INVESTIGATING THE MOTION OF ROLLING OCTAGONAL CYLINDER IN AN INCLINED PLANE

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Introduction:

Our physics textbook discusses only about the rolling of circular body i.e sphere or circular cylinder through the chapter. I questioned myself if the rolling principle was similar to other geometrical shapes. Like a pencil has a hexagonal shape, why can't it be investigated ?[1] Does it have a similar mechanism like a cylinder ? Therefore, I chose to investigate the rolling motion of a pencil (hexagonal shape) which is easily available. After rolling the pencil randomly, I realised that there are some interesting physical phenomena. With limited equipment,the pencil was too small to examine its motion in detail. So, I decided to increase both size and number of sides and concluded to study the motion of rolling octagonal cylinder on an inclined plane.

The rolling of a body down an inclined plane follows the mechanical energy conservation principle as the body when in certain height has some gravitational potential energy which is transformed into its kinetic energy as going down with certain velocity. In ideal condition, as the body rolls the potential energy is transformed into its rotational and translational kinetic energy. However, in realistic cases some of these also go to sound and heat energy.[2] In fact, the interesting thing to notice in rolling octagonal cylinder is how the sides strike on the inclined plane producing a significant amount of sound and probably heat as well. Additionally, things get more interesting

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[1] "Rotational Kinetic Energy: Work and Energy Revisited ...." https://opentextbc.ca/physicstestbook2/chapter/rotational-kinetic-energy-work-and-energy-revisited/. Accessed 4 Jan. 2020.

[2] "Rotational Kinetic Energy: Work and Energy Revisited ...." https://opentextbc.ca/physicstestbook2/chapter/rotational-kinetic-energy-work-and-energy-revisited/. Accessed 4 Jan. 2020.

when the body starts slipping. Thus, throughout this essay I will discuss in regards to the octagonal cylinder:

- Energy transformation during rolling without slipping
- Energy transformation during rolling with slipping
- Comparison and conclusion on the motion of the cylinder.

Firstly, this extended essay has most of the theoretical calculations followed by prior knowledge of momentum, mass, weight, forces, velocities. After enough knowledge in theory, the experiment is carried out considering the uncertainties. Furthermore, there is some data presented in the table extracted from the experiment with all the calculations clearly. Additionally, all the graphs throughout the experiment have been clearly shown. Finally, there are clear calculations of uncertainties in two parts of the whole research paper. In the end, there is a clear appendix showing the data of the entire experiment.

Research Question :

To begin the research, it was important to determine variables for the experiment and to scientifically approach the curiosity the research question was made. Angle of the inclined plane is an independent variable, while dependent variable will be the time it takes to complete the given length and length of the inclined surface and mass, radius of the octagonal cylinder will be the controlled variable for this experiment. Thus, I am curious about, *"How does the angle of the inclined plane affect the linear velocity of the octagonal cylinder ?"*

Theoretical calculation

Calculation of (μ).

When the object is at rest as shown in the figure below:

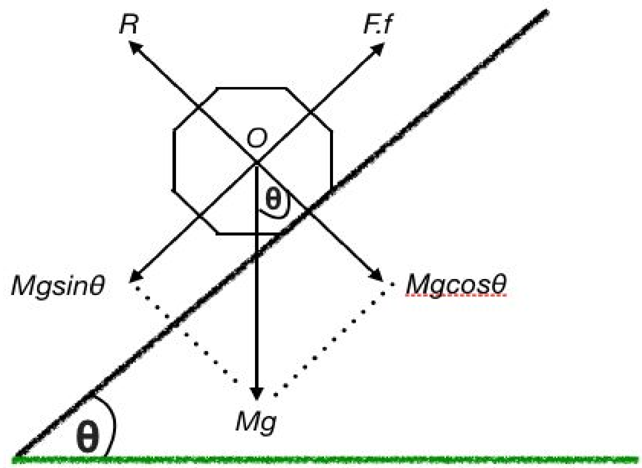


Figure (i) : Side view of Octagon in the incline plane plywood when it is on rest.

From figure (i),

$$F.f = Mg \sin\theta_{min}$$

$$\Rightarrow \mu R = Mg \sin\theta_{min} \qquad [\because F.f = \mu R]$$

And, $R = Mg \cos\theta_{min}$

$$\because \mu Mg cos\theta = Mg \sin \theta_{min}$$

$$\mu = \frac{sin\theta}{cos\theta}$$

$$\mu = \tan\theta_{min}$$

$$\theta_{min} = \arctan(\mu)$$

Where, M : Mass of octagon

θ : Minimum Angle required for the octagon to roll itself

g : Acceleration due to gravity

μ : Static Friction

F.f : Frictional Force acting on the octagon due to surface friction with inclined plane plywood.

R : Normal force acting on the inclined plane acting on the cylinder perpendicular to the plane.

Theoretical calculation of Energy Transformation:

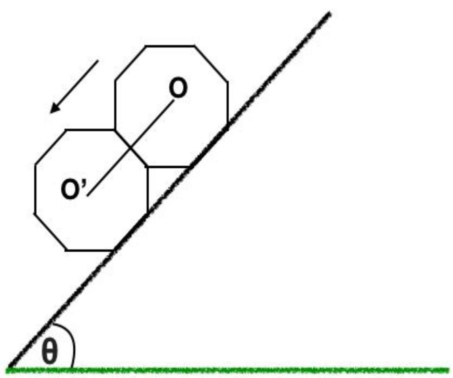
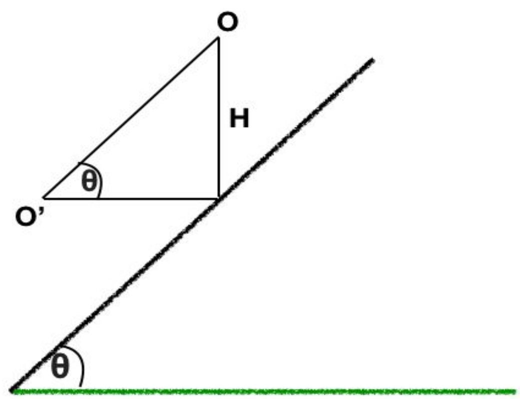


Figure (ii): Side view of the octagon after one strike on the inclined plane



Figure(iii) : Imaginary line drawn to make trigonometric ratios from Fig (ii)

As we know that any rolling body kept on an inclined plane at some angle goes on rolling. Looking in depth from the principle of energy conservation, the rolling body loses its potential energy as it gains kinetic energy (or velocity)[3]. Figure (ii) shows the side view of octagonal cylinder which will be used as rolling body throughout my research paper. The octagon has side length "a".

Let's assume, after one strike, the center moves from O to O' by distance b.

We know from energy conservation principle that:

Loss in potential energy = Gain in kinetic energy

Loss in potential energy = MgH

Where, M is the Mass of the octagonal cylinder and H is the height it falls after one strike.

Using trigonometric equation in figure (iii):

$$\because \sin\theta = \frac{perpendicular}{hypotenuse}$$

So from figure(iii),

$$\Rightarrow \sin\theta = \frac{H}{b}$$

$$\Rightarrow b\sin\theta = H$$

Where, θ: Angle formed after one strike by the slope and imaginary line which is also equivalent to the angle of slope of the inclined plane.

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[3] "conservation of energy | Definition & Examples | Britannica." https://www.britannica.com/science/conservation-of-energy. Accessed 2 Dec. 2019.

b : Distance between the initial and the final centre of the octagon after one strike.

Now, Potential energy (after one turn) = MgH

$= Mgb \sin \theta$

Potential energy (after eight turn/one revolution) = $8Mgb \sin \theta$

When the body strikes the inclined plane 8 times, the object completes a full turn around its rotation axis which passes through O and is parallel to the surface of the inclined plane. So, the equation to calculate the potential energy in one complete rotation.

The octagonal cylinder gains 2 types of kinetic energies i.e. Rotational kinetic energy and Translational kinetic energy.

$$Kinetic\ energy = Translational\ Kinetic\ energy + Rotational\ kinetic\ energy$$

$$= \frac{Mv^2}{2} + \frac{I\omega^2}{2}$$

$$= \frac{M\omega^2 r^2}{2} + \frac{I\omega^2}{2} \left[\because v = \omega r\right]$$

So to calculate the theoretical total kinetic energy we need to find the moment of inertia (I) of the rolling body.

