IMS555 Decision theory

Topic 3

Decision and valuation

Learning Objectives

- After studying this chapter, you should be able to:
  - Describe the concept of "relation" in options and the use of "number" in representing options and alternatives.
  - Differentiate the use of comparative value terms within relation and preference
  - Understand the completeness issue in relation, preference and domain
  - Understand and describe the concept of transitivity
  - Assign and use utilities in making decision

Using Preferences In Decision Making

- Decision making involve making a choice from several alternatives or options.
- The choice is made based on a well-defined preferences over a set of alternatives .
- Example,

"if an alternative is best then choose it"

and the alternative is where best alternative is the one which is better than all other alternative.

Preference Relation

- To define preference in decision theory, preference relations are used.

- Preference relations is about how people rank or order alternatives.

- Alternatives can be ranked based on a set of assumptions, such as :
  - degree of happiness
  - Satisfaction
  - Gratification
  - enjoyment, etc.

The Comparative Value Terms

- In decision theory, to state preference relations, we mainly use three comparative value terms :
  - **strictly preferred to**
  - **indifferent between**
  - **weakly preferred to.**

The Comparative Value Terms

symbol	**represent**	**Meaning**	**Explanation**
$>$	Strict / strong preference	"better than"	**"A>B"** means A is better than B, or A is strictly preferred to B ***Mathematically defined :*** ***A≻B* if and only if *A≽B* and ¬(*B≽A*)** *(A is better than B if and only if A is at least as good as B and **Not** B is at least as good as A)*
$\equiv$ (~)	indifference	"equal in value to"	**"A ≡ B"** or **"A ∼ B"** means A is indifferent to B, or A is equal in value to B ***A~B* if and only if *A≽B* and *B≽A*** *(A is equal in value to B if and only if A is at least as good as B **And** B is at least as good as A)*
$\geq$	weak preference	"at least as good as"	**"A≥B"** means A is at least as good as B, or A is weakly preferred to B ***A≽B* ↔ *A≻B* ∨ *A~B*** *(A is at least as good as B if and only if A is better than B **Or** A is equal in value to B)*

Example of preference

From economy point of view, preference is identified using the frequency of choice.

X = 

Y = 

Suppose a person is asked to choose between apples(X) and durians(Y), repetitively. A decision scientist observing the experiment will conclude that:

- if apples are chosen all of the time,
  - it would mean that **X is strictly preferred than Y**.
- If half of the time durians are chosen (50% durians are chosen, and 50% apples)
  - then **X is indifference to Y**.
- If 51% (or any % more than 50%) of the time apples are chosen,
  - it means that **X is weakly preferred than Y.**

Rational Preference Relation

- A rational/reasonable preference relation is basically based on these premises:
  - Transitivity
  - Completeness

Completeness

- Completeness simply requires that any two alternatives from a set of choice can be compared by the decision maker.

- Completeness:

for all x, y ∈ Z, (where Z is a list of choices and x, y are the choices) preferences are complete if either $x \geq y$ or $y \geq x$

- In other words, if alternative x and alternative y are elements in a set of choice called Z,
  - Completeness happen when x and y can be compared, either $x \geq y$ (x is at least as good as y) or $y \geq x$ (y is at least as good as x).

Completeness

Set Z

A

B

C

Completeness means you are able to state your preference either :

A ≥ B ≥ C

or

A ≥ C ≥ B

or

B ≥ A ≥ C

or

B ≥ C ≥ A

or

C ≥ A ≥ B

or

C ≥ B ≥ A

Completeness

- The reason for completeness property is because of the **Axioms of Order.**
- According to **Axiom of Order**, for preference theory to be useful mathematically, we need to assume continuity.
- Continuity simply means that there are no 'jumps' in people's preferences
- **Completeness** implies that given any alternatives in a set of choices, our preference relation can compare and rank these alternatives.

Question

Are the preference relations below complete?

- $\mathbf{A \geq B \quad C}$
- $\mathbf{A > B > C}$
- $\mathbf{A > B \geq C}$
- $\mathbf{A \geq B \equiv C}$
- $\mathbf{A \equiv B \geq C}$

Answer

Are the preference relations below complete?

- **$A \geq B \quad C$ → $A \geq B$, B ??? C , A ??? C =** 

- **$A > B > C$ → $A>B$, $B>C$, $A>C$ =** 

- **$A > B \geq C$ → $A>B$, $B\geq C$, $A>C$ =** 

- **$A \geq B \equiv C$ → $A \geq B$, $B \equiv C$, $A \geq C$ =** 

- **$A \equiv B \geq C$ → $A \equiv B$, $B \geq C$, $A \geq C$ =** 

Transitivity

- Transitivity implies a relation between three elements such that if it holds between the first and second, and it also holds between the second and third, therefore it must necessarily hold between the first and third.

