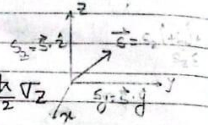
Basics to Advance Quantum Computing

By
Guna Nidhi Poudel

DATE:

$$\psi = e^{i\gamma}\left[\cos\left(\tfrac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\tfrac{\theta}{2}\right)|1\rangle\right]$$

Let us consider the spin matrix



$$S_x = \frac{\hbar}{2}\sigma_x \quad , \quad S_y = \frac{\hbar}{2}\sigma_y \quad ; \quad S_z = \frac{\hbar}{2}\sigma_z$$

$$S_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad S_y = \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad , \quad S_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$(\vec{S}\cdot\hat{x}) = \hbar/2$
$(\vec{S}\cdot\hat{y}) = \hbar/2$
$(\vec{S}\cdot\hat{z}) = \hbar/2$
similarly
$(\vec{S}\cdot\hat{n}) = \hbar/2$

The eigen value of these spin matrix is $\pm\frac{\hbar}{2}$

(a) Eigen value equation of S_x

(i) $\lambda = \frac{\hbar}{2}$

$$\hat{A}\psi = \frac{\hbar}{2}\psi$$

$$\frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{\hbar}{2}\begin{bmatrix} y \\ x \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} x \\ y \end{bmatrix}$$

$y = x$ put $x = 1$
$x = y$

$$\psi = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Normalized wavefunction

$$\psi = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

In Quantum Computing this point is represent by $|+\rangle$.

(ii) $\lambda = -\frac{\hbar}{2}$

$$\hat{A}\psi = -\frac{\hbar}{2}\psi$$

$$\frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{\hbar}{2}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{\hbar}{2}\begin{bmatrix} y \\ x \end{bmatrix} = \frac{-\hbar}{2}\begin{bmatrix} x \\ y \end{bmatrix}$$

$y = -x$ put $x = 1$
$x = -y$ then $y = -1$

$$\psi = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Normalized wavefunction

$$\psi = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

In Quantum Computing this point is reprented by $|-\rangle$.

DATE:

(b) Eigen value equation of S_z

(i) $\lambda = \frac{\hbar}{2}$

$$\hat{A}\psi = \frac{\hbar}{2}\psi$$

$$\frac{\hbar}{2}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\psi = \frac{\hbar}{2}\psi$$

$$\frac{\hbar}{2}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix}x\\y\end{bmatrix}$$

$$\frac{\hbar}{2}\begin{bmatrix}x\\-y\end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix}x\\y\end{bmatrix}$$

$x = x \qquad x = 1$

$y = -y \Rightarrow y = 0$

$$\psi = \begin{bmatrix}1\\0\end{bmatrix}$$

Normalised wavefunction

$$\psi = \frac{1}{\cancel{\sqrt{1}}}\begin{bmatrix}1\\0\end{bmatrix}$$

In Quantum Computing this point is represent by $|0\rangle$.

(ii) $\lambda = -\frac{\hbar}{2}$

$$\hat{A}\psi = -\frac{\hbar}{2}\psi$$

$$\frac{\hbar}{2}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = -\frac{\hbar}{2}\begin{bmatrix}x\\y\end{bmatrix}$$

$$\frac{\hbar}{2}\begin{bmatrix}x\\-y\end{bmatrix} = -\frac{\hbar}{2}\begin{bmatrix}-x\\-y\end{bmatrix}$$

$x = -x \Rightarrow x = 0$

$-y = -y \qquad y = 1$

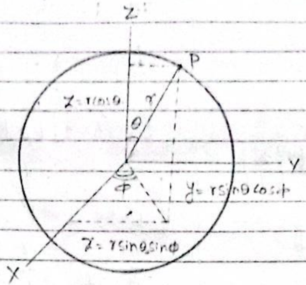
$$\psi = \begin{bmatrix}0\\1\end{bmatrix}$$

Normalised wavefunction

$$\psi = \begin{bmatrix}0\\1\end{bmatrix}$$

In Quantum computing this point is represent by $|1\rangle$.

