BJT Circuit Design Questions and Solutions

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Q1. Calculate the Q-point of the circuit shown in Figure 1. Take $V_{BE}(on) = 0.7V$ and $\beta = 75$.

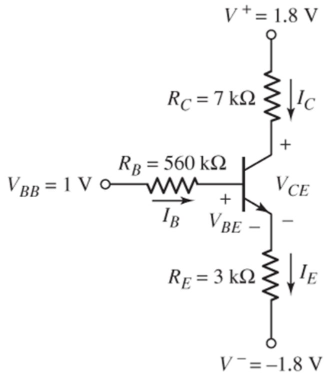


Figure 1: Circuit for Q1

Q1

$$I_B = \frac{V_{BB} - V_{BE(on)} - V^-}{R_B + (1+\beta)R_E} \Rightarrow 2.665\ \mu A$$

$$I_C \simeq I_E = \beta I_B = 75 \times 2.665\ \mu A \Rightarrow 0.20\ mA$$

$$V_{CE} = \underbrace{V^+ - I_C R_C}_{V_C} - \underbrace{(I_E R_E + V^-)}_{V_E}$$

$$= 1.8 - (0.2)(7) - 0.203 \times 3 + 1.8 = 1.59V$$

$$I_{CQ} = 0.20\ mA, \quad V_{CEQ} = 1.59\ V$$

Q2. Draw the output voltage (V_o) versus input voltage (V_I) transfer characteristics for the circuit shown in Figure 2. The circuit parameters are $R_B = 100k\Omega$, $R_C = 5k\Omega$, and $V_{CC} = 5V$. The transistor parameters are $V_{BE}(on) = 0.7V$, $V_{CE}(sat) = 0.2V$ and $\beta = 100$. Show the transition points at which Q1 changes its operating mode. Clearly show your calculations and the assumptions.

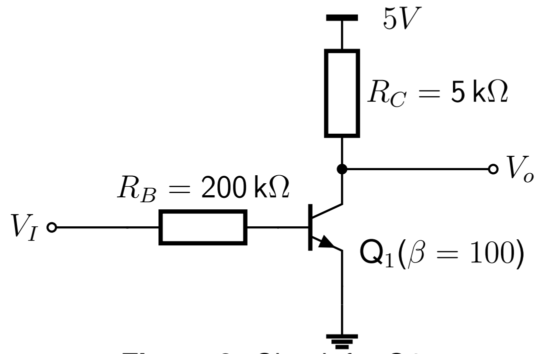
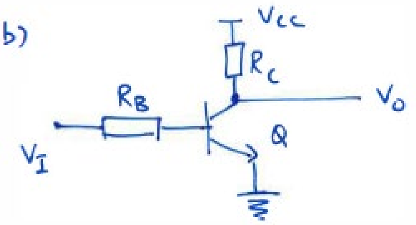


Figure 2: Circuit for Q2

(b)



[13]

When $V_I < 0.7$ [1]

Q is off $I_c \simeq 0$

$V_o = V_{cc} - I_c R_c$ —— ①

$V_o \simeq V_{cc} = 5V$ [1]

When $V_I \geqslant 0.7V$ [1]

$I_B = \dfrac{V_I - 0.7}{R_B}$ [1]

$\therefore I_c = \beta I_B = \beta \dfrac{(V_I - 0.7)}{R_B}$ [1]

$V_o = 6.75 - 2.5 V_I$

From ①

$V_o = V_{cc} - \beta (V_I - 0.7) \dfrac{R_c}{R_B}$ —— ② [1]

When Q is in saturation $V_{CE} = 0.2V$ [1]

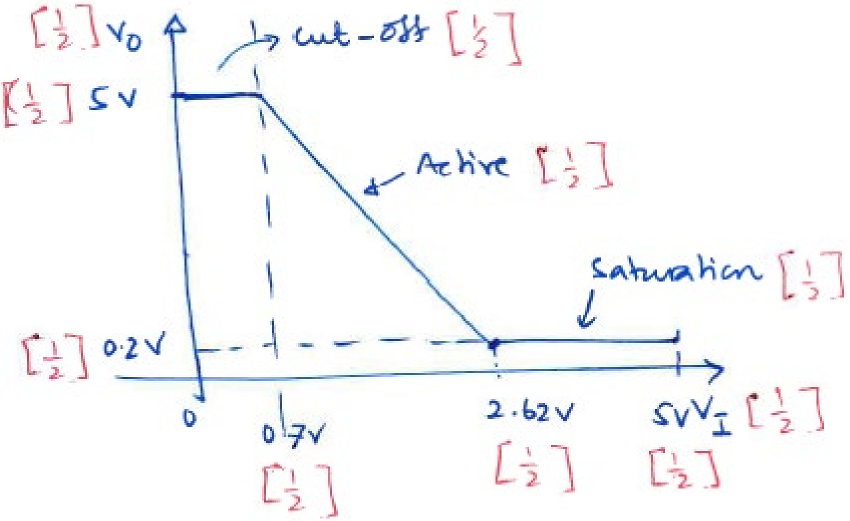
$V_{CE} = V_o$

using ② we can find V_I when $V_o = 0.2$ V.

$$0.2 = 5 - 100 \times \frac{5k\Omega}{200k\Omega} (V_I - 0.7)$$

$V_I = 2.62$ V [1]

Q is in active mode when $0.7 \leqslant V_I < 2.62$ V



Q3. For the circuit in Figure 3 determine V_B and I_E such that $V_B = V_C$. Assume $\beta = 90$. What value of V_B results in $V_{CE} = 2V$?

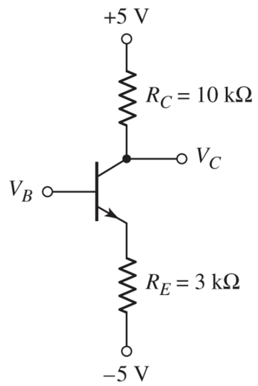
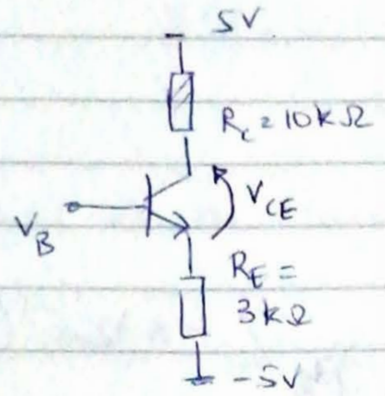


Figure 3: Circuit for Q3



(a) $I_c = \dfrac{5 - V_B}{10}$; $I_E = \dfrac{(V_B - 0.7) + 5}{3}$

$I_c = \dfrac{\beta}{1+\beta} I_E = \dfrac{90}{91} I_E$

$$\frac{5 - V_B}{10} = \frac{90}{91} \times \frac{V_B + 4.3}{3}$$

$$V_B = -2.136 \text{ V}$$

$$I_E = \frac{(-2.136 + 4.3)}{3} = 0.721 \text{ mA}$$

(b) Apply KVL to C-E path, ~~then~~

$$5 - (-5) = V_{CE} + I_E R_E + I_c R_C$$

$$10 = 2 + I_E R_E + \frac{\beta}{1+\beta} I_E R_C$$

$$= 2 + I_E \left[3 + \frac{90}{91} \times 10 \right]$$

$$I_E = 0.621 \text{ mA}$$

$$V_B = V_E + 0.7 = 0.621 \times 3 - 5 + 0.7 = -2.44 \text{ V}$$

Q4. Consider the circuit shown in Figure 4. The transistor parameters are $\beta = 150$ and $V_{BE}(on) = 0.7V$. The circuit parameters are $R_E = 2k\Omega$ and $R_C = 10k\Omega$. Design a bias circuit such that output voltage (v_o) is zero. What are the values of I_{CQ} and V_{CEQ}?

