closed trailing edge.

where t is the maximum thickness-to-chord ratio, x is the chordwise coordinate normalised by the chord length, and y_t is the half-thickness distribution measured normal to the chord line. For the NACA 0012 airfoil, $t = 0.12$ is the thickness ratio and x is the chordwise coordinate normalised by the chord length.

Cosine spacing was employed in the chordwise direction in order to cluster points near the leading edge, where curvature and aerodynamic gradients are largest. The upper and lower surfaces were constructed by adding and subtracting the half-thickness distribution respectively.

Figure 1 shows the resulting NACA 0012 geometry.

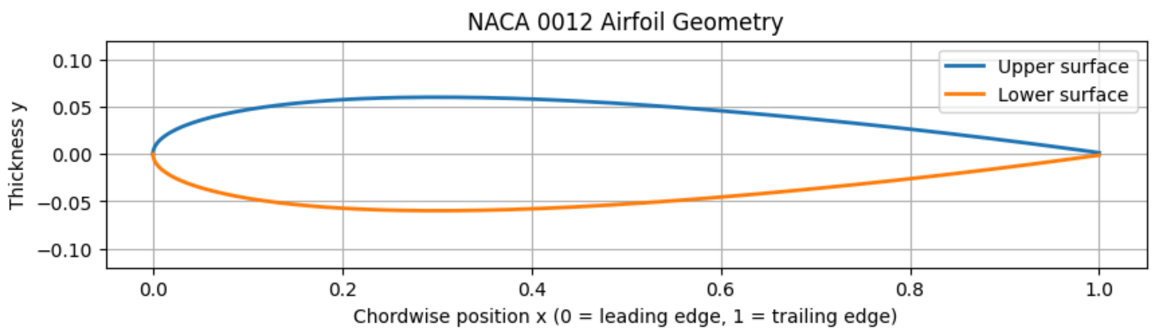

Figure 1: Generated geometry of NACA 0012 airfoil using standard NACA thickness distribution and cosine spacing

3. Lift Modelling Using Thin Airfoil Theory

For small angles of attack and inviscid, incompressible flow, thin airfoil theory predicts a linear relationship between lift coefficient and angle of attack for a two-dimensional airfoil.

For a symmetric airfoil with no camber, such as the NACA 0012, the lift coefficient is given by

$$C_L = 2\pi\alpha$$

where α is the angle of attack in radians, defined as the angle between the chord line and the free-stream velocity vector.

This relationship indicates that lift increases linearly with angle of attack in the attached-flow regime prior to stall. Thin airfoil theory also predicts that the aerodynamic centre is located at approximately the quarter-chord position for symmetric airfoils. Consequently, thin airfoil theory provides a useful theoretical baseline against which simplified aerodynamic models can be compared.

A lift curve was generated over angles of attack from −5° to 15° using thin airfoil theory and is shown in Figure 2. The expected linear behaviour is observed.