Calculation of Moment of inertia of octagon (I):

Finding angles for the rotation of the body is not enough for the body to continue its rotation. The body is only able to rotate once or twice but not able to reach infinite rolling. Infinite rolling is very important for this research as we are calculating the energy transformation. Rotational kinetic energy is able to make the body roll with infinite rolling. Moment of inertia could be one of the keys to be able to gain infinite rolling. So, the moment of inertia of the octagon is calculated.

From the calculation performed as shown in Appendix A, the moment of inertia of the octagonal cylinder about its center is:

$$I = \frac{2}{3}(5 + 3\sqrt{2}) * Ma^2$$

Similarly, the moment of inertia about its edge is:

$$\therefore I_{edge} = I + M\left(\frac{a}{2sin(\frac{a}{2})}\right)^2$$

Now, we have the moment of Inertia, the side length of the octagonal cylinder, Mass of the cylinder, we can calculate both linear and angular velocities by performing the experiment and finding the energy loss.

Experimental setup and methodology.

Apparatus required

1. Plywood of 76.5cm
2. A wooden octagonal cylinder
3. Two stands
4. Protractor
5. One rod to hold the wooden plank
6. Camera to record the video
7. Computer with logger pro installed

Methodology

Figure (iv) : Experimental setup

1. The experiment set up is to be fixed with the help of two stands and a rod in between to place the wooden plank in the inclined angle.
2. Measure the 76.5cm and mark with a pencil by drawing one straight line in the plywood.
3. Place the octagon on the line and release it to roll down the inclined plane.
4. For the angle between (2º-16º), an angle must be made by slightly lifting the octagon with the finger which is calculated in the table.
5. Take the help of another person to record the video in 40K HD of complete rolling of the octagon in the inclined plane.
6. Take 8 trials at every angle.
7. Analyze the video in logger pro to know the start time and end time. Note the time in excel.

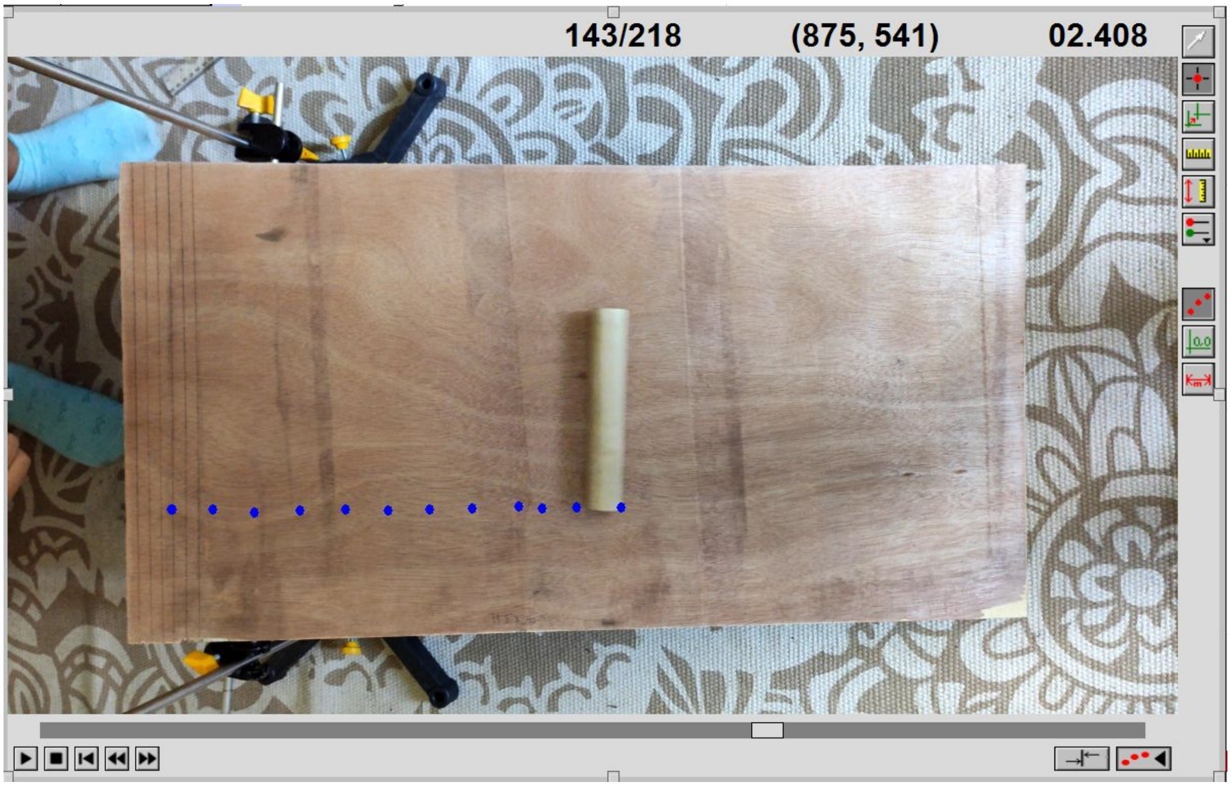


Figure (v): Screenshot showing the sample in the logger pro of video analysis.

Evaluations and Improvements:

Weakness:

- The octagon was handmade, which is why it was not uniform and smooth like a pencil. Try to find an octagon with a perfect shape made by machines.
- It was very difficult to have a perfect rolling of the octagon because the release was not perfect in the plywood from the beginning.
- Slipping was not observed with our device because of bad video resolution. So, a better camera might give better results.

Limitations:

- Devices like motion sensors could not be used in this particular case. It was because the rays of motion detector (echolocation) would not be reflected back correctly as the shape is octagon and rolling on an inclined plane.

Strength :

- The experiment processed in the logger pro has a decimal point of 4 which is very precise and accurate.

Test angles at which the body starts to roll itself

Angles at a specific range are required for the experiment to be carried out. So, I tried to find the angle in which the object starts to rotate itself by changing the height and also the angle at which there is rolling without slipping. Lifting up the wooden plank would give us a limited range of angels which will not be enough for the experiment to be concluded. It is possible to use initial push on the object to roll it down the inclined plane however the initial push would not be constant throughout the whole experiment which would create the human error. So, I found a relation between the angle of the inclined plane and angle of the octagon itself which helps the octagon to rotate on its own.

Side of octagon	Minimum angle of rolling infinitely ± 1º
1	16º
2	17º
3	18º
4	16º
5	16º
6	18º
7	17º
8	17º

Table (i): showing minimum angles for each side at which the body goes infinite rolling

Taking the average of the minimum angle at which rotates :

$$Angle_{min} = \frac{16+17+18+16+16+18+17+17}{8} \pm \frac{18-16}{2}$$

$$= 16.875 \pm 1^{o}$$

$$\approxeq 17^{o} \pm 1^{o}$$

Side of octagon	Maximum angle of rolling without slipping (± 1°)
1	64°
2	64°
3	63°
4	63°
5	65°
6	63°
7	65°
8	64°

Table (ii) : showing maximum angles for each side at which the body goes infinite rolling without slipping.

$$Angle_{max} = \frac{64+64+62+65+65+62+62+64}{8} \pm \frac{65-63}{2}$$

$$= 63.875^{o} \pm 1^{o}$$

$$\approx 64^{o} \pm 1^{o}$$

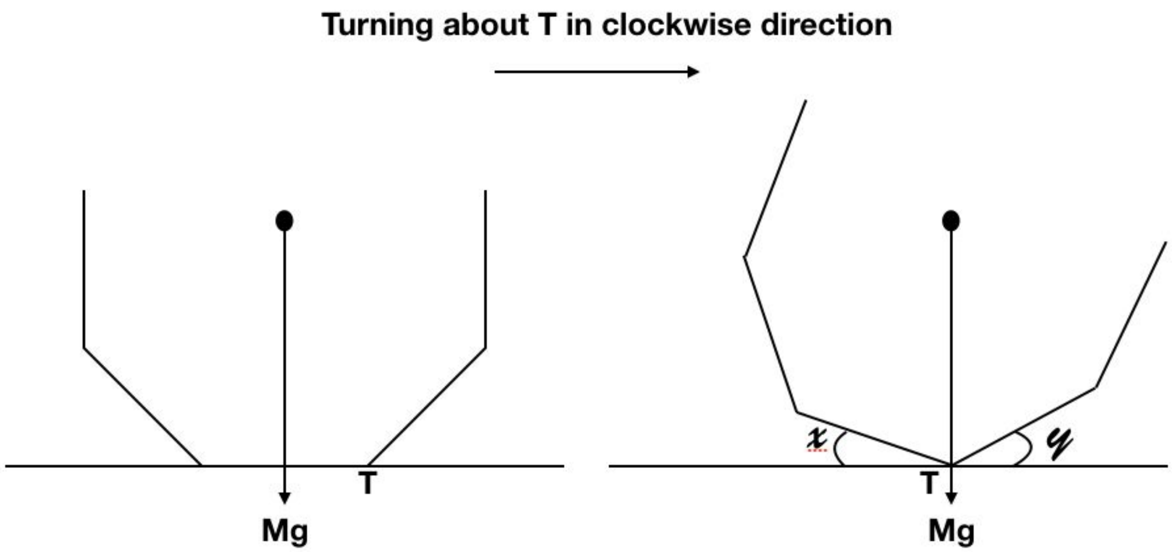
Thus, it can be concluded that the range at which experiment can be carried is [$17^{o} \pm 1^{o}$, $64^{o} \pm 1^{o}$] . So the range is of 47° ± 3°. When we have a higher number of trials, we can have accurate results which can draw us on to reliable conclusions. However, the range of 47° ± 3° might not be accurate enough to define the motion of the octagonal cylinder.