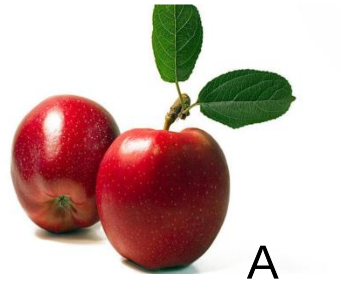
- Axiom of Transitivity
  - if alternative A is preferred to alternative B, and B is preferred to C, then A is preferred to C.

- **for all a, b, c $\in$ X, (where X is a set of choices and a, b and c are alternatives in the set), preferences are *transitive* if**

$a \geq b$ and $b \geq c$; and $a \geq c$.

Transitivity

Set Z



A

B

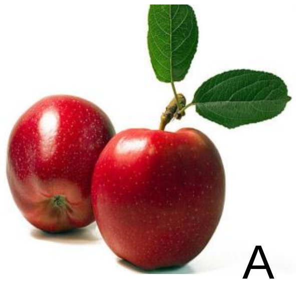
C

if A is preferred to B, and B is to C, therefore A is preferred to C.

- **if A > B and B > C; and A > C** $\rightarrow$ transitive
- **if A > C and C > B; and A > B** $\rightarrow$ transitive
- **if B > C and C > A; and B > A** $\rightarrow$ transitive
- **if B > A and A > C; and B > C** $\rightarrow$ transitive
- **if C > A and A > B; and C > B** $\rightarrow$ transitive
- **if C > B and B > A; and C > A** $\rightarrow$ transitive

Transitivity

Set Z



A

B

C

if A is preferred to B, and B is to C, therefore A is preferred to C.

- **if A > B and B > C; and C > A** → Not transitive
- **if A > C and C > B; and B > A** → Not transitive
- **if B > C and C > A; and A > B** → Not transitive
- **if B > A and A > C; and C > B** → Not transitive
- **if C > A and A > B; and B > C** → Not transitive
- **if C > B and B > A; and A > C** → Not transitive

Cycles in preferences or cyclic preferences

Question

if A > B and B > C; and C > B

- Is the preference relation above transitive?

Answer

if $A > B$ and $B > C$; and $C > B$

- Is the preference relation above transitive?

Answer: Not transitive because there is cyclic preference between B and C

Completeness and Transitivity

- When a preference order is both **transitive and complete**, then it is a standard practice to call it **a *rational preference relation***, **and the people who comply with it are *rational agents***.[1]
- Transitivity of preferences is a fundamental principle in rational decision making.
- The transitivity of preference assumption is meant to rule out irrational preference cycles.
- Cycles in preferences seem irrational.
- Any claim of violations of transitivity by decision maker requires sufficient evidence to eliminate any doubt.

[1] Source: Regenwetter, Michel, Dana, Jason. (2011). Transitivity of Preferences. Psychological Review 118(1), 42–56.

Relations & Numbers

- **Numerical assignment** can also be used to **represent the values of the alternatives** that decision maker decides between.

Relations & Numbers

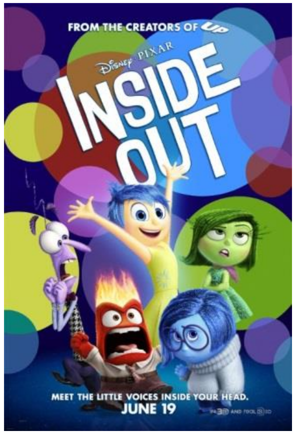
A

B

C

Relation	**Number**
The preference relation of the the house or car may be expressed as; Handphone A better than Handphone B Handphone B better than Handphone C Handphone A better than Handphone C	Using numerical value, the expression would be changed to; Handphone A = **90** Handphone B = **75** Handphone C = **65**
Choice : Handphone A WHY ? : **Handphone A better** than two other handphones	Choice: Handphone A WHY ? : **Handphone A has a higher value** than either Handphone B or Handphone C

Numerical Preferences



A B C

- Another example.
- The numerical values are given as follows;

Movie	Rating
Movie A	****
Movie B	*****
Movie C	***

- **Which is the preferred movie?**
- **B since the numerical value representing the rating is highest.**

Numerical Preferences

- From the numerical value, it is good enough to obtain a preference and indifference representation.

- **Problem**: **the meaning of number representation is unclear** since the number associated with each alternative is **arbitrary** (subject to individual will or judgment).
- Despite the arbitrary in number assignment, numeric values are still easy to use in decision-making.

Utility Theory

- The assigning numerical value to alternative or outcome is called **utility**.

- Utility is estimated based on the decision maker's personal judgment.

- Utility theory only explain how utilities are used to make choices, but cannot explain why a decision maker values one alternative over another alternatives.

- Utility enable us to make decision by assessing number(utility) of each alternatives.