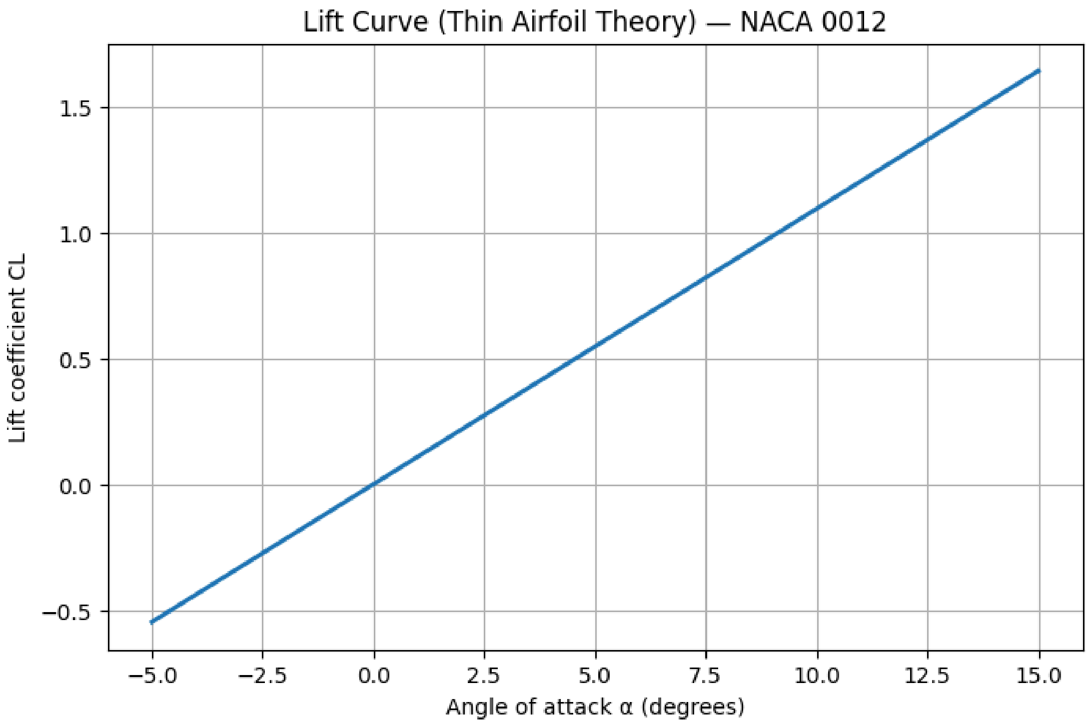

Figure 2: Lift coefficient variation with angle of attack for NACA 0012 airfoil predicted using thin airfoil theory

4. Pressure Coefficient Modelling

In order to investigate how lift arises from surface pressure distributions, a low-fidelity pressure coefficient model was implemented to represent chordwise trends on the upper and lower surfaces of the airfoil.

The model was designed to reproduce the dominant qualitative features of airfoil pressure distributions:

- a strong suction peak near the leading edge on the upper surface,
- higher pressure on the lower surface near the leading edge,
- and recovery of pressure towards the trailing edge.

The pressure coefficient distributions were parameterised as smooth functions of chordwise position and angle of attack. Increasing angle of attack increases the magnitude of the pressure difference between the upper and lower surfaces, particularly near the leading edge.

The model is not derived from a full potential-flow or Navier–Stokes formulation and is therefore intended to provide qualitative and semi-quantitative insight rather than high-fidelity aerodynamic prediction.

An angle of attack of 5° was selected as a representative operating condition within the linear lift regime, where flow remains attached and thin airfoil theory assumptions remain valid. Figure 3 shows the pressure coefficient distribution at this angle of attack.

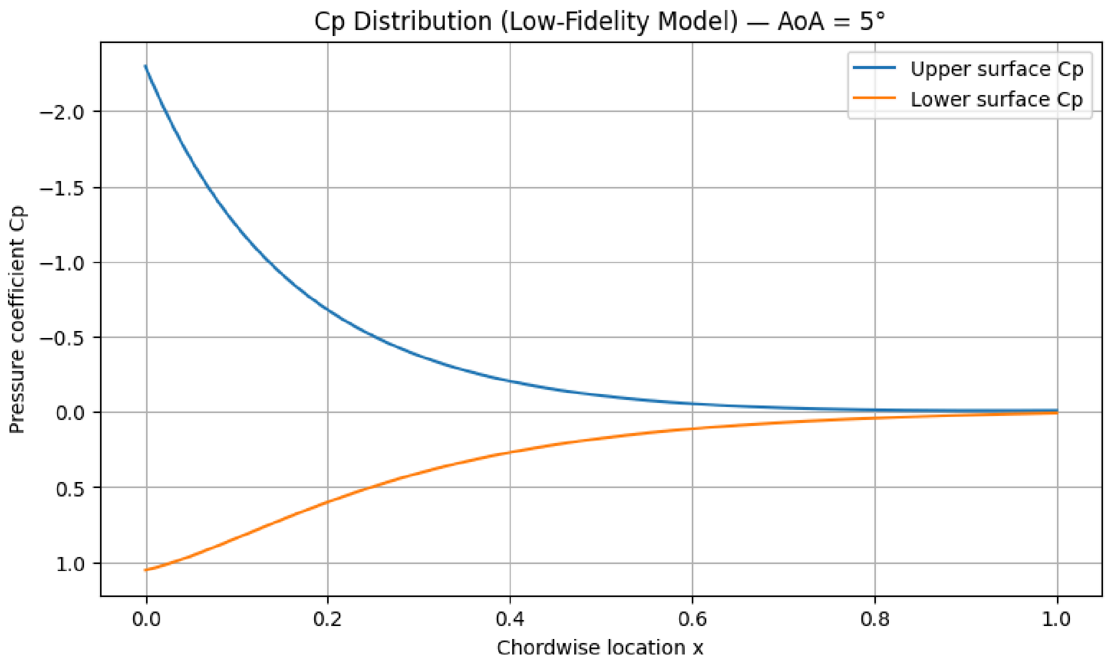

Figure 3: Chordwise pressure coefficient distribution for NACA 0012 airfoil at angle of attack of 5° using low-fidelity model

A strong suction peak is observed near the leading edge on the upper surface, while the lower surface exhibits higher pressure in the same region. Both distributions recover smoothly towards the trailing edge.