Analyzing the range of angles (above experiment):

1. Since we have a small range of angles at which we have to perform the experiment. As a solution to this, during the experiment it was observed that an initial push on the rolling body makes the body roll infinitely. However, it is hard to give a constant initial push during each trial.
2. But instead of ‘literal initial push’ we can turn the body at some angle and make the body roll itself. So we can find a theoretical relation between the angle between the inclined plane and the ground and the angle at which we have to turn the body to make it roll infinitely.

Analyzing the angles to be tilted in some special angles for torque due to weight to make the body turn around.

Relation between angle of inclination (θ), and the initial push equivalent angle "y".



Fig(vi) : Side view of the octagon when in rest

Fig(vii) : Side view of octagon tilting in the opposite direction of the rolling to excel the movement of octagon to reach infinite rolling.

When the angle of inclination is 0 on the ground $x + y = 45$

When the body does not roll, it stands as it is at point T (which we can call critical point)

When $y > x$ then the body rolls to the left or backwards.

When $x > y$, then the body rolls to the right or forward.

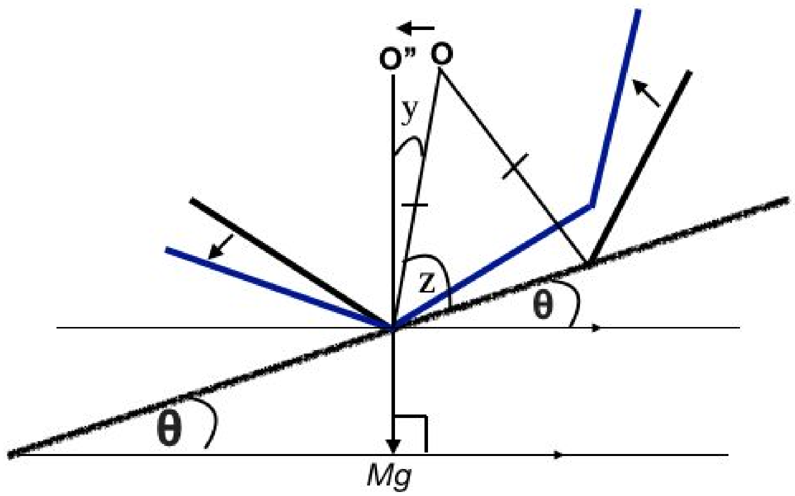


Fig (viii): Side view of the tilting octagon to reach the minimum angle to reach infinite rolling.

From the figure above,

$$y + z + \theta = 90^\circ$$

Using geometry, $z = \frac{135}{2}$

$$\therefore y = 22.5^\circ - \theta \text{ ---------(A)}$$

So at any angle bigger than y, where, $y = 22.5^\circ - \theta$, the body will turn by itself.

When $\theta = 0^\circ$, $y = 22.5^\circ$

As discussed above, at any angle larger than 22.5º the body turns to the other side. So from equation (A), we can conclude that the relation between 'y' and θ holds true only when θ is less than 22.5º. Thus at angle 0º to 22.5º, we can find minimum angle "y" at which we have to turn the body, so that body rolls by itself.

Now, we can claim that we will have bigger range for our experiments and we can somehow give “minimum constant push by rotating body at some angle "y" .

S.No	θ (º)	y (º)	Remarks
1.	0	22.5	Only one turn
2.	1	21.5	Only 2 turns
3.	2	20.5	Infinite rolling
4.	3	19.5	Infinite rolling
5.	4	18.5	Infinite rolling
6.	5	17.5	Infinite rolling
7.	6	16.5	Infinite rolling
8.	7	15.5	Infinite rolling
9.	8	14.5	Infinite rolling
10.	9	13.5	Infinite rolling
11.	10	12.5	Infinite rolling
12.	11	11.5	Infinite rolling
13.	12	10.5	Infinite rolling
14.	13	9.5	Infinite rolling
15.	14	8.5	Infinite rolling
16.	15	7.5	Infinite rolling
17.	16	6.5	Infinite rolling

Table (iii) : The angles required to be tilted for infinite rolling without any external force.

According to our theoretical assumption (A), for any angle "y" bigger than 22.5º, the body must undergo infinite rolling. The angles from (2- 22.5) will only go infinite rolling if tilted to a particular angle mentioned on the table. However, from experimental value listed above table, the body didn't roll at [0º, 1º]. The reason behind this is, at [0º, 1º] body does not gain enough potential energy to gain infinite rolling with a certain velocity. Even if we say that the body at [0º, 1º] gains enough kinetic energy to undergo infinite rolling, it might not be enough as some of the potential energy is transferred also into the sound energy and heat energy. So we can conclude that at [0º, 1º] the body does not gain enough energy to overcome collectively sound, heat, rolling and translational kinetic energy.

The initial push equivalent angle has given the experiment a bigger range of angles i.e. [2º, $64^{\circ} \pm 1^{\circ}$] for my experiment that will definitely result in larger accuracy than what I would have concluded from a small range of data.

Assumptions:

Let us say for the angle of the inclined plane $\theta = 10^{\circ}$, Then using equation (A), $y = 12.5^{\circ}$; means if we rotate the body at any angle larger than 12.5º then, the body must turn by itself. We might observe that the body might stop after some turns or might require a larger angle of turning due to some reasons. As mentioned earlier I am interested in energy loss during rolling, so to study the loss at larger accuracy, the body must roll infinitely or at least close to infinite rolling.

Calculations of Theoretical velocity

Calculation of theoretical change in energy at the experimentally measured angles.

When the body is at the top of the point from where it is allowed to fall, it initially has the maximum potential energy and “0” kinetic energy as it is not moving. The maximum potential energy along the way transforms into kinetic energy as it gains some velocity. When the octagonal body is at the top, its potential energy is $Mg\Delta H$ where, ΔH is the height at which it is placed above the ground. In my experimental setup, ΔH= radius of cylinder + length of ply*sine(angle)

Using the energy conservation principle;

Initial potential (maximum P.E) = Total kinetic energy

∴ Maximum potential energy = Translational kinetic energy + Rotational kinetic energy

$$= \frac{Mv^2}{2} + \frac{I\omega^2}{2}$$

$$Mg\Delta H = \frac{Mv^2}{2} + \frac{Iv^2}{2r^2} \qquad [\because v = \omega r]$$

We know,

$$I_{edge} = \frac{2}{3}(5 + 3\sqrt{2}) * Mr^2 + M\left(\frac{r}{2sin(\frac{\alpha}{2})}\right)^2$$

$$\Rightarrow Mg\Delta H = \frac{Mv^2}{2} + \frac{\left(\frac{2}{3}(5+3\sqrt{2})*Mr^2+M\left(\frac{r}{2sin(\frac{\alpha}{2})}\right)^2\right)v^2}{2r^2}$$

$$\Rightarrow 2g\Delta H = v^2\left[1 + \frac{\frac{2}{3}(5+3\sqrt{2})r^2+\frac{r^2}{4sin^2(22.5)}}{r^2}\right] \quad [\because \alpha = 45^o]$$

$$\Rightarrow 2g\Delta H = v^2\left[1 + \frac{2}{3}(5 + 3\sqrt{2}) + \frac{1}{4sin^2(22.5)}\right]$$

$$\Rightarrow 2g\Delta H = v^2k$$

$$\because k = 1 + \frac{2}{3}(5 + 3\sqrt{2}) + \frac{1}{4sin^2(22.5)} = 8.8688672$$

$$\because v = \sqrt{\frac{2g\Delta H}{k}}$$

This v is the theoretical velocity for any height, ΔH.