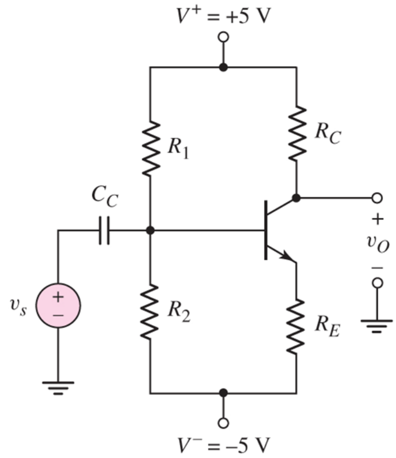
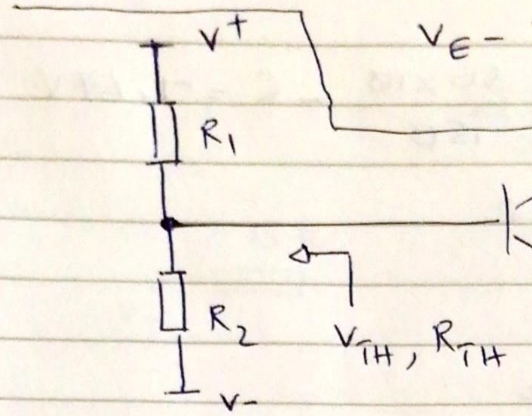


Figure 4: Circuit for Q3

Q3: since $v_0 = 0$, $V_C = 0$, Then $I_C = \frac{5-0}{R_C} = 0.5 \text{ mA}$

$I_E \simeq I_C = 0.5 \text{ mA}$



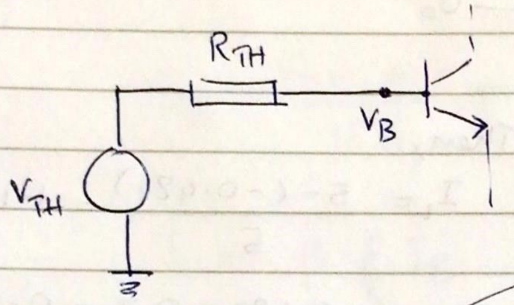
$V_E - V^- = I_E R_E \Rightarrow V_E = -4V$

$(V_{CEQ} = V_C - V_E = 4V \,;\, I_{CQ} = 0.5 \text{ mA})$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{TH} = \frac{R_2}{R_1+R_2}\left[V^+ - V^-\right] + V^-$$

$$V_{TH} = \frac{R_{TH}}{R_1}\left[V^+ - V^-\right] + V^-$$



$$I_B = \frac{V_{TH} - V_B}{R_{TH}}$$

For bias-stability $R_{TH} = 0.1(1+\beta)R_E$

$= 0.1 \times 151 \times 2 \text{ k}$

$R_{TH} \simeq 30 \text{ k}\Omega$

$$\frac{I_C}{\beta} = \frac{V_{TH} - V_B}{R_{TH}}$$

$$\frac{0.5m}{150} = \left[\frac{30k[5+5]}{R_1} - 5 - (-3.3)\right] \Big/ 30k$$

$$\frac{0.5m}{150} \times 30k = \frac{30k \times 10}{R_1} - 5 + 3.3$$

$$\frac{30k \times 10}{R_1} = \frac{0.5 \times 3\cancel{0}}{15\cancel{0}} + 1.7 = 1.8$$

$$R_1 = \frac{30k \times 10}{1.8} = 167 \text{ k}\Omega$$

$$R_2 = \frac{R_1 R_{TH}}{R_1 - R_{TH}} = \frac{167 \times 30}{167 - 30}k = 36 \text{ k}\Omega$$

Q5. For each transistor in the circuit in Figure **??**, the parameters are: $\beta = 125$, $V_{BE(on)} = 0.7\ V$, and $r_o = \infty$. (a) Determine the Q-points of each transistor; (b) Find the overall small-signal voltage gain $A_v = V_o/V_s$. (c) Determine the input resistance R_i and the output resistance R_o.

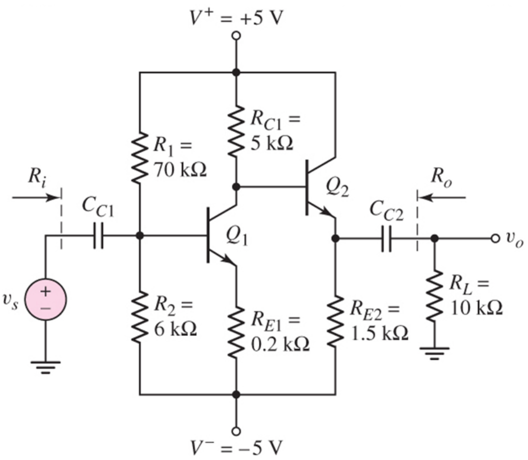
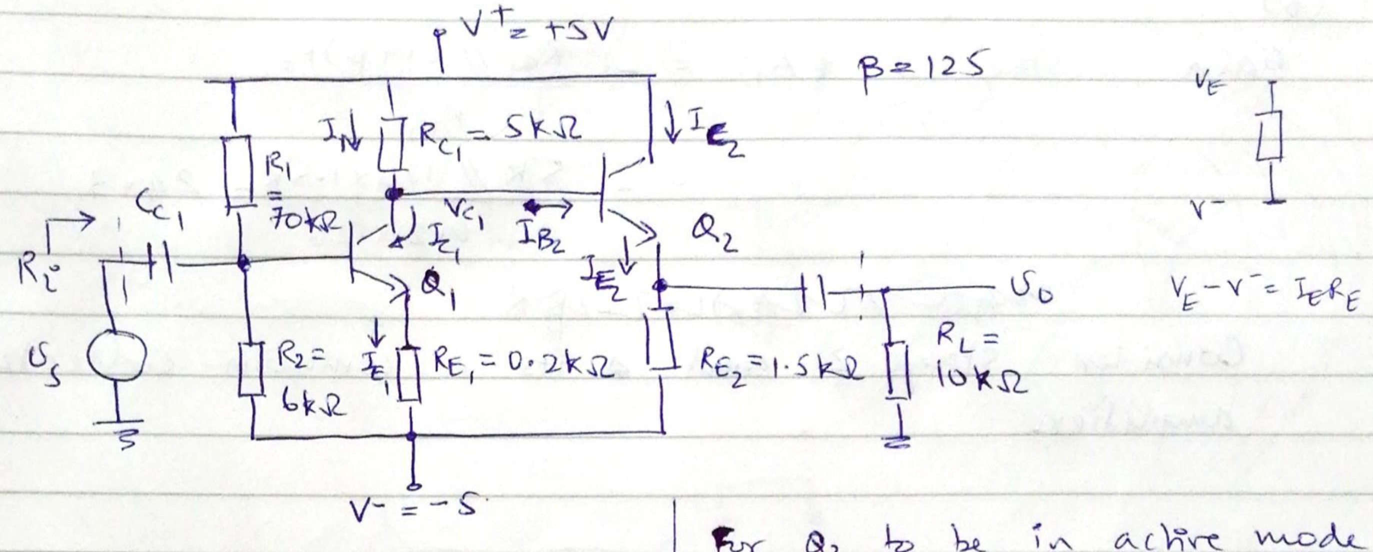


Figure 5: Circuit for Q4

Q4:



For DC Analysis

$$V_{TH} = \frac{R_2[V^+ - V^-]}{R_1 + R_2} + V^-$$

$$V_{TH} = \frac{6 \times 10}{76} - 5 = -4.21V$$

$$R_{TH} = R_1 // R_2 = 5.53\,k$$

Applying KVL to B-E loop of Q_1

$$V_{TH} - V^- = I_{B_1} R_{TH} + V_{BE(ON)} + I_{E_1} R_{E_1}$$

$$-4.21 + 5 = I_{B_1}(5.53) + 0.7 + 126\, I_B (0.2)$$

$$I_{B_1} = 2.9\ \mu A$$

Then, $I_{C_1} = \beta I_{B_1} = 0.36\ mA$ //

~~For Q_2 to be in active mode~~

$$\cancel{V_{E_2} = V_{C_1} + 0.7}$$

$$\cancel{5V = V_{C_2} + 0.7 \Rightarrow V_{C_1} = 4.3V}$$

~~Applying KCL to Q_1 collector junc.~~

$$\cancel{\frac{V^+ - V_{C_1}}{R_{C_1}} = I_{B_2} + I_{C_1}}$$

$$\cancel{\frac{5 - 4.3}{5k} + I_{B_2} = 0.36m}$$

$$\cancel{I_{B_2} = 0.22\ mA}$$

For Q_2 to be in active mode

$$V_{E_2} = V_{C_1} - 0.7$$

Therefore $I_{E_2} = \frac{V_{C_1} - 0.7 + 5}{R_{E_2}}$

Applying KCL to Q_1 collector junction

$$\frac{V^+ - V_{C_1}}{R_{C_1}} = I_{B_2} + I_{C_1}$$

$$\frac{V^+ - V_{C_1}}{R_{C_1}} = \frac{I_{E_2}}{1+\beta} + I_{C_1}$$

$$\frac{5 - V_{C_1}}{5k} = \frac{V_{C_1} - 0.7}{126 \times 1.5k} + 0.36m$$

$$V_{C_1} = 3.00\ V$$

Now, $I_{E_2} = (3 - 0.7 + 5)/1.5$

$$I_{E_2} = 4.87\ mA$$

Then $I_{C_2} = \beta I_{E_2}/(1+\beta) = \underline{\underline{4.82\,mA}}$

$$V_{CEQ_2} = 5 - V_{E_2} = 5 - (3 - 0.7)$$

$$V_{CEQ_2} = 2.7\ V\ //$$

$$V_{CEQ_1} = V_{C_1} - (I_{E_1} R_{E_1} - 5)$$

$$= 3 - (0.36 \times 0.2 - 5)$$

$$\underline{\underline{V_{CEQ_2} = +2.072\ V}}$$

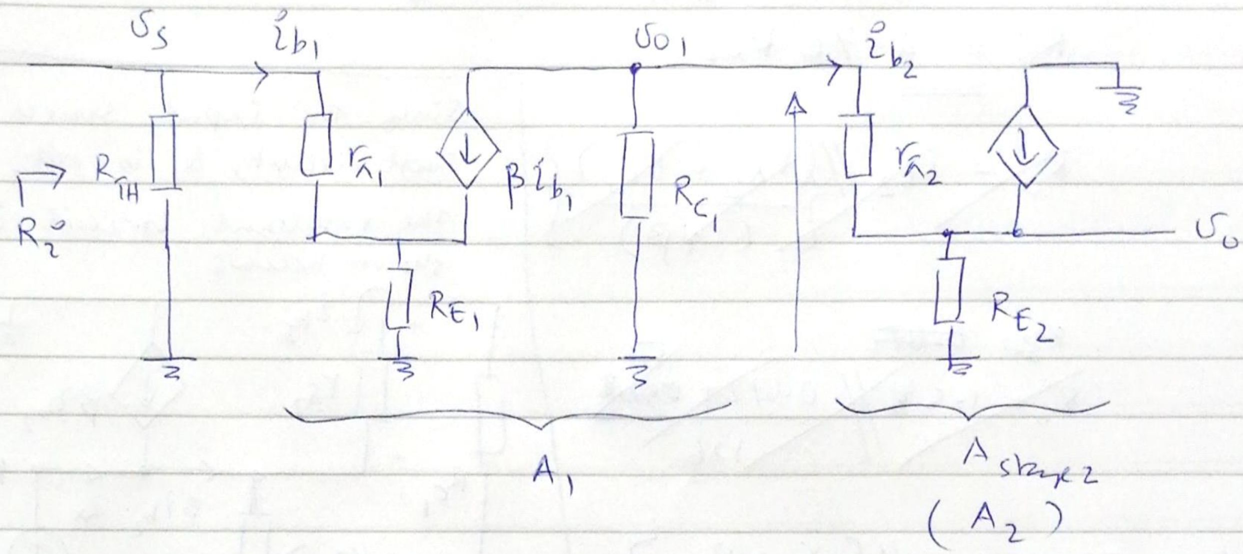
Q4

$\omega = \frac{1}{LC}$

(b)

Gain estimation

$$A_{overall} = \underbrace{A_{stage1}}_{(A_1)} \times \underbrace{A_{stage2}}_{(A_2)}$$



$$v_s = i_{b_1}\left[r_{\pi_1} + (1+\beta)R_{E_1}\right]$$

Apply KCL to node v_{o_1}:

$$\cancel{\frac{v_{o_1}}{R_{C_1}} + \underbrace{\frac{v_{o_1} - v_o}{r_{\pi_2}}}_{= i_{b_2}} + \beta i_{b_1} = 0 \quad — \quad (1)}$$

Apply KCL to node v_o:

$$\frac{v_o}{R_{E_2}} - \beta i_{b_2} - i_{b_2} = 0 \Rightarrow \quad v_o = R_{E_2}(1+\beta)\, i_{b_2}$$

$$v_{o_1} = v_o + i_{b_2} r_{\pi_2}$$

$$A_2 = \frac{v_o}{v_{o_1}} = \frac{(1+\beta)R_{E_2}}{r_{\pi_2} + (1+\beta)R_{E_2}} = \frac{126 \times 1.5k}{0.672 + 126 \times 1.5} \simeq 1$$

$$A_1 = -\beta \frac{(R_{C_1} // R_{i_2})}{\underbrace{r_{\pi_1} + (1+\beta)R_{E_1}}_{9k}}$$

input impedance of stage 2

$R_{i_2} = r_{\pi_2} + (1+\beta)R_{E_2}$

$\simeq 0.672 + 126 \times 1.5k$

$R_{i_2} = 189.672\ k$

$A_1 = -17.79$

$A = -17.79$

(c) $R_i = R_{TH} // [r_{\pi_1} + (1+\beta) R_{E_1}]$

$= 5.53k // [9k + 126 \times 0.2k]$

$R_i = 4.76\,k\Omega$

$\cancel{R_o = R_{E_2} // \left\{ \frac{(r_{\pi_2} + R_{C_1})}{(1+\beta)} \right\}}$

$\cancel{R_3 = 0.672}$

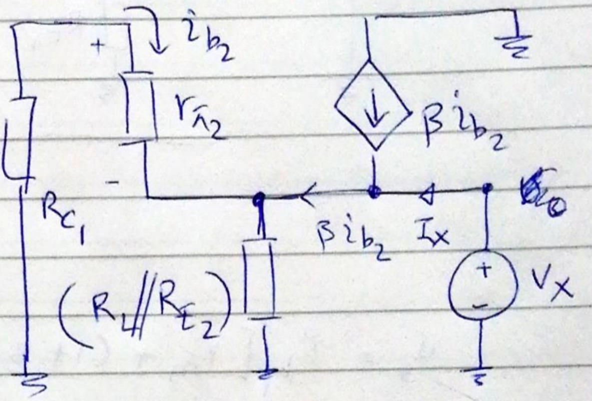
$\cancel{R_o = 1.5k // \frac{0.672 + 5k}{126}}$

$R_o = R_L // R_{E_2} // \left\{ \frac{r_{\pi_2} + R_{C_1}}{(1+\beta)} \right\}$

$= 10k // 1.5k // \left(\frac{0.672 + 5k}{126} \right)$

$R_o = 43.5\,\Omega$

Since the input source is short circuit, Q_1 is off. The resultant circuit is shown below:



$\cancel{v_o = (1+\beta) i_{b_2} R_{E_2}}$

$\cancel{v_o = i_{b_2} [r_{\pi_2} + R_{C_1}]}$

$R_o = \frac{V_x}{I_x}$

$I_x + \beta i_{b_2} = \frac{V_x}{R_{E_2}} + \frac{V_x}{r_{\pi_2} + R_{C_1}} + \frac{V_x}{R_L}$

$I_x + \beta \left[\frac{-V_x}{r_{\pi_2} + R_{C_1}} \right] = \frac{V_x}{R_{E_2}} + \frac{V_x}{r_{\pi_2} + R_{C_1}} + \frac{V_x}{R_L}$

$I_x = V_x \left[\frac{1}{R_{E_2}} + \left\{ \frac{(1+\beta)}{r_{\pi_2} + R_{C_1}} \right\} + \frac{1}{R_L} \right]$

$R_o = \frac{V_x}{I_x} = R_L // R_{E_2} // \left\{ \frac{r_{\pi_2} + R_{C_1}}{1+\beta} \right\}$

$I_x = V_x \left[\frac{1}{1.5k} + \frac{1}{10k} + \frac{126}{0.672 + 5k} \right]$

$I_x = V_x \times 22.98m$

$R_o = 43.5\,\Omega$

Q6. The parameters of the transistor in the circuit in Figure 6 are $\beta = 150$ and $V_A = \infty$. Determine R_1 and R_2 to obtain a stable-bias with the Q-point in the center of the load line. Determine the small signal voltage gain $A_v = v_o/v_i$.