Spherical Co-ordinate System



Position vector of $P = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

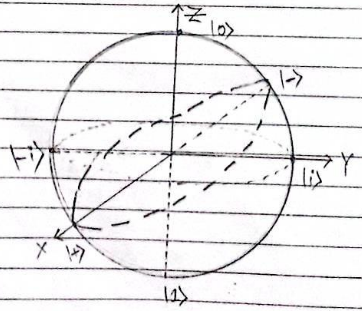
$$= r\sin\theta\cos\phi\,\hat{i} + r\sin\theta\sin\phi\,\hat{j} + r\cos\theta\,\hat{k}$$

$$\therefore \vec{r} = r\sin\theta\cos\phi\,\hat{i} + r\sin\theta\sin\phi\,\hat{j} + r\cos\theta\,\hat{k}$$

$$= r\,[\sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k})$$

$$\frac{\vec{r}}{r} = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$$

$$\boxed{\hat{n} = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}}$$



Measure of spin in any direction

$$\hat{A} = \vec{S} \cdot \hat{n}$$

$$= (S_x \hat{i} + S_y \hat{j} + S_z \hat{k}) \cdot (\sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k})$$

$$= S_x \sin\theta\cos\phi + S_y \sin\theta\sin\phi + S_z \cos\theta$$

$$= \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\sin\theta\cos\phi + \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\sin\theta\sin\phi + \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\cos\theta$$

$$= \frac{\hbar}{2}\begin{bmatrix} \cos\theta & \sin\theta(\cos\phi - i\sin\phi) \\ \sin\theta(\cos\phi + i\sin\phi) & -\cos\theta \end{bmatrix}$$

$$= \frac{\hbar}{2}\begin{bmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{bmatrix}$$

(1) Eigen value $\lambda = \frac{\hbar}{2}$

$$\frac{\hbar}{2}\begin{bmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{\hbar}{2}\begin{bmatrix} x\cos\theta + y e^{-i\phi}\sin\theta \\ x e^{i\phi}\sin\theta - y\cos\theta \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$x\cos\theta + y e^{-i\phi}\sin\theta = x$$

$$y e^{-i\phi}\sin\theta = x(1-\cos\theta) = x \cdot 2\sin^2\theta/2$$

$$y e^{i\phi}\, 2\sin\theta/2 \cos\theta/2 = 2x\sin^2\theta/2$$

$$y e^{-i\phi} \cos\theta/_2 = x \sin\theta/_2$$

$$y = \frac{\sin\theta/_2}{\cos\theta/_2} e^{i\phi} x$$

Wavefunction $|\psi\rangle = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ \frac{\sin\theta/_2}{\cos\theta/_2} e^{i\phi} x \end{bmatrix} \doteq \begin{bmatrix} 1 \\ \frac{\sin\theta/_2}{\cos\theta/_2} e^{i\phi} \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta/_2 \\ \sin\theta/_2 \, e^{i\phi} \end{bmatrix} = \cos\frac{\theta}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{i\phi} \sin\theta/_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \cos\theta/_2 \, |0\rangle + e^{i\phi} \sin\theta/_2 \, |1\rangle$$

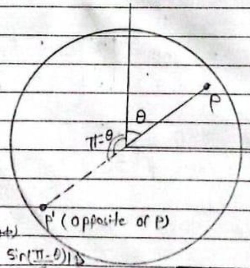
Similarly

② Eigen value $\lambda = -\frac{\hbar}{2}$

$$|\psi\rangle = \cos\theta/_2 \, |0\rangle - e^{i\phi} \sin\theta/_2 \, |1\rangle$$

$\phi \to \pi + \phi$

$\theta \to \pi - \theta$



$|\psi\rangle = \cos\left(\frac{\pi-\theta}{2}\right)|0\rangle + e^{i(\pi+\phi)} \sin\left(\frac{\pi-\theta}{2}\right)|1\rangle$

$$\boxed{|\psi\rangle = \cos\theta/_2 \, |0\rangle - e^{i\phi} \sin\theta/_2 \, |1\rangle}$$

The opposite points on Bloch Sphere represents basis sets.