Experimental Raw and Processed data

Raw Data

Time Taken = Final Time - Start Time

Primary data from the experiment extracted from logger pro for time taken to roll.

Example can be seen in page 7.

Angle (∂)	Time Taken(∆ T) ± 0.033 s
2	4.55
3	4.02
4	3.54
5	3.27
6	3.11
7	2.94
8	2.97
9	2.65
10	2.59

Table (iv) : Table showing the time taken at specific angles.
*Note: Remaining data are presented in Appendix B.

Processed Data:

Length = 76.4± 1 cm = 0.764± 0.001 m			
Angle(∂) (± 1°)	Time Taken(Δ T)± 0.033 in seconds	Experimental Linear Velocity(V)	$\Delta V = \frac{V_{max} - V_{min}}{2}$
17	1.173	0.651321398	0.019376869
18	1.158	0.659758204	0.019871336
19	1.055	0.724170616	0.023852267
20	0.971	0.786817714	0.028073513
21	0.925	0.825945946	0.030885006
22	0.985	0.775634518	0.027294694
23	0.889	0.859392576	0.033395003
24	0.866	0.88221709	0.035164381
25	0.862	0.886310905	0.035486588
26	0.854	0.894613583	0.036144589
27	0.818	0.93398533	0.039347544
28	0.801	0.95380774	0.041011837
29	0.782	0.976982097	0.043001495
30	0.752	1.015957447	0.046454637

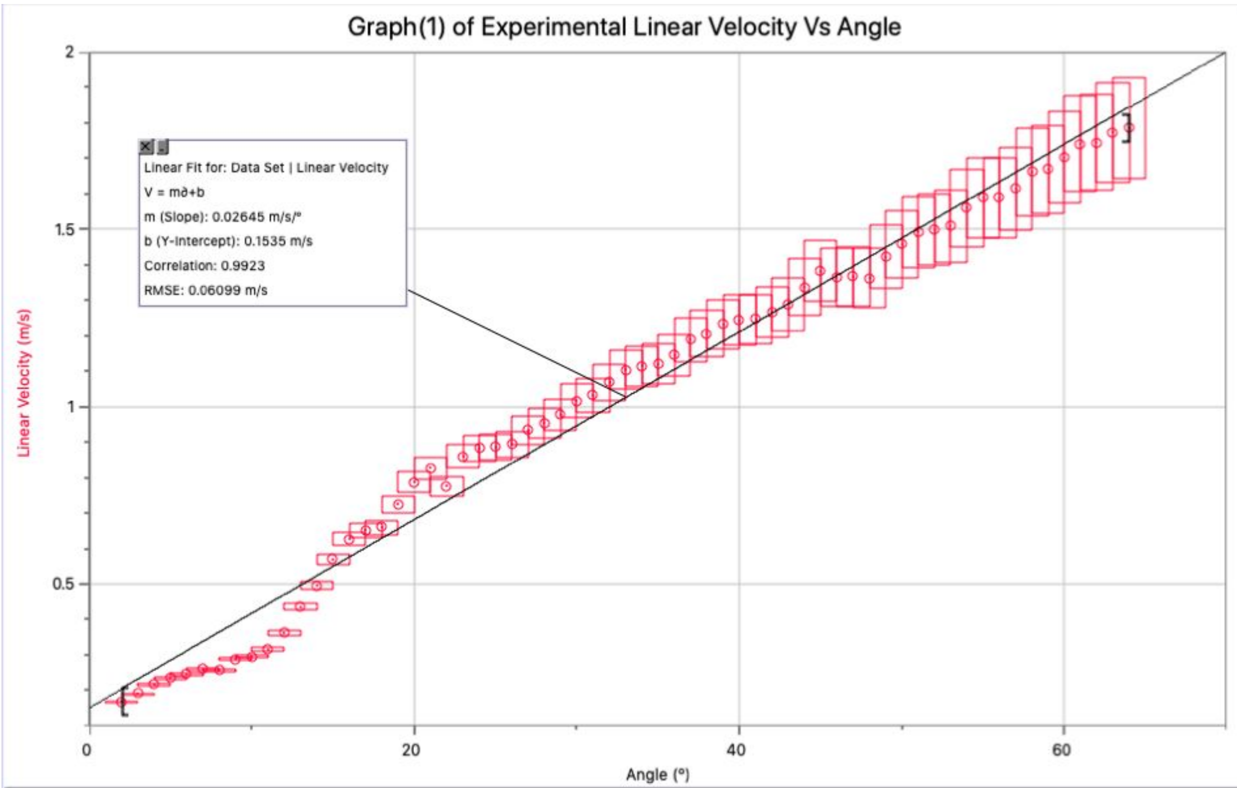
Table (v) : Table showing the velocity from the time taken from logger pro with uncertainty.

*Note: Remaining data are presented in Appendix C

Graphs

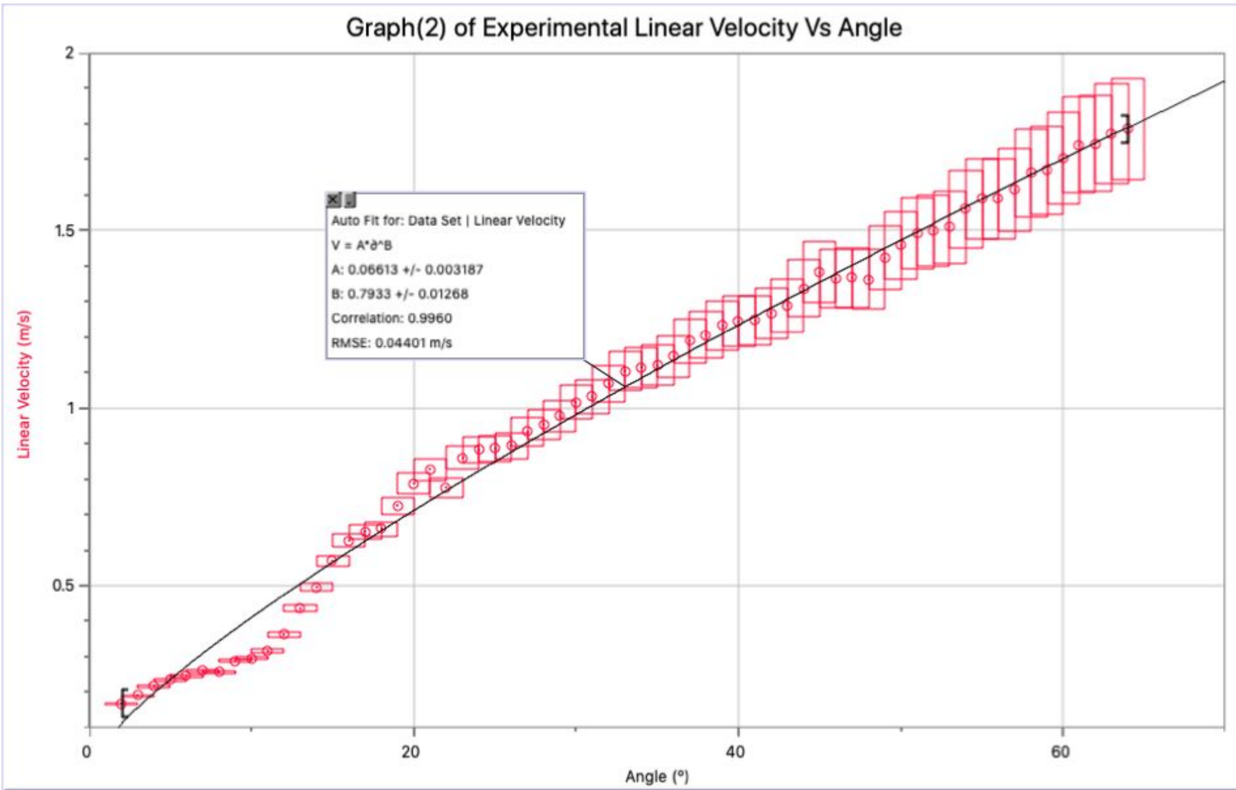
To determine the relationship between a dependent (linear velocity) and independent variables (angle), graphs 2 and 3 were plotted in logger pro. Uncertainties in the experiments were calculated as shown in Table A and plotted in graphs using the square boxes around each point in the graphs as shown below. For further analysis, the determination of the relationship between these variables was crucial. However, it is visually evident that a significant number of values on both relations are ignored. For example, In graph 1, starting 13 values do not fall under the line of best-fit line, despite a good correlation (0.9923) as given by logger pro. This highlights the error associated with experiments which will be discussed later.