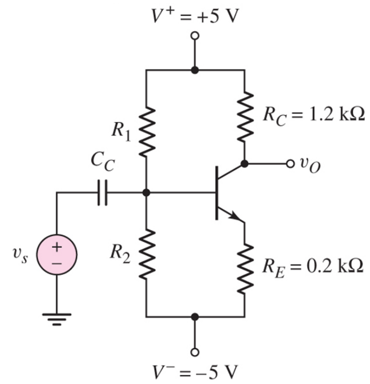


Figure 6: Circuit for Q6

Q6:

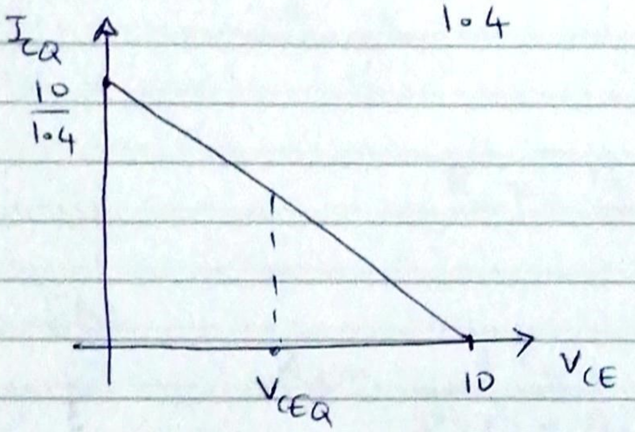
Applying KVL to CE branch of the transistor

$$V^+ - V^- = I_{CQ} R_C + I_E R_E + V_{CEQ}$$

Assume that $I_E \simeq I_C$

$$5 - (-5) = I_{CQ}(1.2 + 0.2) + V_{CEQ}$$

$$I_{CQ} = \frac{10}{1.4} - \frac{V_{CEQ}}{1.4}$$



$V_{CEQ} = 5\,V$ (centre of the load line)

$\therefore\ I_{CQ} = 3.57$ mA

$I_{BQ} = 0.024$ mA

$$R_1 // R_2 = R_{TH} = 0.1 \times (1+\beta) R_E = 0.1 \times 151 \times 0.2 = 3.02\,k\Omega$$

$$V_{TH} = \frac{R_{TH}}{R_1}(V^+ - V^-) + V^- = \frac{3.02 \times 10}{R_1} - 5$$

Applying KVL to BE loop

$$V_{TH} - V^- = I_{BQ} R_{TH} + V_{BE(on)} + (1+\beta) I_{BQ} R_E$$

$$\frac{3.02 \times 10}{R_1} - \cancel{5} = 0.024 \times 3.02 + 0.7 + 151 \times 0.024 \times 0.2 + \cancel{5}$$

$$R_1 = 20.1\ k\Omega$$

$$R_{TH} \Rightarrow \frac{20.1 R_2}{20.1 + R_2} = 3.02 \Rightarrow R_2 = 3.55\ k\Omega$$

(b) $A_v = \dfrac{-\beta R_C}{r_\pi + (1+\beta) R_E}$ or using $A_v \cong -\dfrac{R_C}{R_E}$

$r_\pi = \dfrac{150 \times 0.026}{3.57} = 1.09\ k\Omega$ $= -\dfrac{1.2}{0.2}$

$A_v = \dfrac{-150 \times 1.2}{1.09 + 151 \times 0.2} = -5.75$ $\boxed{A_v = 6}$

Q7. Assume the transistor in the circuit in Figure 7 has parameters $\beta = 120$. (a) Design a bias-stable circuit such that $VCEQ = 5.20V$. (b) Determine the small signal trans-resitance function $R_m = v_o/i_s$. The Early voltage for the BJT is 100V. Using the results of part (a), determine the variation in R_m if $100 \leq \beta \leq 150$.

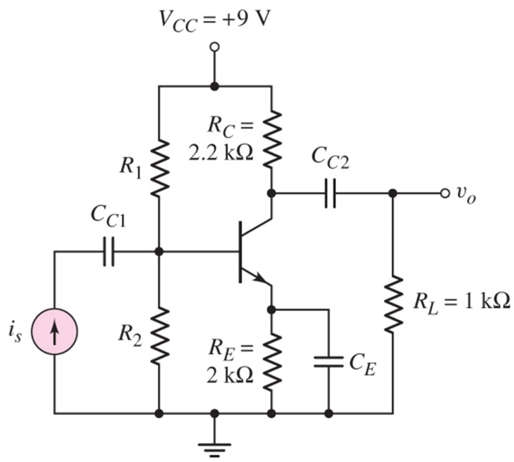


Figure 7: Circuit for Q7

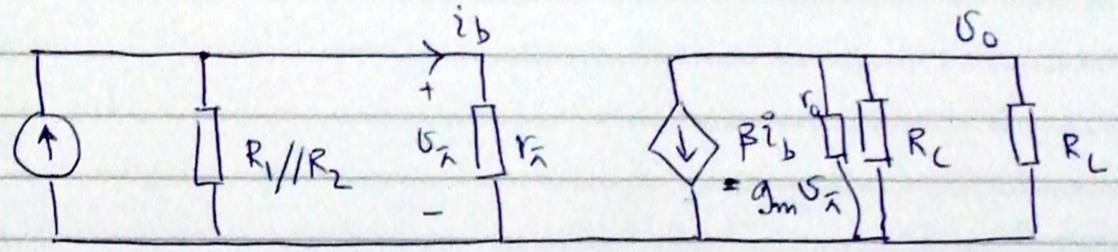
Q7:

(a) Follow steps in Q6(a) for this part.

$R_1 = 80.5\ k\Omega \qquad R_2 = 34.6\ k\Omega \qquad R_{TH} = 24.2\ k\Omega$

$I_{CQ} = 0.905\ mA\ , \ I_{BQ} = 0.007\ mA \qquad V_{TH} = 2.707\ V$

(b) Small-signal model



$$r_\pi = \frac{120 \times 0.026}{0.905} \qquad g_m = \frac{I_{CQ}}{V_T} = \frac{0.905}{0.026} \qquad r_o = \frac{V_A}{0.905} = 110$$

$$= 3.448\ k\Omega \qquad = 34.81\ mA/V$$

$$v_\pi = i_s\,(R_1 // R_2 // r_\pi) = i_s\,(80.5 // 34.6 // 3.448)$$

$$v_\pi = i_s \times 3.02$$

$$v_o = -g_m v_\pi\,(R_C // r_o // R_L)$$

$$v_o = -34.81 \times i_s \times 3.02 \times (2.2 // 110 // 1)$$

$$R_m = \frac{v_o}{i_s} = -71.8\ V/mA$$

(c) For $\beta = 100$ For $\beta = 150$

$I_{BQ} = \dfrac{V_{TH} - 0.7}{R_{TH} + (1+\beta)R_E}$ $I_{BQ} = 0.923$ mA

$= 0.008873$ mA

$I_{CQ} = 0.8873$ mA

$r_\pi = 2.93$ kΩ

$g_m = 34.13$ mA/V

$r_o = \cancel{100}$ 113 kΩ

$R_m = -61.$ V/mA $R_m = -87.2$ V/mA

So, $61. \leq |R_m| \leq 87.2$

Q8. For the transistors in the circuit in Figure 8, the parameters are $\beta = 100$ and $V_{BE(on)} = 0.7V$. The Early voltage is $V_A = \infty$ for Q1 and Q2, and is $V_A = 50V$ for Q3 and Q4. (a) Design resistor values such that $I_3 = 400\mu A$ and $V_{CE1} = V_{CE2} = 10V$. (b) Find A_d, A_{cm}, and CMRR (in dB) for one-sided output at v_{O2}. (c) Determine the differential- and common-mode input resistances.