Quantum Gates

DATE:

Taylor Series

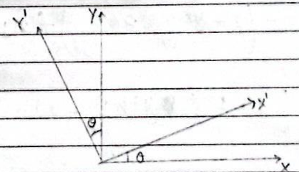
$$f(x)\Big|_{x=a} = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \dots$$

$$f(x+a) = f(x) + a\frac{\partial f}{\partial x} + \frac{a^2}{2!}\frac{\partial^2 f}{\partial x^2} + \frac{a^3}{3!}\frac{\partial^3 f}{\partial x^3} + \dots$$

$$f(x+a) = f(x) + a\frac{\partial f}{\partial x} \quad (\text{small } a \approx 0)$$

$$\boxed{f(x+a, y+b) = f(x,y) + a\frac{\partial f}{\partial x} + b\frac{\partial f}{\partial y}}$$

Rotation Axes



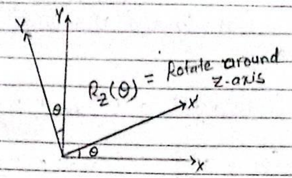
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x' = x\cos\theta + y\sin\theta = x + y\theta$$

$$y' = -x\sin\theta + y\cos\theta = -x\theta + y = y - x\theta$$

DATE:

Spin rotation matrix



$$f'(x',y') = R_z(\theta)\, f(x,y)$$

Again

$$f'(x',y') = f(x+y\theta,\, y-x\theta)$$

$$= f(x,y) + y\theta\frac{\partial f(x,y)}{\partial x} + (-x\theta)\frac{\partial f(x,y)}{\partial y}$$

$$= \left(1 + y\theta\frac{\partial}{\partial x} - x\theta\frac{\partial}{\partial y}\right) f(x,y)$$

$$f' = \left[1 + \frac{1}{i\hbar}\left(y\, i\hbar\frac{\partial}{\partial x} - x\, i\hbar\frac{\partial}{\partial y}\right)\theta\right] f(x,y)$$

$$= \left[1 + \frac{1}{i\hbar}\left(y(-P_x) - x(-P_y)\right)\right] f(x,y)$$

$$= \left[1 + \frac{1}{i\hbar}\left(xP_y - yP_x\right)\right] f(x,y)$$

$$f' = \left[1 + \frac{1}{i\hbar}\hat{L}_z\theta\right] f(x,y)$$

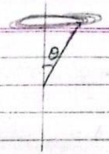
$$f' = \left[1 - \frac{i}{\hbar}\hat{L}_z\theta\right] f(x,y)$$

$\vec{L} = \vec{r}\times\vec{p}$

$\vec{L} = \hat{i}(yP_z - zP_y) + \hat{j}(zP_x - xP_z) + \hat{k}(xP_y - yP_x)$

$\vec{L} = \hat{i}L_x + \hat{j}L_y + \hat{k}L_z$

for n- rotation



$$f' = \left[1 - \frac{i\hat{L}_z \theta}{\hbar}\right]^n f$$

$$= \lim_{n\to\infty} \left(1 - \frac{i\hat{L}_z \theta}{\hbar}\right)^n f$$

$$f' = \underbrace{e^{-\frac{i\hat{L}_z \theta}{\hbar}}}_{\text{Rotational operator in Z-axis}} f$$