Graph (i): Showing the possible relation between Linear experimental Velocity and Angle



According to graph (i), The relationship between angle and linear velocity is given by the equation: $v = m\partial + b$, where m= 0.02646 and b=0.1535 are constants. Similarly, V is linear velocity and ∂ is the angle in degrees.

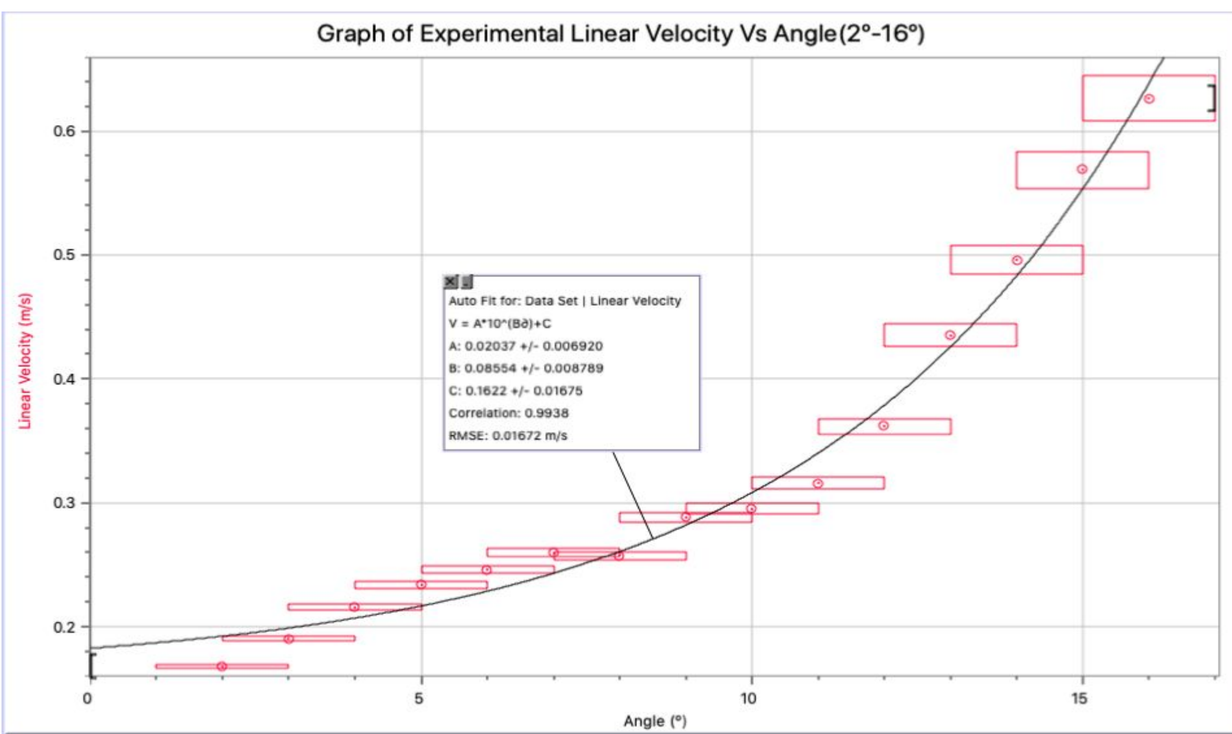
Graph (ii): Showing the possible relation between Linear experimental Velocity and Angle (ii)



The relationship between angle and linear velocity is given by the equation: $v = A \times \partial^B$, where A= 0.06613 and B=0.7933 are constants. Similarly, V is velocity and ∂ is the angle in degrees. This graph is another possible relation because the graph has two possible relations that are linear and power form. This graph represents the power form relation between angles and velocity with a correlation of 0.9960 which is greater than the correlation of the equation in graph 2.

The experimental graphs, one linear and other power of 10, despite considerable correlations, visually showed that some significant values were not considered in both the graph. For example, the curve/line does not pass from the values 2º to 13º on both the graphs (2&3). So further analyzing the equations of these graphs results in larger inaccuracy. To avoid this inaccuracy, the graph was divided into two parts one from the angle between (2º-16º) with an exponential relation and the other graph which is from the angle between (17º-64º) and linearize them if required. Following these graphs, the experiment will have two uncertainties which will be added to make a more accurate calculation and help us draw a better conclusion.

Graph (iii): Relationship between Linear Velocity and Angle (2º to 16º)

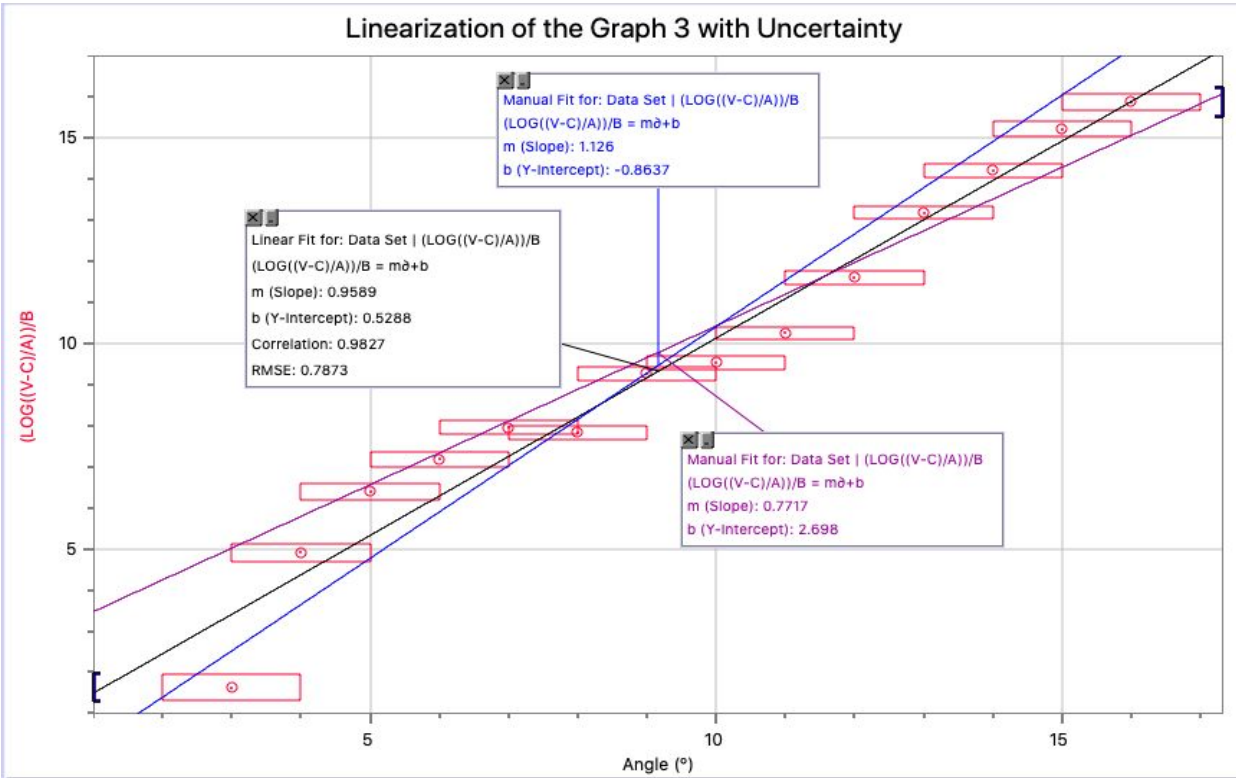


The graph (iii) is the correlation between the angle and velocity from angle between (2º-16º) which only consists of pure exponential relation. V represents velocity and constants A has value of 0.02037 and B has value of 0.08554.

A = 0.02037				
B = 0.08554				
C = 0.1622				
Angle	$\frac{Log\ (V-C) - log\ A}{B}$ (K)	K_{max}	K_{min}	$\Delta K = \frac{K_{max}-K_{min}}{2}$
2	-6.390203565	-5.253998862	-7.856270548	1.301135843
3	1.630879684	1.951180582	1.289003017	0.331088783
4	4.908059777	5.122779542	4.683856561	0.219461491
5	6.385872361	6.57314749	6.191423904	0.190861793
6	7.179403337	7.355815624	6.99663996	0.179587832
7	7.948994131	8.117136459	7.775091995	0.171022232
8	7.829114783	7.998406584	7.653982778	0.172211903
9	9.255676359	9.4143951	9.091835228	0.161279936
10	9.522052923	9.679517883	9.359547474	0.159985205
11	10.24965246	10.40479165	10.08962291	0.157584367
12	11.59009836	11.74506968	11.4302474	0.15741114
13	13.17489177	13.33660558	13.00785718	0.164374202
14	14.19466363	14.36495546	14.01846118	0.173247142
15	15.20057449	15.38294334	15.01141012	0.185766611
16	15.87040647	16.06302764	15.67018822	0.19641971

Table (vi) : Table of linearized values for calculation of uncertainty

Graph (iv) : This graph is a linearization of the graph (iii) to calculate the uncertainty.