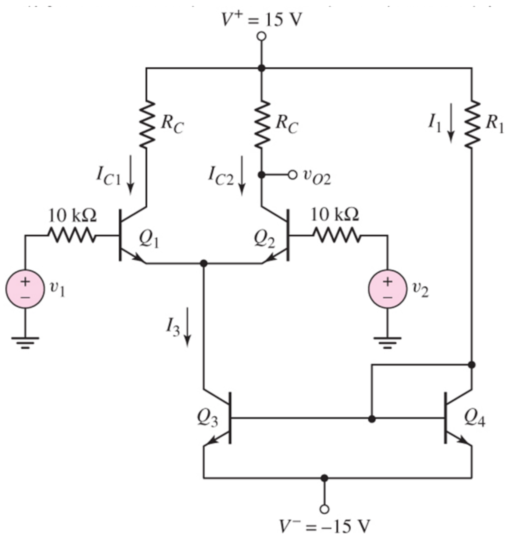


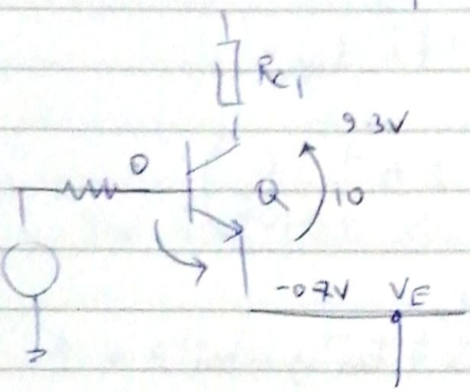
Figure 8: Circuit for Q8

Q8:

(a) Neglecting the base currents

$$I_1 = I_3 = 400\,\mu A$$

$$R_1 = \frac{30 - 0.7}{400\mu} = 73.25\ k\Omega$$



$$V_E = -0.7\ V$$

Since $V_{CE_1} = 10\ V$

$$V_C = 9.3\ V$$

$$R_C = \frac{15 - 9.3}{0.2} = 28.5\ k\Omega$$

(b)

$$r_\pi = \beta V_{T}/I_C$$

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$r_\pi = \frac{100 \times 0.026}{0.2}$$

$$r_\pi = 13\ k\Omega$$

$$= \frac{100 \times 28.5}{2(13 + 10)}$$

$$A_d = 62$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B} \times \frac{1}{1 + \frac{2r_o(1+\beta)}{r_\pi + R_B}}$$

$$r_o(Q_3) = \frac{V_A}{I_Q} = \frac{50}{0.4} = 125\ k\Omega$$

$$A_{cm} = -\frac{100 \times 28.5}{13 + 10} \times \left\{ \frac{1}{1 + \frac{2 \times 125 \times 101}{13 + 10}} \right\}$$

$$= -0.113$$

$$CMRR_{dB} = 20 \log_{10}\left(\frac{62}{0.113}\right) = 54.8 \text{ dB}$$

(c) $$R_{id} = 2(r_\pi + R_B) = 46\text{ k}\Omega$$

$$R_{icm} = \frac{1}{2}\left[r_\pi + R_B + 2(1+\beta)r_o\right]$$

$$= 12.6 \text{ M}\Omega$$

Q9. The loop-gain function of an amplifier is given by

$$T(f) = \frac{\beta(100)}{\left(1 + j\frac{f}{10^3}\right)^3}$$

determine the value of β at which the amplifier becomes unstable.

Q9:

$$T(f) = \frac{\beta(100)}{\left(1 + j\frac{f}{10^3}\right)^3}$$

loop gain can be written as:

$$T(f) = \frac{\beta(100)}{\left[\sqrt{\left\{1 + \left(\frac{f}{10^3}\right)^2\right\}}\right]^3} \angle -3\tan^{-1}\left(\frac{f}{10^3}\right)$$

The freq at which the phase becomes -180° is

$$-3\tan^{-1}\left(\frac{f_{180}}{10^3}\right) = -180^\circ$$

$$f_{180^\circ} = 10^3 \times \tan\left(\frac{180^\circ}{3}\right)$$

$$= 1.73 \times 10^3 \text{ Hz}$$

Now, for sability

$$|T(f_{180^\circ})| = \frac{\beta(100)}{\left[1 + \left(\frac{f_{180}}{10^3}\right)^2\right]^{3/2}} < 1$$

$$100\beta < 1 \times \left[1 + (1.73)^2\right]^{3/2}$$

$$\underline{\underline{\beta < 8 \times 10^{-2}}}$$

If $\underline{\underline{\beta < 0.08}}$ system is stable.

Q10. The loop gain function of a feedback amplifier is given by

$$T(f) = \frac{\beta(3000)}{\left(1 + j\frac{f}{10^3}\right)\left(1 + j\frac{f}{10^5}\right)^2}$$

(a) determine the value of f at which the phase becomes -180. (b) Then, determine the β at which the amplifier becomes unstable.

Q10: $$T(f) = \frac{\beta(3000)}{\left(1+j\frac{f}{10^3}\right)\left(1+j\frac{f}{10^5}\right)^2}$$

Phase of the loop-gain $= -\tan^{-1}\left(\frac{f}{10^3}\right) - 2\tan^{-1}\left(\frac{f}{10^5}\right)$

~~&~~ Phase becomes $-180°$ when $f = f_{180°}$

$$\therefore \quad -180 = -\tan^{-1}\left(\frac{f_{180°}}{10^3}\right) - 2\tan^{-1}\left(\frac{f}{10^5}\right)$$

Using the trigonometric relationship,

$$x + y + z = xyz \quad \text{if} \quad \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$$

Then, $$\frac{f_{180}}{10^3} + \frac{2f_{180°}}{10^5} = \frac{2f^3_{180°}}{10^{13}} \Rightarrow \frac{1}{10^3} + \frac{2}{10^5} = \frac{2f^2_{180°}}{10^{13}}$$

$$f_{180°} = 100995 \quad kHz$$

Now,

$$|T(f_{180°})| = \frac{\beta(3000)}{\sqrt{1+\left(\frac{f_{180°}}{10^3}\right)^2} \times \left[1+\left(\frac{f_{180°}}{10^5}\right)^2\right]}$$

$$= \frac{\beta(3000)}{\underbrace{\sqrt{1+(100.995)^2}}_{\simeq 100.1} \times \underbrace{\left[1+1.00995^2\right]}_{\simeq 2.02}} = \beta \times \overset{14.83}{\cancel{59.99}}$$

$$= \beta \times \cancel{60}\, 14.83$$

For stability $|T(f_{180°})| < 1$

$\cancel{\beta < 1/14.8 = 0.0678}$ //

$$\beta < \frac{1}{14.8} = 0.067$$ //

Q11. Consider a three-pole amplifier with a loop gain function given by

$$T(f) = \frac{250}{\left(1 + j\frac{f}{10^3}\right)\left(1 + j\frac{f}{10^5}\right)^2}$$

. Show that the system is unstable. (b) Stabilize the circuit by inserting a new dominant pole. Assume the original poles are not altered. At what frequency must the new pole be placed to achieve a phase margin of 60^o.

Q11:

$$T(f) = \frac{250}{\left(1+j\frac{f}{10^3}\right)\left(1+j\frac{f}{10^5}\right)^2}$$

The phase angle of $T(f)$ becomes -180° when $f = 100.995$ kHz (refer Q 10)

NOW,

$$|T(f_{180^\circ})| = \frac{250}{\sqrt{\left\{1+(100.995)^2\right\}} \times \left[1+ 1.00995^2\right]}$$

≈ 100.1 $\qquad \approx 2.02$

$= 1.24 > 1 \rightarrow$ System is unstable.

To achieve a phase margin of 60°, a phase of 120° should be achieved close to $f_{120^\circ} = 10^3$.