To investigate the effect of angle of attack on surface pressure, the upper-surface pressure distributions were evaluated at 0°, 5° and 10°, as shown in Figure 4. Increasing angle of attack leads to a clear increase in the magnitude of the leading-edge suction peak, while the downstream pressure recovery remains similar in shape.

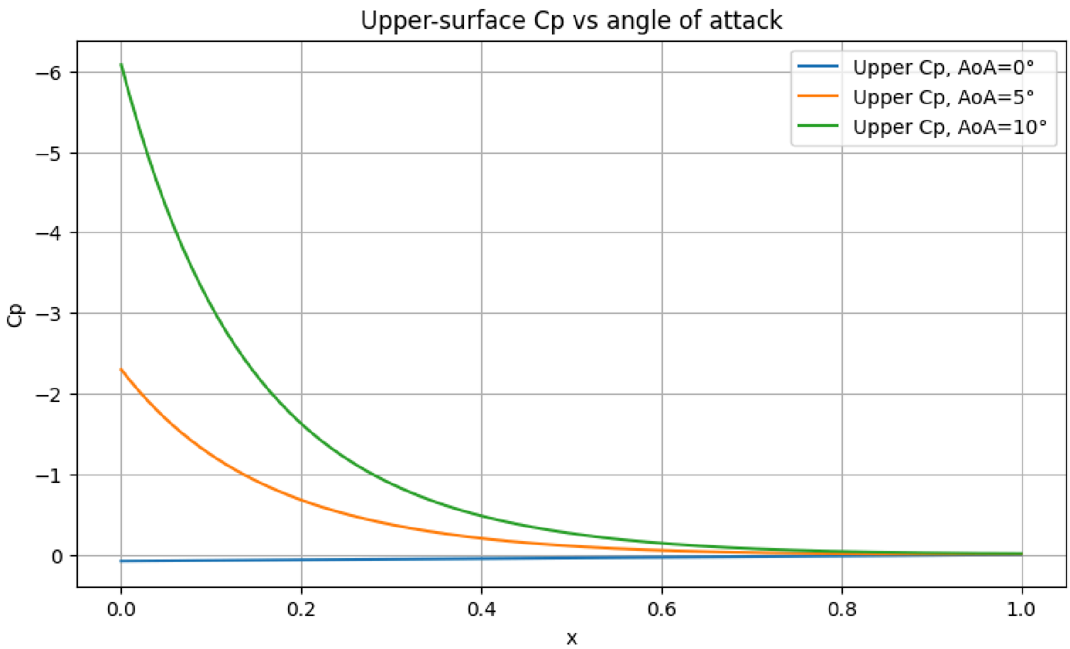

Figure 4: Upper-surface pressure coefficient variation with angle of attack for NACA 0012 airfoil

5. Lift from Pressure Integration and Validation

The sectional lift coefficient was obtained by integrating the pressure coefficient difference between the lower and upper surfaces along the chord:

$$C_L = \int_0^1 (C_{p,lower} - C_{p,upper})\, dx$$

This formulation directly relates lift generation to the surface pressure field and allows the low-fidelity pressure model to be compared with classical aerodynamic theory. For small angles of attack, the pressure-derived normal force coefficient closely approximates the lift coefficient, and the result is therefore reported here as C_L.

A lift curve was generated by evaluating the pressure distributions over angles of attack from −5° to 15° and performing pressure integration at each angle. The resulting lift curve is shown in Figure 5 together with the thin airfoil theory prediction.