Similary for Spin rotation

$$f' = e^{-\frac{i\hat{S}_z \theta}{\hbar}} f \qquad S_z = \frac{\hbar}{2}\sigma_z$$

$$= \exp\left(-i\frac{\hbar}{2}\frac{\sigma_z \theta}{\hbar}\right) f$$

$$f' = \exp\left(\frac{-i\sigma_z \theta}{2}\right) \cdot f$$

Spin rotation Matrix

$$f' = \exp\left(-i\sigma_z \frac{\theta}{2}\right) f$$

$$R_z = \exp\left(-i\sigma_z \frac{\theta}{2}\right)$$

$$\boxed{R_n(\theta) = \exp\left[-i(\vec{\sigma}\cdot\hat{n})\frac{\theta}{2}\right]}$$

This is the Spin rotation matrix.

$$U = e^{i\delta} \exp\left[-i(\vec{\sigma}\cdot\hat{n})\theta/2\right]$$

DATE:

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$\sigma^2 = I$

$\sigma^3 = \sigma^2 . \sigma = \sigma$

$\sigma^n = \begin{cases} I & (n = \text{even}) \\ \sigma & (n = \text{odd}) \end{cases}$

$$e^{i\sigma\theta} = 1 + i\sigma\theta + \frac{(i\sigma\theta)^2}{2!} + \frac{(i\sigma\theta)^3}{3!} + \cdots$$

$$= 1 + i\sigma\theta + \frac{i^2\theta^2}{2!} I + \frac{i^2\sigma\theta^3}{3!} + \cdots$$

$$= I\left[1 - \frac{\theta^2}{2!} + \cdots\right] + i\sigma\left[\theta - \frac{\theta^3}{3!} + \cdots\right]$$

$$= I\cos\theta + i\sigma\sin\theta$$

$$\left\{ e^{-i(\vec{\sigma}.\hat{n})\frac{\theta}{2}} = I\cos\frac{\theta}{2} - i(\vec{\sigma}.\hat{n})\sin\theta/2 \right\}$$

$$R_{\hat{n}}(\theta) = \exp\left\{-i(\vec{\sigma}.\hat{n})\frac{\theta}{2}\right\} = I\cos\frac{\theta}{2} - i(\vec{\sigma}.\hat{n})\sin\frac{\theta}{2}$$

U-Rotational matrix (Spin rotation) $= e^{i\delta}\left[I\cos\theta/_2 - i(\vec{\sigma}.\hat{n})\sin\theta/_2\right]$ ⊛

for rotation matrix.

let $\delta = \pi/_2$

$e^{i\pi/2} = \cos\pi/_2 + i\sin\pi/_2$

$= 0 + i.1$

$\boxed{e^{i\pi/2} = i}$

$\theta = \pi, \delta = \pi/_2$

$$= e^{i\pi/2} \cdot \left[\cos\pi/_2 - i(\vec{\sigma}.\hat{n})\sin\pi/_2\right]$$

$$= i\left[0 - i(\vec{\sigma}.\hat{n}).1\right]$$

$\boxed{(\vec{\sigma}.\hat{n}) = \psi}$ ⇐ Important

DATE:

$$U = \cancel{\hat{R}} = (\vec{\sigma} \cdot \hat{n})$$

Rotation matrix $U = (\sigma_x \hat{i} + \sigma_y \hat{j} + \sigma_z \hat{k}) \cdot (\hat{n})$

$\hat{n} = (1,0,0)$	$\hat{n} = (0,1,0)$	$\hat{n} = (0,0,1)$
σ_x	σ_y	σ_z
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
$-\boxed{X}-$	$-\boxed{Y}-$	$-\boxed{Z}-$
X-Gate	Y-Gate	Z. Gate

When, $\delta = \pi/2$, $\theta = \pi$, $\hat{n} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ from eq^n ⊛

$$U = i \left[I \cdot 0 - i (\sigma_x \hat{i} + \sigma_y \hat{j} + \sigma_z \hat{k}) \cdot \left(\frac{1}{\sqrt{2}} \hat{i} + 0 \hat{j} + \frac{1}{\sqrt{2}} \hat{k} \right) \right]$$

$$= \frac{\sigma_x}{\sqrt{2}} + \frac{\sigma_z}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$-\boxed{H}-$