Assume new ~~pole is~~ dominant pole freq is f_{PD}

$$f_{PD} \ll 10^3 \text{ Hz}$$

$$|T_{PD}(f_{120})| = 1 = \frac{250}{\sqrt{1+\left(\frac{10^3}{f_{PD}}\right)^2}\sqrt{1+\left(\frac{10^3}{10^3}\right)^2}\left[1+\left(\frac{10^3}{10^5}\right)^2\right]}$$

$$1 = \frac{250}{\sqrt{1+\left(\frac{10^3}{f_{PD}}\right)^2} \times \underbrace{1.414}_{\sqrt{2}} \times 1}$$

$$f_{PD} = 5.65 \text{ Hz}$$

$$\sqrt{2} = \frac{250}{\sqrt{1+\left(\frac{10^3}{F}\right)^2}}$$

$$\frac{250}{\sqrt{2}} = \sqrt{1+\left(\frac{10^3}{F}\right)^2}$$

$$\frac{250^2}{2} = 1+\left(\frac{10^3}{F}\right)^2$$

$$\frac{250^2}{2} - 1 = \left(\frac{10^3}{F}\right)^2$$

$$\sqrt{\frac{250^2}{2} - 1} = \frac{10^3}{F}$$

$$F = \frac{10^3}{\sqrt{\frac{250^2}{2} - 1}}$$

Q12. The transistor parameters for the circuit in Figure 9 are: $\beta = 200, V_{BE(on)} = 0.7V$, and $V_A = 80V$. (a) Determine the differential-mode voltage gain $A_d = v_{o3}/v_d$ and the common-mode voltage gain $A_{cm} = v_{o3}/v_{cm}$. (b) Determine the output voltage v_{o3} if $v_1 = 2.015 \sin \omega t\ V$ and $v_2 = 1.985 \sin \omega t\ V$. Compare this output to the ideal output that would be obtained if $A_{cm} = 0$. (c) Find the differential-mode and common-mode input resistances. (d) carry out SPICE simulations and verify results in (a) and (d).

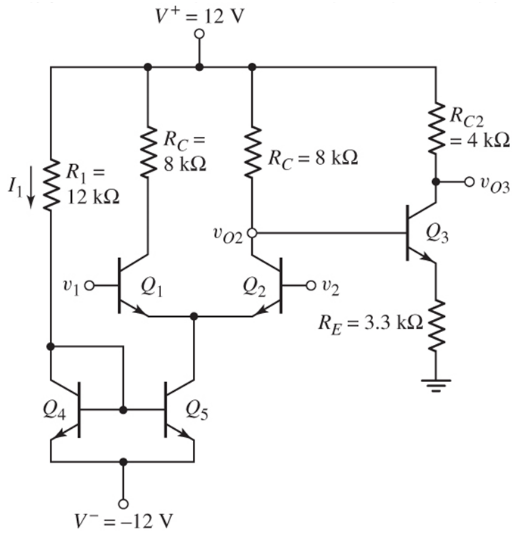


Figure 9: Circuit for Q9

Q12:

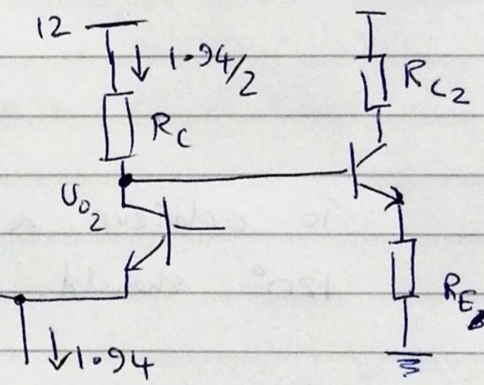
(a) $$A_{d_1} = \frac{v_{o2}}{v_d} = \frac{1}{2} g_{m_2} (R_c \parallel R_{i_3})$$

↳ input impedance of Q3

$$I_1 = \frac{12 - 0.7 - (-12)}{R_1} = \frac{23.3}{12} = 1.94 \text{ mA}$$

$$g_{m_2} = \frac{I_Q}{V_T} = \frac{\frac{1}{2} \times 1.94}{0.026} = 37.3 \text{ mA/V}$$

$$r_{\pi_3} = \frac{200 \times 0.026}{I_{E_3}}$$



?

$$v_{o_2} = 12 - \frac{1}{2} \times 1.94 \times 8$$

$$v_{o_2} = 4.24 \text{ V}$$

Assuming $I_{C_3} \simeq I_{E_3} = \frac{4.24 - 0.7}{R_E}$

$$= 1.07 \text{ mA}$$

$$r_{\pi_3} = 4.86 \text{ k}\Omega$$

$$\therefore R_{i_3} = r_{\pi_3} + (1+\beta) R_E = 668 \text{ k}\Omega$$

$$A_{d_1} = \frac{1}{2} \times 37.3 \times (8 \parallel 668) = 147.4$$

$$A_3 = \frac{-\beta R_{C_2}}{r_{\pi_3} + (1+\beta) R_E} = -\frac{200 \times 4}{4.86 + 201 \times 3.3} = -1.197$$

$$\therefore A_d = \frac{v_{o3}}{v_d} = 147.4 \times -1.197 = -176$$

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + 2/\beta} \times \frac{\left(1 + \frac{V_{CE2}}{V_A}\right)}{\left(1 + \frac{V_{CE1}}{V_A}\right)}$$ → For the current mirror to get accurate bias currents.

$I_1 = 1.94$ mA, $I_{c_1} = \frac{2.16}{2}$ mA

$$A_{cm_1} = \frac{-g_{m_2}(R_c // R_{i3})}{1 + \frac{2(1+\beta)R_o}{r_{\pi_2}}}$$

$$R_o = r_{o5} = \frac{V_A}{I_{c5}} = \frac{80}{1.94} = 41.2\ k\Omega$$

$$r_{\pi_2} = \frac{200 \times 0.026}{\frac{1}{2} \times 1.94} = 5.36\ k\Omega$$

$$A_{cm_1} = \frac{-(37.3)(8 // 668)}{1 + \frac{2 \times 201 \times 41.2}{5.36}} = -0.09539$$

$$A_{cm} = A_{cm_1} \times A_2 = -0.09539 \times -1.197$$

$$\underline{\underline{A_{cm} = 0.114}}$$

(b) $v_d = v_1 - v_2 = 2.015 \sin \omega t - 1.985 \sin \omega t$

$= 0.03 \sin \omega t$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2 \sin \omega t$$

$$v_{o_3} = A_d v_d + A_{cm} v_{cm}$$

$$= (-176 \times 0.03 + 0.114 \times 2$$

$$v_{o3} = -5.052 \sin \omega t //$$

Ideally for an amplifier, $A_{cm} = 0$

$$v_{o_3\,ideal} = -5.28 \sin \omega t //$$

(c) $R_{id} = 2 r_{\pi_2} = 10.72\ k\Omega //$

$2R_{icm} = 2(1+\beta)R_o // (1+\beta) r_o$ $\longrightarrow$

$r_o = \frac{V_A}{I_{c_2}} = \frac{80}{\frac{1}{2} \times 1.94} = 82.5\ k\Omega$ $\qquad \underline{\underline{R_{icm} = 4.015\ M\Omega}}$

Q13. For the circuit in Figure 10, the transistor parameters are $\beta = 100$ and $VA = \infty$. The bias currents in the transistors are indicated on the figure. (a) Determine the input resistance R_i, the output resistance R_o, (b) Draw the small-signal model and determine the voltage gain $A_v = vo/vin$.