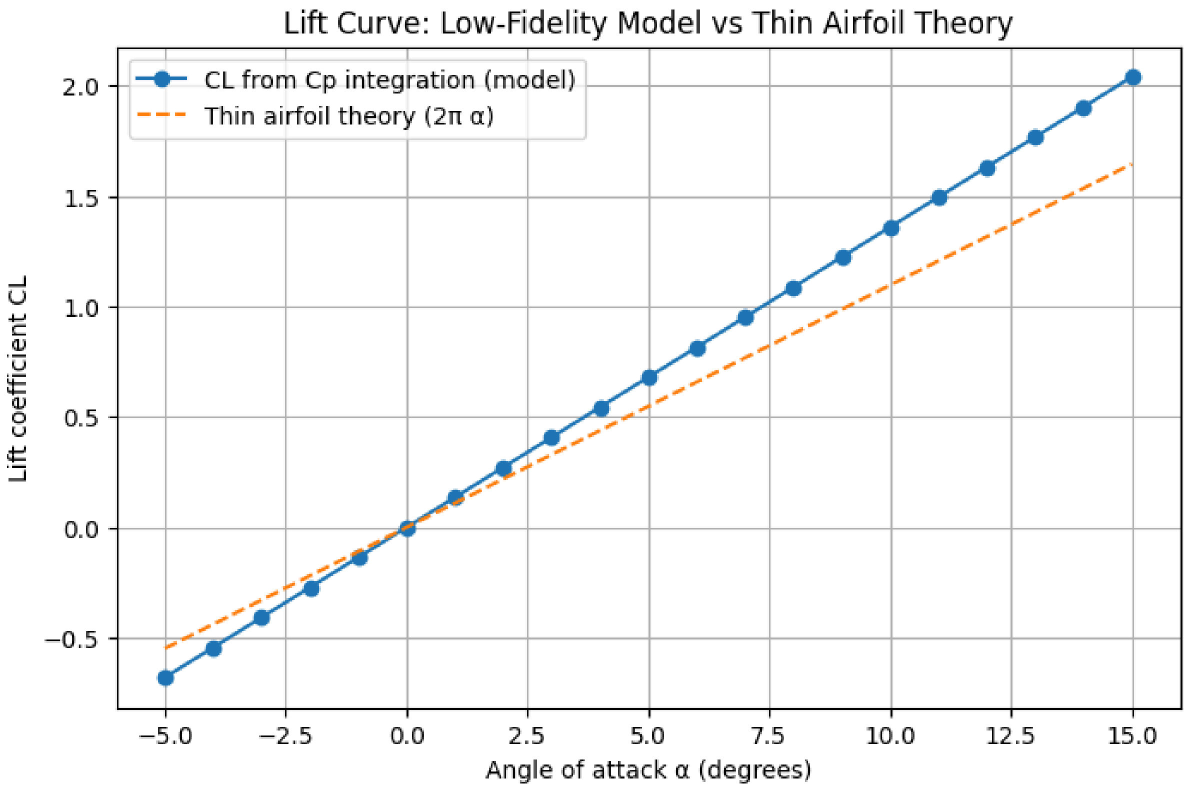

Figure 5: Lift coefficient variation with angle of attack obtained from pressure integration compared with thin airfoil theory for NACA 0012 airfoil

This consistency in trend provides confidence that the low-fidelity model captures the dominant aerodynamic mechanisms governing lift generation.

At an angle of attack of 5°, the low-fidelity model predicts a lift coefficient of $C_L = 0.680$ while thin airfoil theory predicts $C_L = 0.548$.

The model therefore slightly over-predicts lift. This difference is primarily attributed to the simplified representation of the leading-edge suction peak in the pressure distribution model, which produces a stronger pressure differential than predicted by thin airfoil theory.

Despite this difference in magnitude, the expected near-linear dependence of lift on angle of attack is preserved, and the overall trend is consistent with classical aerodynamic theory.

6. Discussion

The results demonstrate that the dominant mechanism of lift generation for the symmetric NACA 0012 airfoil in attached flow is the increase in suction on the upper surface near the leading edge as angle of attack increases.

The pressure distribution plots show that variations in pressure over the forward portion of the airfoil contribute most strongly to the overall lift coefficient, while downstream pressure recovery plays a secondary role. This observation is consistent with aerodynamic theory, where leading-edge suction is a primary contributor to lift generation for thin airfoils operating within the linear regime.

The lift curve obtained from pressure integration exhibits an approximately linear relationship with angle of attack over the investigated range. This behaviour agrees with thin airfoil theory and with experimental observations for symmetric airfoils at small angles of attack prior to stall.

The study also highlights the importance of validating simplified aerodynamic models against established theory in order to identify modelling bias and to ensure that the predicted trends remain physically meaningful.

7. Limitations

The present model is subject to several important limitations:

- The flow is assumed to be inviscid.
- Boundary-layer development and flow separation are neglected.
- The airfoil is treated as a two-dimensional section.
- The pressure distributions are prescribed using a low-order parametric model rather than being obtained by solving the governing flow equations.

In addition, exact symmetry of the pressure distribution at zero angle of attack is not explicitly enforced. Consequently, the model is intended primarily for conceptual analysis and trend investigation rather than for detailed aerodynamic design or performance prediction.

8. Conclusion

A low-fidelity computational framework has been developed to analyse the aerodynamic behaviour of a NACA 0012 airfoil. Airfoil geometry was generated using standard NACA formulations with cosine spacing, and both lift characteristics and surface pressure distributions were investigated.

The study demonstrated how lift can be obtained from chordwise pressure integration and how pressure distributions evolve with angle of attack. Comparison with thin airfoil theory confirmed that the expected linear lift trend is captured, while highlighting the limitations associated with simplified pressure modelling.