Assigning utility

- Utility must be dimensionless because utility is used in calculation.

- Research in Decision Theory highlighted basic sources of utility:
  - Utilities are constructed, at the moment and on the spot, in a manner that serves the purpose of the immediate decision context (Slovic 1995)

  - Prospect theory (Kahneman &Tversky 1979)postulates that utility are determined by comparing an outcome to some contextually dependent reference point.

Assigning utility

Decision theorists distinguish three kinds of utility scales:

- Ordinal
- In an ordinal scale, preferred outcomes are assigned higher numbers, but the numbers don't tell us anything about the differences or ratios between the utility of different outcomes.
- 12 is better than 6
- Interval
- An interval scale gives us more information than an ordinal scale.
- Example:
- 1-very poor, 2-poor, 3-average, 4-good, 5-very good.
- Ratio
- Numerical utility assignments on a ratio scale give us the most information of all. *(But not everybody value these ratio scale the same.We will talk about this in topic 5).*
- Example :
- Price/cost ($), distance (km), weight (kg)

Utility Notation

- Notation : **U(x)** ; U mean utility, x is the alternative/outcome.

- Example, **U(x) = 6** means Utility of alternative/outcome x is 6.

Utility must represent preference

- In economics, a utility function in a preference is such as :

$$\mathbf{u(A) \geq u(B)} \text{ if and only if } \mathbf{A \geq B}$$

A

B

If you prefer apple (A) than orange (B) , then utility for A should be higher than utility for B, i.e. U(A) = 10, U(B) = 6

Therefore, **if A ≥ B then u(A) ≥ u(B)**

Utility

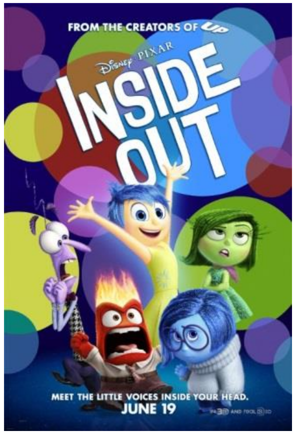
- Another example.
- The numerical values are given as follows;
  - Nokia 70
  - Sony Ericsson 80
  - Samsung 95

Utility - ordinal

- **Which is the preferred choice?**
- **Samsung since its numerical value or utility is highest.**

- **Why is Samsung given highest utility?**
- **Only the decision maker knows why.**

Utility



A

B

C

Rating of movie:
1-very poor
2-poor
3-average
4-good
5-very good

- Another example.
- The numerical values are given as follows;
  - Movie A 4
  - Movie B 5
  - Movie C 3

Utility - interval

- **Which is the preferred choice?**
- **B since its numerical value representing the rating is highest .**

Preference and **difficult choice**

- There are many cases in which **no alternative is found as best**, since the "best" is "shared" by two or more alternatives

- Example, and the decision is to have one preferred out of two.
  - Handphone A and Handphone B are equally good **(A ≡ B)**
  - Handphone A is better than Handphone C **(A > C)**
  - Handphone C is better than Handphone C **(B > C)**
  - So, which one to choose, A or B?

- When faced with the above situation where **(A ≡ B),** it means the **decision making is under uncertainty**.

Maximization (Preference and difficult choice)

- The decision maker needs to be careful if there are two alternatives with equal maximal value as in the following example;
  - Handphone A **80**
  - Handphone B **80**
  - Handphone C **70**

- If more than one alternative has the highest utility, **maximization of the expected utility** can be used in choosing one from both alternatives.

- Therefore, preference ordering over the two alternatives is determined by the **expected utility** associated with A and B.

- **Maximization** means that choosing the alternative with the best or highest expected outcome, without regard to cost or expense.

Conclusion

- The use of relation and numerical models to represent option in making decision
- Preference can be stated using comparative value terms namely >, ≥ and ≡ .
- Completeness of the preference relation follows from the assumption that element of preference has nonempty values
- A preference relation “>” is transitive if and only if it holds for all elements A, B, and C of its domain that if A>B and B>C, then A>C
- Preference can be represented using number.
- Assigning number to alternative/outcome based on preference is called utility
- Maximization of expected utility can be used as a strategy to choose between 2 alternatives with same highest utility

Fun Discussion

- A farmer with his wolf, goat, and cabbage come to the edge of a river they wish to cross. There is a boat at the river's edge, but of course, only the farmer can row. The boat can only handle one animal/item in addition to the farmer. If the wolf is ever left alone with the goat, the wolf will eat the goat. If the goat is left alone with the cabbage, the goat will eat the cabbage. In what order should the farmer bring all his possessions to get across the river?

Discussion

- In decision theory, besides maximization, there are also strategies called satisficing. Search in the Internet and discuss the difference between maximizing and satisficing.