Error and uncertainties(1)

Maximum Slope: 1.126

Minimum Slope: 0.7717

Line of best fit: 0.9589

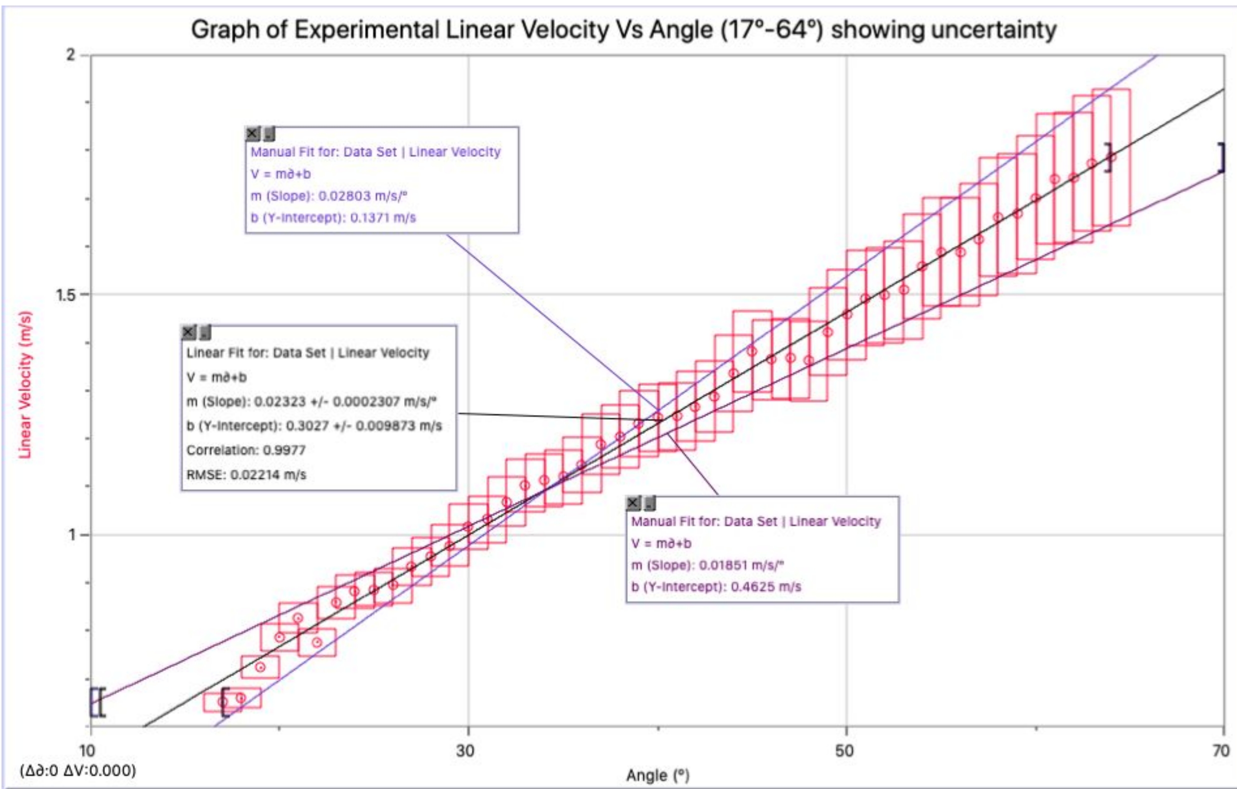
∴ Uncertainty of experiment = $(\Delta U_1) = \frac{Maximum\ Slope - Minimum\ Slope}{2}$

$$= \frac{1.126 - 0.7717}{2}$$

$$= 0.17715$$

Percentage Uncertainty of Experiment $(\Delta U_1) = \frac{0.17715}{0.9589} * 100\% = 18.48\%.$

Graph (v) : Relationship between Linear Velocity and Angle (17º to 64º)



Error and uncertainties (2)

This graph shows a relation between angle and linear velocity of angles ranged from (17º-64º). This graph is a continuation of graph (iii) but with a linear relation.

From the graph,

Maximum Slope = 0.02803

Minimum Slope = 0.01851

Line of best fit = 0.02323

∴ Uncertainty of experiment $(\Delta U_2) = \frac{Maximum\ Slope - Minimum\ Slope}{2}$

$$= \frac{0.02803 - 0.01851}{2}$$

$$= 0.00476$$

Percentage Uncertainty $(\Delta U_2) = \frac{0.00476}{0.02323} * 100\% = 20.49\%$.

Therefore, Percentage Uncertainty of the experiment (ΔU) = ΔU_1 + ΔU_2

$$= 18.48\% + 20.49\%$$

$$= 38.97\%$$

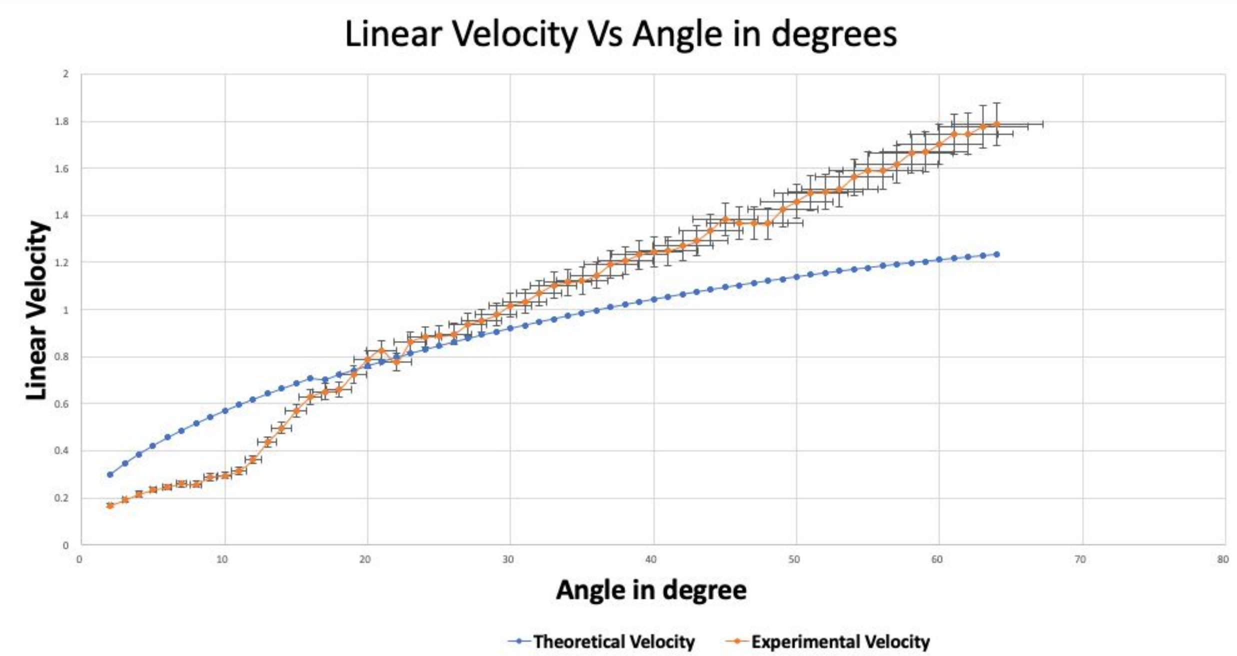
Analysis and conclusion

The experiment has an error percentage of 38.97% by adding up two errors, as the experimental analysis was performed in two parts. This was done to have precise calculations of errors.

Furthermore, the experiment can be continued out by the calculation of g, which has not been done in this research paper. It could help us discover more about acceleration due to gravity on slopes. The strikes of octagon to roll results in inelastic collision which can have possible relation with momentum. More research can be performed on this topic. The heat and sound energy could also be calculated.

Comparing this experiment with another experiment which is cited below. The other experiment was carried out with cylinder, which has the exponential graph for the angle until 65 degrees while on this research paper we have exponential relation only until 16 degrees and linear from (17º-64º) degrees. How can shape cause this difference despite having factors like friction, plywood, energy and velocity ?