H-Gate

$$\equiv \frac{1}{\sqrt{2}} \sigma_x + \frac{1}{\sqrt{2}} \sigma_y.$$

DATE:

When $\gamma = 0$, $e^{i\gamma} = e^{i0} = \cos 0^\circ + i \sin 0^\circ = 1$

$$U = I \cos\theta/_2 - i(\vec{\sigma}.\hat{n}) \sin\theta/_2$$

for Z-axis rotation $\quad n = (0,0,1)$

$$U_Z = R_Z = I\cos\theta/_2 - i\sigma_Z \sin\theta/_2$$

$$= \cos\theta/_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i\sin\theta/_2 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta/_2 - i\sin\theta/_2 & 0 \\ 0 & \cos\theta/_2 + i\sin\theta/_2 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-i\theta/_2} & 0 \\ 0 & e^{i\theta/_2} \end{bmatrix} = e^{i\theta/_2} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

Multiplying by $(e^{i\theta/_2})$ ← Global phase

$$\approx \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

$\theta = 2\pi$ $\quad$ $\theta = \pi/_2$ $\quad$ $\theta = \pi/_4$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad$ $S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ $\quad$ $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/_4} \end{bmatrix}$

$\boxed{I}$ $\quad$ $\boxed{S}$ $\quad$ $\boxed{T}$

$S^{+} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ $\quad$ $T^{+} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\pi/_4} \end{bmatrix}$

$\boxed{S^{+}}$ $\quad$ $\boxed{T^{+}}$

Pauli's Gates (X, Y, Z, H)

(i) X- Gate

X	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	0	1
$\langle 1\|$	1	0

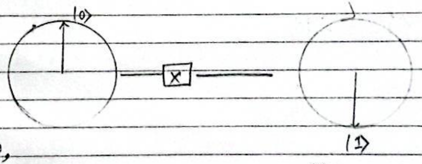
$$\equiv X = |0\rangle\langle 1| + |1\rangle\langle 0|$$

$X|0\rangle =$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \equiv |1\rangle$$

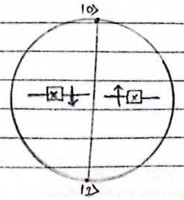
$$\left[|0\rangle\langle 1| + |1\rangle\langle 0|\right]|0\rangle = |0\rangle\overbrace{\langle 1|0\rangle}^{\to 0} + |1\rangle\underbrace{\langle 0|0\rangle}_{1} = |1\rangle$$

$$|0\rangle \text{———} \boxed{X} \text{———} |1\rangle$$



Similarly,

$$X|1\rangle = |0\rangle$$



(ii) Y-Gate

$$\begin{array}{c|cc} Y & |0\rangle & |1\rangle \\ \hline \langle 0| & 0 & -i \\ \langle 1| & i & 0 \end{array} \equiv Y = -i\,|0\rangle\langle 1| + i\,|1\rangle\langle 0|$$

$$Y|0\rangle = i\,|1\rangle$$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ i \end{bmatrix} \equiv i\,|1\rangle$$

$$\left[-i\,|0\rangle\langle 1| + i\,|1\rangle\langle 0|\right]|0\rangle$$

$$= -i\,|0\rangle\overbrace{\langle 1|0\rangle}^{0} + i\,|1\rangle\underbrace{\langle 0|0\rangle}_{1}$$

$$= i\,|1\rangle$$

$$|0\rangle \longrightarrow \boxed{Y} \longrightarrow i\,|1\rangle$$

Similarly

$$|1\rangle \longrightarrow \boxed{Y} \longrightarrow -i\,|0\rangle$$

$$Y|1\rangle = -i|0\rangle$$

(iii) Z-gate (phase change)