Overall, the project demonstrates how mathematical modelling and numerical implementation can be used to gain physical insight into aerodynamic behaviour and provides a foundation for future extension to higher-fidelity panel methods, CFD simulations or experimental studies.

References

Abbott, I. H., & von Doenhoff, A. E. (1959). *Theory of Wing Sections*. Dover Publications.

Anderson, J. D. (2010). *Fundamentals of Aerodynamics* (5th ed.). McGraw-Hill.

Public Domain Aeronautical Software (PDAS). NACA 4-digit airfoil thickness equations. https://www.pdas.com/naca456thick4.html

Airfoil Tools. NACA 0012 airfoil data. http://airfoiltools.com/airfoil/details?airfoil=n0012-il

Appendix A - Key Implementation Functions

The following code excerpts illustrate the principal numerical components used in the aerodynamic modelling framework. The complete implementation is available via the accompanying repository: https://github.com/rosh03a/naca0012-aerodynamic-analysis.git

A.1 Imports

```
import numpy as np
import matplotlib.pyplot as plt
```

A.2 NACA 0012 geometry (cosine spacing)

```
def naca_00xx_thickness(x, t=0.12):
   """
   Half-thickness distribution for a NACA 4-digit symmetric airfoil
   x: chordwise coordinate normalised by chord (0..1)
   t: thickness ratio (NACA 0012 -> t=0.12)
   Returns: y_t(x) half thickness
   """
   x = np.clip(x, 0, 1)  # safety
   yt = 5*t*(0.2969*np.sqrt(x) - 0.1260*x - 0.3516*x**2 + 0.2843*x**3 -
0.1015*x**4)
   return yt

def naca0012_coordinates(n_points=200): #Generate upper and lower surface
coordinates for NACA 0012 using cosine spacing (this gives more points near the
leading edge where curvature is highest)

   beta = np.linspace(0, np.pi, n_points)
   x = 0.5*(1 - np.cos(beta))   #cosine spacing from 0..1

   yt = naca_00xx_thickness(x, t=0.12)
   x_upper, y_upper = x,  yt
   x_lower, y_lower = x, -yt

   return x_upper, y_upper, x_lower, y_lower
```

A.3 Low-fidelity pressure coefficient model + lift from pressure integration

```
def Cp_distribution_low_fidelity(x, alpha_deg): #Low-fidelity Cp model for a
symmetric airfoil at small angles of attack
   """
   Designed to look physically plausible:
   - mild leading-edge suction peak
   - recovery toward trailing edge
   - upper/lower separation increases with AoA
   """
   alpha = np.deg2rad(alpha_deg)

   #Smooth suction shape (bounded): high near LE, decays downstream
   #avoids the extreme 1/sqrt(x) singularity
   suction_shape = np.exp(-4*x) * (1 + 3*(1 - x))

   #Scale with AoA (tuned for realistic Cp range)
   strength = 1.2 * alpha

   #Baseline + AoA-induced delta
   delta = strength * suction_shape

   #Cp definition from a "velocity-like" model, but bounded
   Cp_upper = 1 - (1 + 2.0*delta)**2
   Cp_lower = 1 - (1 - 2.0*delta)**2

   #Mild recovery (optional)
   recovery = 0.08*(1 - x)
   Cp_upper += recovery
   Cp_lower += recovery

   return Cp_upper, Cp_lower

def CL_from_pressure(x, Cp_upper, Cp_lower):
    #CL = integral_0^1 (Cp_lower - Cp_upper) dx

   return np.trapezoid(Cp_lower - Cp_upper, x)
```

A.4 Example: operating point evaluation at 5° (reported values)

```
x = np.linspace(0, 1, 400)
alpha_test = 5

Cp_u, Cp_l = Cp_distribution_low_fidelity(x, alpha_test)
CL_pressure = CL_from_pressure(x, Cp_u, Cp_l)
CL_theory = CL_thin_airfoil(alpha_test)

print(f"AoA = {alpha_test}°")
print(f"CL from pressure integration (model) = {CL_pressure:.3f}")
print(f"CL from thin airfoil theory          = {CL_theory:.3f}")
```

A.5 Example: lift curve generation (model vs theory)

```
alphas = np.linspace(-5, 15, 21)

CL_model = []
for a in alphas:
    Cpu, Cpl = Cp_distribution_low_fidelity(x, a)
    CL_model.append(CL_from_pressure(x, Cpu, Cpl))

CL_model = np.array(CL_model)
CL_theory_curve = CL_thin_airfoil(alphas)
```