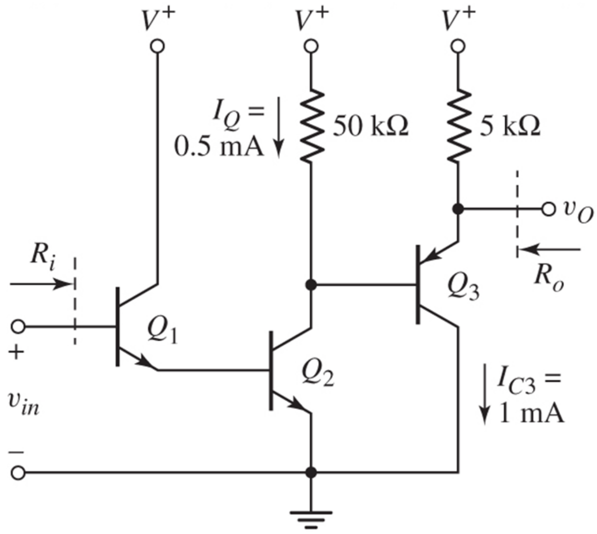
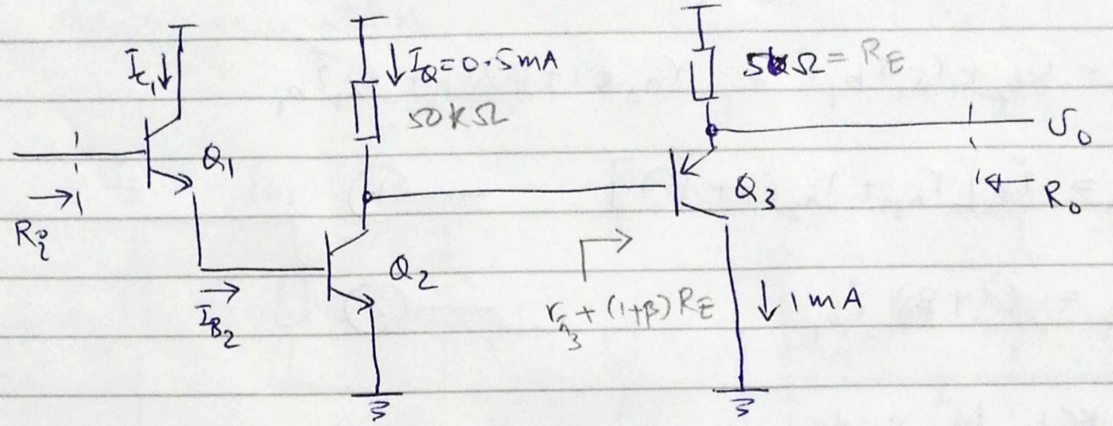
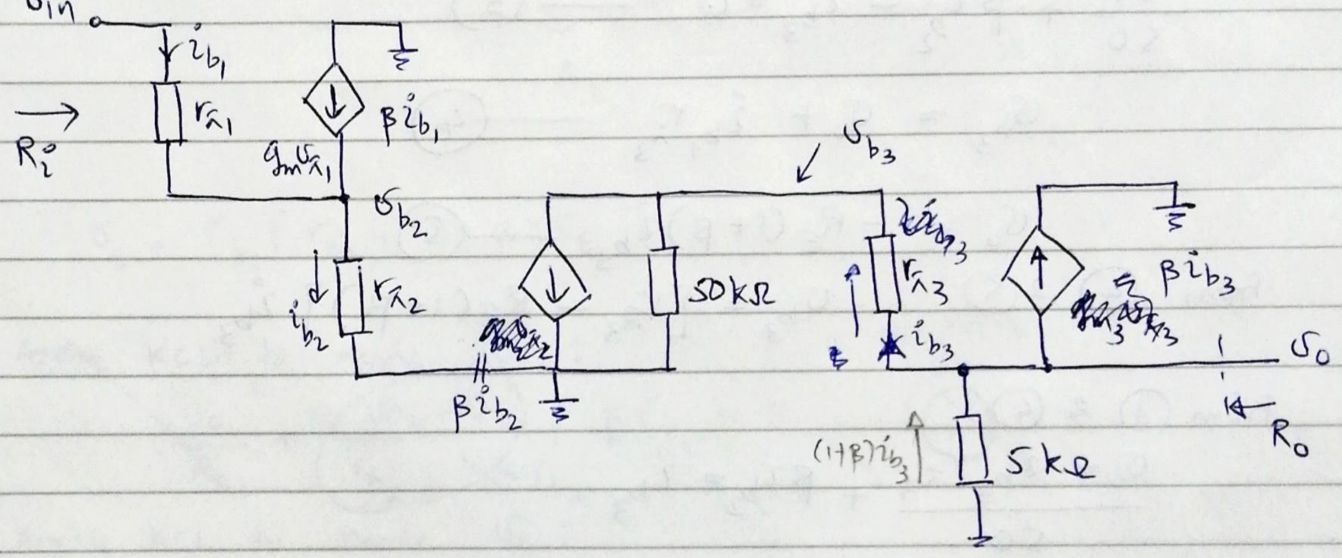


Figure 10: Circuit for Q13

Q 13:





(a) $R_i = r_{\pi_1} + (1+\beta) r_{\pi_2}$

$r_{\pi_1} = \frac{100 \times 0.026}{I_{C_1}}$?

$I_{C_1} \simeq I_{E_1} = I_{B_2} = \frac{I_Q}{2} = \frac{0.5}{100}$

$r_{\pi_1} = 520 \text{ k}\Omega$

$r_{\pi_2} = \frac{100 \times 0.026}{I_Q} = 5.2 \text{ k}\Omega$

$\therefore R_i = 520 + 101 \times 5.2 = 1.05 \text{ M}\Omega$

$R_o = 5 \,//\, \left(\frac{50 + r_{\pi_3}}{1+\beta} \right)$ ⟹

$r_{\pi_3} = \frac{100 \times 0.0026}{1} = 2.6 \text{ k}\Omega$

short circuit input source

$R_o = 0.472 \text{ k}\Omega$

From the small-signal model

$$v_{in} = v_{b_2} + r_{\pi_1} i_{b_1} = r_{\pi_2}(1+\beta) i_{b_1} + r_{\pi_1} i_{b_1}$$

$$v_{in} = i_{b_1}[r_{\pi_1} + r_{\pi_2}(1+\beta)] \quad \text{—— (1)}$$

$$i_{b_2} = (1+\beta) i_{b_1} \quad \text{—— (2)}$$

Apply KCL to node v_{b_3}

$$\frac{v_{b_3}}{50} + \beta i_{b_2} - i_{b_3} = 0 \quad \text{—— (3)}$$

$$v_{b_3} = v_o + i_{b_3} r_{\pi_3} \quad \text{—— (4)}$$

$$v_o = -R_E(1+\beta) i_{b_3} \quad \text{—— (5)}$$

From (4) & (5) $\quad v_{b_3} = [r_{\pi_3} - R_E(1+\beta)] i_{b_3}$

From (3) & (4)

$$\frac{v_o + i_{b_3} r_{\pi_3}}{50} + \beta i_{b_2} = i_{b_3}$$

$$\frac{v_o}{50} + \beta i_{b_2} = i_{b_3}\left(1 - \frac{r_{\pi_3}}{50}\right)$$

$$\frac{v_o}{50} + \beta(1+\beta) i_{b_1} = \underbrace{\frac{-v_o}{(1+\beta)R_E}}_{\text{from (5)}}\left(1 - \frac{r_{\pi_3}}{50}\right)$$

$$v_o\left\{\frac{1}{50} + \frac{1}{(1+\beta)R_E}\left(1 - \frac{r_{\pi_3}}{50}\right)\right\} = \frac{-\beta(1+\beta) v_{in}}{r_{\pi_1} + r_{\pi_2}(1+\beta)}$$

$$v_o\left\{\frac{1}{50} + \frac{1}{101\times5}\left(1 - \frac{2.6}{50}\right)\right\} = \frac{-100\times101\ v_{in}}{520 + 5.2\times101}$$

$$\frac{v_o}{v_{in}} = -442$$

Q14. Three voltage amplifiers are in cascade as shown in Figure 11 with various amplification factors. Determine the value of β such that the closed-loop voltage gain is $A_{vf} = V_o/V_s = 120$. Find the percent change in A_{vf} if each individual amplifier gain decreases by 10 percent.