Graph (vi): Graph showing experimental and theoretical linear velocity



The theoretical value is less than experimental because;

1. Slipping should be considered.
2. The formula is based on an assumption of the perfect rolling of an object which is not achieved in this practical experiment carried out. In this practical experiment, angular velocity is negligible and the translational velocity is higher due to slipping.

3. The slipping of an object has not been considered in the theoretical formulas. This experiment covers more translational than the angular velocity. The theoretical formula definitely considers more rotational velocity than the experimental ones. The angular rotational velocity is very less in the experimental calculation.
4. Heat and sound energy is not considered in the theoretical calculations.

Personal Note:

The angle of the inclined plane and the velocity of octagonal cylinder is directly proportional which is evident from the experiment. The relation between angle and velocity is exponential from the angle (2º-16º) and linear from (17º-64º). There was a very big difference in theoretical and experimental values mainly because of slipping. It could be important in our life for vehicles to know the velocity when it travels uphills and downhills. This experiment stimulated the hills with an inclined plane and octagon with tyres of vehicles. The video was recorded and imported into logger pro to record the taken to travel in the inclined plane. I personally learned the practical knowledge of translational, theoretical velocity, potential and kinetic energies, moment of inertia involved. I could answer the difference in the expected and real values of the experiment which was the main obstacle throughout the experiment.

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International Baccalaureate Organization, 2008, pp. 1–16.

Appendix

Appendix A: Calculation for Moment of Inertia of octagonal cylinder about the edge

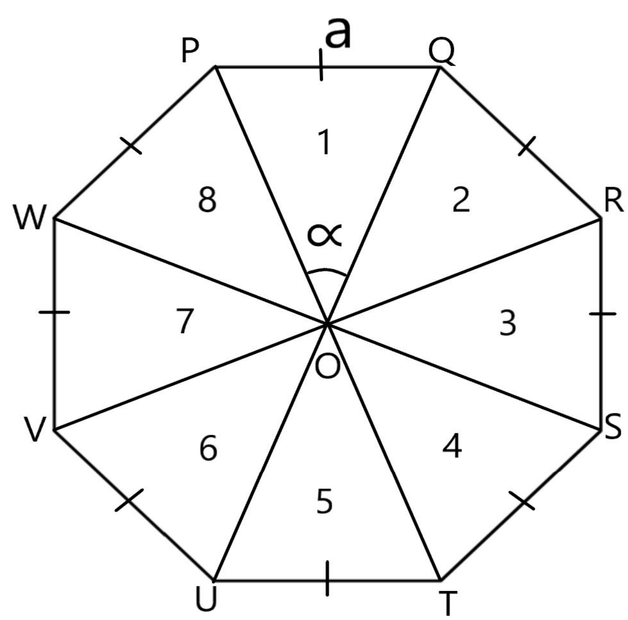

Figure (ix) : Side view of the octagonal cylinder of side length a.

As we know all the triangles 1 to 8 are congruent. So, the study of any one of them will give the same result. To simplify we will calculate the moment of inertia of 1 of these triangles and multiply it by 8 to get the moment of inertia of the octagon PQRSTUVW with center O and radius “*r*”.

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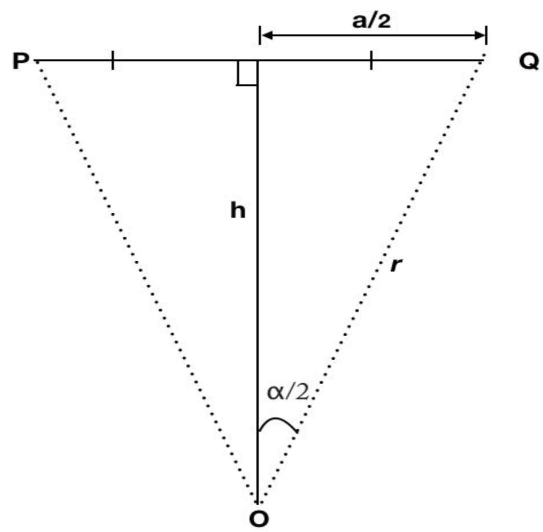

Figure (x): Triangle PQO

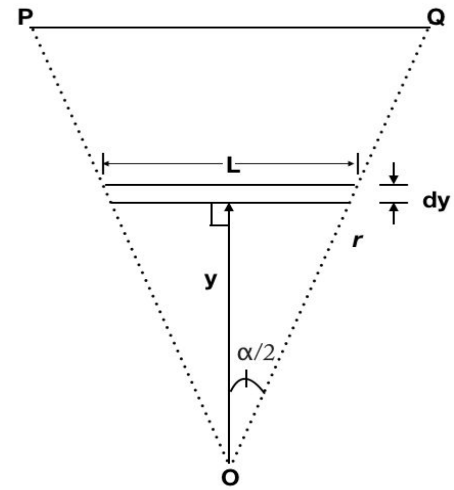

Figure (xi):

Using trigonometric relation in the small triangle of the figure (x);

$tan(\frac{\alpha}{2}) = \frac{Perpendicular}{Base} = \frac{a/2}{y}$

$\therefore a = 2ytan(\frac{\alpha}{2})$ --------(i)

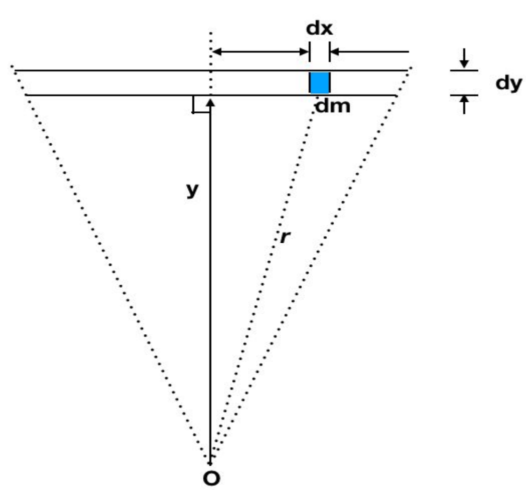

Figure (xii): the Closer look of figure 3 and consideration of small mass of strip.

Moment of Inertia of "dm" mass is: $dI = dm \times r^2$

$dI = dm * (x^2 + y^2)$ ----------(ii)

If m be the mass of "strip" shown in figure 3 and if it has length 'l' then,

$\therefore$ $\frac{m}{l}$ is mass per unit length.

$\therefore dm = \frac{m}{l} \times dx$ ----------(iii)

Using (i) and (iii)

$dm = \frac{m}{2ytan(\frac{\alpha}{2})} \times dx$ ---------(iv)

Using (ii) and (iv),

$dI = \frac{m}{2ytan(\frac{\alpha}{2})} * dx * (x^2 + y^2)$

Integrating,

$$\int_0^I dI = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{m}{2ytan(\frac{\alpha}{2})} * dx * (x^2 + y^2)$$

$$I = \frac{m}{ytan(\frac{\alpha}{2})} * \frac{L}{2} * \left[\frac{L^2}{12} + y^2\right]$$

This is the moment of inertia of the strip shown in Figure (xii).

Assuming the triangle PQO is made up of a number of similar strips.

Now, consider the mass of the strip shown in figure (xii) be dm.