Z	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	1	0
$\langle 1\|$	0	-1

$$\equiv \quad Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$Z|0\rangle = |0\rangle$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$\left[|0\rangle\langle 0| - |1\rangle\langle 1|\right]|0\rangle$$

$$= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\cancel{\langle 1|0\rangle}^{\to 0}$$

$$= |0\rangle$$

$$|0\rangle \,—\boxed{Z}—\, |0\rangle$$

Similarly

$$|1\rangle \,—\boxed{Z}—\, -|1\rangle$$

$$Z|1\rangle = -1|1\rangle$$

DATE:

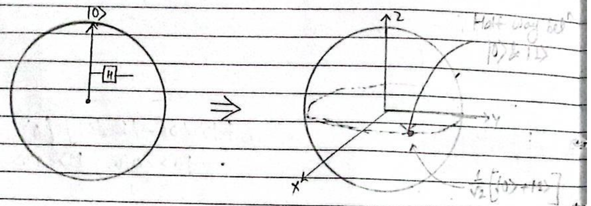
(iv) Hadmard Gate (H-Gate)

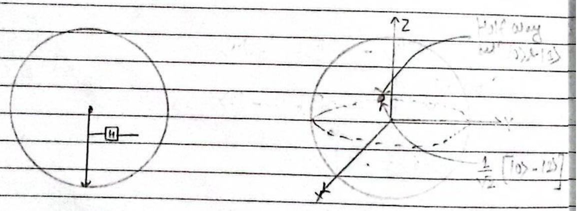
(Superposition state)

H	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\langle 1\|$	$1/\sqrt{2}$	$-1/\sqrt{2}$

$$\equiv H = \frac{1}{\sqrt{2}}\left[|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right]$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right] \quad , \quad H|1\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$$





In General

$$H|x\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + (-1)^{x}|1\rangle\right]$$

Single Qubit Quantum Gates

Gate	Matrix	Projection operator	
$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\|0\rangle\langle 1\| + \|1\rangle\langle 0\|$	$X\|0\rangle = \|1\rangle$ $X\|1\rangle = \|0\rangle$
$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$-i\|0\rangle\langle 1\| + i\|1\rangle\langle 0\|$	$Y\|0\rangle = i\|1\rangle$ $Y\|1\rangle = -i\|0\rangle$
$-\boxed{Z}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\|0\rangle\langle 0\| - \|1\rangle\langle 1\|$	$Z\|0\rangle = \|0\rangle$ $Z\|1\rangle = -1\|1\rangle$
$-\boxed{H}-$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\left[\|0\rangle\langle 0\| + \|0\rangle\langle 1\| + \|1\rangle\langle 0\| - \|1\rangle\langle 1\|\right]$	$H\|0\rangle = \frac{1}{\sqrt{2}}[\|0\rangle + \|1\rangle]$ $H\|1\rangle = \frac{1}{\sqrt{2}}[\|0\rangle - \|1\rangle]$
$-\boxed{I}-$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\|0\rangle\langle 0\| + \|1\rangle\langle 1\|$	$I\|0\rangle = \|0\rangle$ $I\|1\rangle = \|1\rangle$
$-\boxed{S}-$	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$\|0\rangle\langle 0\| + i\|1\rangle\langle 1\|$	$S\|0\rangle = \|0\rangle$ $S\|1\rangle = i\|1\rangle$
$-\boxed{T}-$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	$\|0\rangle\langle 0\| + e^{i\pi/4}\|1\rangle\langle 1\|$	$T\|0\rangle = \|0\rangle$ $T\|1\rangle = e^{i\pi/4}\|1\rangle$
$-\boxed{S^\dagger}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$	$\|0\rangle\langle 0\| - i\|1\rangle\langle 1\|$	$S^\dagger\|0\rangle = \|0\rangle$ $S^\dagger\|1\rangle = -i\|1\rangle$
$-\boxed{T^\dagger}-$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{-i\pi/4} \end{bmatrix}$	$\|0\rangle\langle 0\| + e^{-i\pi/4}\|1\rangle\langle 1\|$	$T^\dagger\|0\rangle = \|0\rangle$ $T^\dagger\|1\rangle = e^{-i\pi/4}\|1\rangle$