Ejercicio-Ejemplo (15 min)

[E1] A mass m is connected to a vertical revolving axle by two strings of length l, each making an angle of 45° with the axle, as shown. Both the axle and mass are revolving with angular velocity ω. Gravity is directed downward.

a) Draw a clear force diagram for m.

b) Find the tension in the upper string, $T_{\rm up}$, and lower string, $T_{\rm low}$.

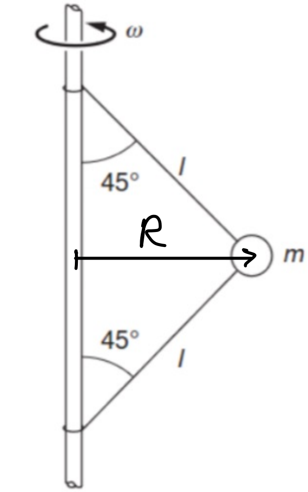


Fig. 1: Prob. 1.

$R = l\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} l$

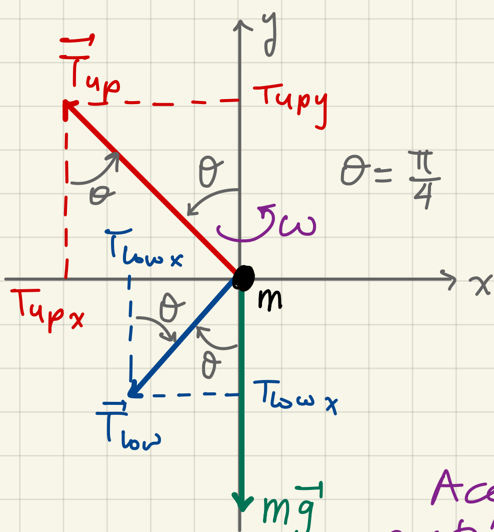
Sol:

a) Identifiquemos las fuerzas sobre m:

1. Tensión superior, $\vec{T}_{up}$. $\vec{T}_{up} = T_{upx}\,\hat{\imath} + T_{upy}\,\hat{\jmath} = -T_{up}\sin(\frac{\pi}{4})\hat{\imath} + T_{up}\cos(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}T_{up}\hat{\imath} + \frac{\sqrt{2}}{2}T_{up}\hat{\jmath}$

2. Tensión inferior, $\vec{T}_{low}$. $\vec{T}_{low} = T_{lowx}\,\hat{\imath} + T_{lowy}\,\hat{\jmath} = -T_{low}\sin(\frac{\pi}{4})\hat{\imath} - T_{low}\cos(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}T_{low}\hat{\imath} - \frac{\sqrt{2}}{2}T_{low}\hat{\jmath}$

3. Peso, $m\vec{g}$ $\qquad m\vec{g} = -mg\hat{\jmath}$



b) Planteamos 2da ley de Newton:

$$F_{Tx} = \sum_m F_x = -\frac{\sqrt{2}}{2}T_{up} - \frac{\sqrt{2}}{2}T_{low} = ma_x$$

$$F_{Ty} = \sum_m F_y = \frac{\sqrt{2}}{2}T_{up} - \frac{\sqrt{2}}{2}T_{low} - mg = ma_y.$$

Análisis cinemático: M.C.U $\rightarrow$ vel. angular ω.

Acel. centrípeta $\rightarrow a_x = -R\omega^2$, $\quad a_y = 0 \leftarrow$ No hay movimiento vertical.

$R = \frac{\sqrt{2}}{2} l \Rightarrow a_x = -\frac{\sqrt{2}}{2} l\omega^2$

Teniendo en cuenta las aceleraciones:
$$\begin{cases} -\frac{\sqrt{2}}{2}T_{up} - \frac{\sqrt{2}}{2}T_{low} = -m\frac{\sqrt{2}}{2}L\omega^2 \\ \frac{\sqrt{2}}{2}T_{up} - \frac{\sqrt{2}}{2}T_{low} - mg = 0 \end{cases}$$

Resolvemos: $\oplus$ las dos Ecs. $-\sqrt{2}\,T_{low} - mg = -\frac{\sqrt{2}}{2}mL\omega^2$

$$\Rightarrow T_{low} = \frac{1}{2}m\left(L\omega^2 - \sqrt{2}g\right)$$

$\ominus$ Las dos Ecs. $-\sqrt{2}\,T_{up} + mg = -\frac{\sqrt{2}}{2}mL\omega^2$

$$\Rightarrow T_{up} = \frac{1}{2}m\left(L\omega^2 + \sqrt{2}g\right)$$