$dI = \frac{dm}{ytan(\frac{\alpha}{2})} * \frac{L}{2} * \left[\frac{L^2}{12} + y^2\right]$ ----------(v)

The moment of inertia of the total mass of the triangular object to be M, then mass per unit area of the triangle is:

$\frac{M}{Area} = \frac{M}{0.5*h*a}$ --------(vi)

From figure (x):

$tan(\frac{\alpha}{2}) = \frac{Perpendicular}{Base} = \frac{a/2}{h}$

$h = \frac{a}{2tan(\frac{\alpha}{2})}$ ------(vii)

Substituting h in (vi),

$\frac{M}{Area} = \frac{M}{0.5*\frac{a}{2tan(\frac{\alpha}{2})}*a}$

$\therefore \frac{M}{Area} = \frac{4Mtan(\frac{\alpha}{2})}{a^2}$ -------(viii)

∴ Mass of the strip in figure (xii) is:

$dm = \frac{M}{Area} * area\ of\ strip$

$dm = \frac{M}{Area} * dA$

$dm = \frac{M}{Area} * L * dy$

$\therefore dm = \frac{4Mtan(\frac{\alpha}{2})}{a^2} * L * dy$ [using (viii)]

And put L from (i)

$\therefore dm = \frac{4Mtan(\frac{\alpha}{2})}{a^2} * 2ytan(\frac{\alpha}{2}) * dy$ -------(ix)

Using (i) and (v),

$$dI = \frac{dm}{ytan(\frac{\alpha}{2})} * \frac{2ytan(\frac{\alpha}{2})}{2} * \left[\frac{(2ytan(\frac{\alpha}{2}))^2}{12} + y^2\right]$$

$$dI = dm * \left[\frac{y^2(tan(\frac{\alpha}{2}))^2}{3} + y^2\right]$$

$$dI = dm * y^2 * \left[\frac{(tan(\frac{\alpha}{2}))^2}{3} + 1\right]$$ -----(x)

Using (ix) and (x),

$$dI = \frac{4Mtan(\frac{\alpha}{2})}{a^2} * 2ytan(\frac{\alpha}{2}) * dy * y^2 * \left[\frac{(tan(\frac{\alpha}{2}))^2}{3} + 1\right]$$

$$dI = \frac{8Mtan^2(\frac{\alpha}{2})}{a^2} * y^3 * dy * \left[\frac{(tan(\frac{\alpha}{2}))^2}{3} + 1\right]$$

$$dI = K_1 * \frac{8M}{a^2} * y^3 * dy$$

Where $K_1 = tan^2(\frac{\alpha}{2}) * \left[\frac{tan^2(\frac{\alpha}{2})}{3} + 1\right]$

Now, Integrating,

$$\int_0^I dI = \int_0^h K_1 * \frac{8M}{a^2} * y^3 * dy$$

$$I = K_1 * \frac{8M}{a^2} * \frac{h^4}{4}$$

Putting the value of h from (vii),

$$I = K_1 * \frac{2M}{a^2} * \frac{a^4}{16*tan^4(\frac{\alpha}{2})}$$

$$I = K_1 * \frac{Ma^2}{8*tan^4(\frac{\alpha}{2})}$$

$$I = \frac{K_1}{8*tan^4(\frac{\alpha}{2})} * Ma^2$$

$I = K_2 * Ma^2$ (where $K_2 = \frac{K_1}{8*tan^4(\frac{\alpha}{2})}$)

This is the moment of Inertia for one triangle of an octagon.

$\therefore I_{octagon} = 8 * K_2 * Ma^2$

Simplifying K_2

$$K_2 = \frac{tan^2(\frac{\alpha}{2}) * \left[\frac{tan^2(\frac{\alpha}{2})}{3} + 1\right]}{8 * tan^4(\frac{\alpha}{2})}$$

Simplifying,

$K_2 = \frac{2}{24}(5 + 3\sqrt{2})$

$\therefore I_{octagon} = \frac{2}{3}(5 + 3\sqrt{2}) * Ma^2$

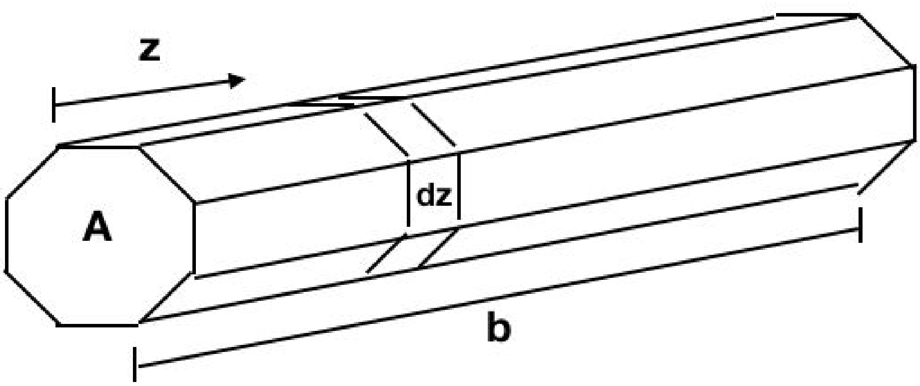

Figure (xii) : Octagonal cylinder

If the octagonal cylinder has mass M, volume V, length “b” as shown in figure 5 of density “ρ”

Let the mass of an octagonal ring of width dz at distance z from the side be dm. And its volume is dV.

$dI = \frac{2}{3}(5 + 3\sqrt{2}) * dm * a^2$

$dm = dV * \rho$

$dm = A * dz * \frac{M}{V}$

$dm = A * dz * \frac{M}{A*b}$

$dI \ = \frac{2}{3}(5 + 3\sqrt{2}) * A * dz * \frac{M}{A*b} * a^2$

Integrating,

$\int_0^I dI \ = \int_0^b \frac{2}{3}(5 + 3\sqrt{2}) * A * dz * \frac{M}{A*b} * a^2$

$I \ = \frac{2}{3}(5 + 3\sqrt{2}) * A * \frac{M}{A*b} * a^2 * b$

$I \ = \frac{2}{3}(5 + 3\sqrt{2}) * Ma^2$

This I mentioned above is the moment of inertia of Octagonal cylinder of mass ‘M’ and side length as about its center $"O"$ as shown in figure (ix).

Throughout the experiment, the body rolls around its edge. So the moment of inertia of the body around its edge is:

$\therefore I_{edge} = I + Mr^2$ [$\because$Theorem of parallel axis][4]

From, figure 2,

$sin(\frac{\alpha}{2}) = \frac{a/2}{r}$

$\Rightarrow r = \frac{a}{2sin(\frac{\alpha}{2})}$

$\therefore I_{edge} = I + M(\frac{a}{2sin(\frac{\alpha}{2})})^2$

[4] "Perpendicular and Parallel Axis Theorem: Videos and Solved ... - Toppr." https://www.toppr.com/guides/physics/system-of-particles-and-rotation-dynamics/theorems-of-parallel-and-perpendicular-axis/. Accessed 9 Jul. 2019.

Appendix B: Raw Data Table

Angle (°)	Time (**Δ**)	Angle (°)	Time (**Δ**)	Angle (°)	Time (**Δ**)
11	2.422	29	0.782	47	0.559
12	2.111	30	0.752	48	0.561
13	1.754	31	0.740	49	0.538
14	1.541	32	0.715	50	0.524
15	1.34	33	0.693	51	0.5120
16	1.22	34	0.686	52	0.511
17	1.173	35	0.682	53	0.506
18	1.158	36	0.668	54	0.490
19	1.055	37	0.643	55	0.482
20	0.971	38	0.634	56	0.482
21	0.925	39	0.621	57	0.472
22	0.985	40	0.614	58	0.461
23	0.889	41	0.613	59	0.458
24	0.866	42	0.603	60	0.449
25	0.862	43	0.593	61	0.439
26	0.854	44	0.572	62	0.438
27	0.818	45	0.553	63	0.431
28	0.801	46	0.560	64	0.427

Appendix C: Processed Data Table 2

Length = 76.4± 1 cm = 0.764± 0.001 m			
Angle(∂) (± 1°)	Time Taken(T)± 0.033	Experimental Linear Velocity(V)	$\Delta V = \frac{V_{max}-V_{min}}{2}$
31	0.74	1.032432432	0.047954619
32	0.715	1.068531469	0.05132514
33	0.693	1.102453102	0.054597366
34	0.686	1.113702624	0.055705023
35	0.682	1.120234604	0.05635332
36	0.667	1.145427286	0.058889069
37	0.643	1.188180404	0.063321047
38	0.634	1.205047319	0.065114131
39	0.62	1.232258065	0.068060065
40	0.614	1.244299674	0.069384717
41	0.613	1.246329527	0.069609282
42	0.603	1.266998342	0.071916741
43	0.593	1.28836425	0.07434194
44	0.572	1.335664336	0.079855356
45	0.553	1.381555154	0.085394969
46	0.56	1.364285714	0.083288295
47	0.559	1.366726297	0.083584404
48	0.561	1.361853832	0.082993769
49	0.537	1.422718808	0.090523769
50	0.524	1.458015267	0.095042061
51	0.512	1.4921875	0.09952258
52	0.51	1.498039216	0.100300313
53	0.506	1.509881423	0.101883605
54	0.49	1.559183673	0.108610349
55	0.481	1.588357588	0.112693504
56	0.481	1.588357588	0.112693504
57	0.473	1.615221987	0.116521034

58	0.46	1.660869565	0.123173564
59	0.458	1.668122271	0.12424782
60	0.449	1.70155902	0.129261714
61	0.439	1.740318907	0.135200051
62	0.438	1.744292237	0.135816465
63	0.431	1.77262181	0.140252815
64	0.428	1.785046729	0.14222143