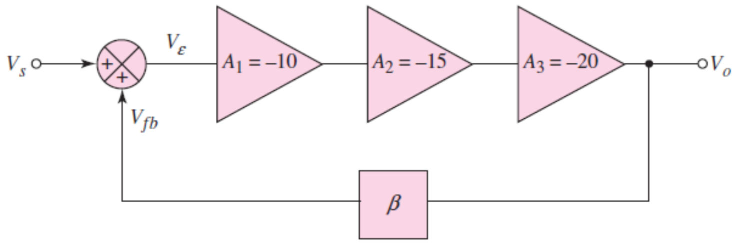


Figure 11: Circuit for Q13

Q14

$$V_o = -10 \times -15 \times -20 \; V_\epsilon = -3000 \; V_\epsilon$$

$$V_\epsilon = \beta V_o + V_s$$

$$V_o = -3000(\beta V_o + V_s)$$

$$A_{vf} = \frac{V_o}{V_s} = \frac{-3000}{1+3000\beta}$$

$$-120 = -\frac{3000}{1+3000\beta}$$

$$\underline{\underline{\beta = 0.008}}$$

A 10% decrement in stage ~~make~~ $\underline{A_{openloop}}$ results in

$$A_{openloop} = -9 \times -13.5 \times -18$$
$$= -2187$$

$$A_{vf} = \frac{-2187}{1+2187 \times 0.008} = -118.24$$

$$\% \text{ change} = \frac{120-118.24}{120}$$

$$\underline{\underline{= 1.47\%}}$$

Q15. The BJT in the circuit of Figure 12 has $\beta = 100$ and $V_{BE} = 0.7$ V. Find the DC voltage at the collector and the DC collector current.

Replacing the transistor by its T Model, draw the small-signal equivalent circuit of the amplifier. Analyse the resulting circuit to determin the voltage gain v_o/v_i.

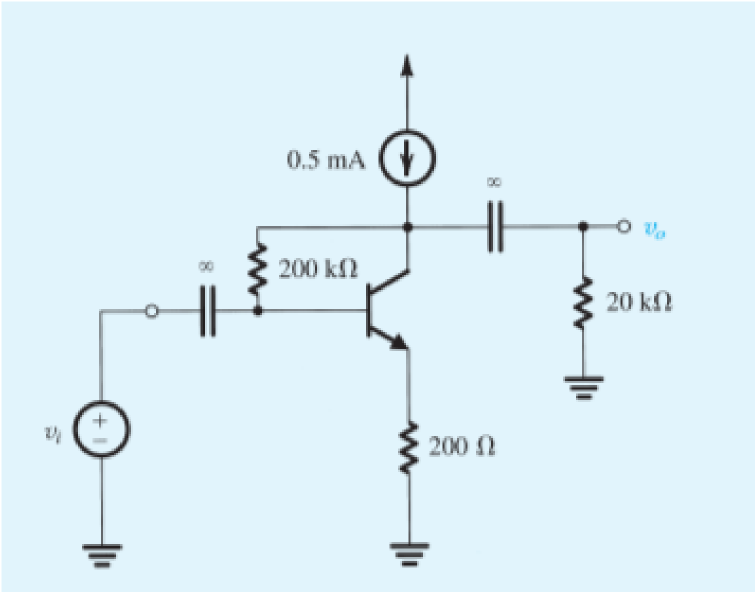
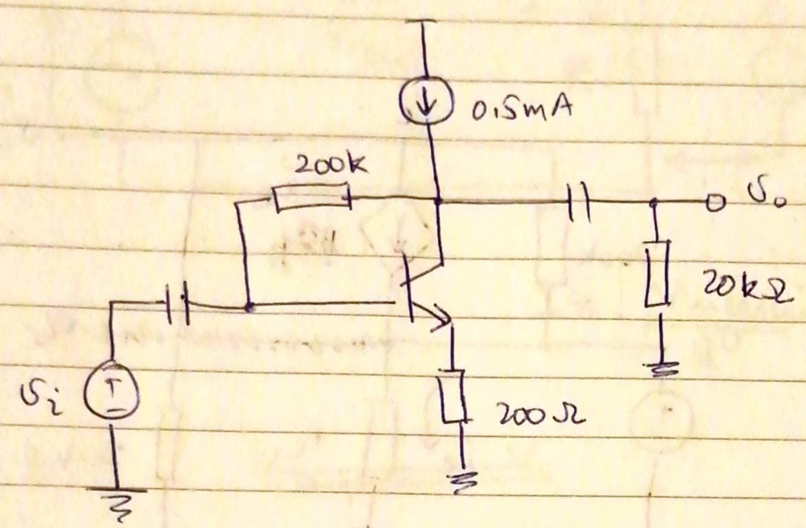


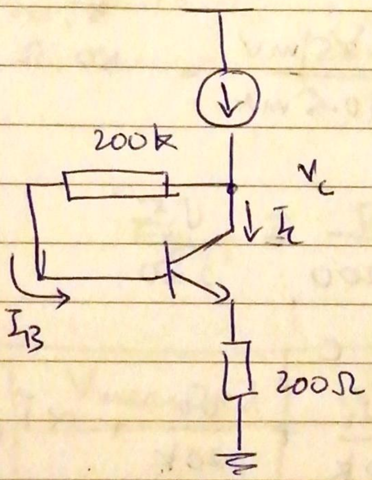
Figure 12: Circuit for Q15

$\pm 0.08mm$

Q16



DC circuit



$$I_c + I_B = 0.5 \quad \text{—— } (1) \quad \Rightarrow \quad I_c + \frac{I_c}{\beta} = 0.5$$

$$100 I_B + I_B = 0.5$$

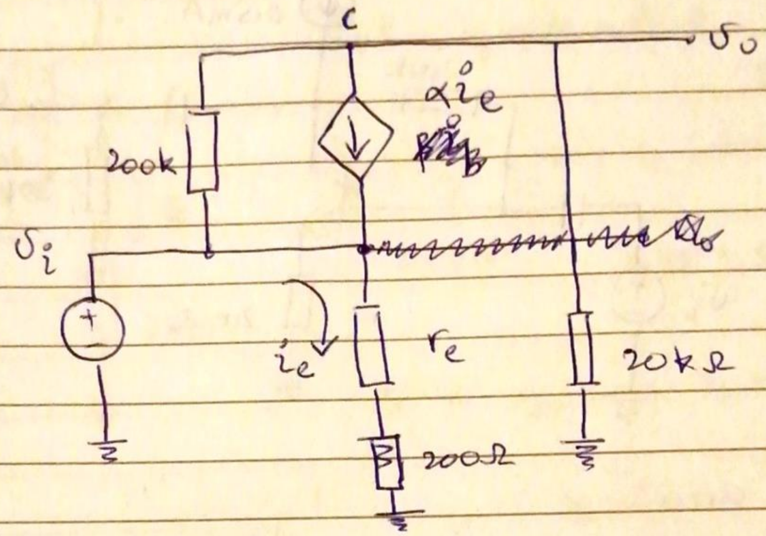
$$I_c = \frac{\beta}{\beta + 1} \times 0.5$$

$$I_B = \frac{0.5m}{101} = 4.95\ \mu A$$

$$I_c = 0.495\ mA$$

$$I_c = 100 \times 4.95\ \mu A$$
$$= 0.495\ mA$$

$$V_c = 200 \times I_E + 0.7 + I_B \times 200$$
$$= 200 \times 0.495 \times 10^{-3} + 0.7 + 4.95 \times 10^{-6} \times 200 \times 10^{3}$$
$$= \cancel{200}\ 0.1 + 0.7 +$$
$$V_c \cong 1.79\ V$$



$$r_e = \frac{V_T}{I_E} \simeq \frac{25\,mV}{0.5\,mA} = 50\ \Omega$$

$$i_e = \frac{v_i}{r_e + 200} = \frac{v_i}{250}$$

KCL at node C

$$\frac{v_o - v_i}{200k} + \frac{v_o}{20k} + \alpha i_e = 0$$

$$\frac{v_o - v_i}{200k} + \frac{v_o}{20k} + \alpha \frac{v_i}{250} = 0$$

$$v_o\left[\frac{1}{200k} + \frac{1}{20k}\right] = v_i\left[\frac{1}{200k} - \frac{0.99}{250}\right]$$

$$\frac{v_o}{v_i} = -71.9$$

Q16. For the circuit shown in Figure 13, the transistor parameters are β = 100 and VA=∞. (a) Determine the dc voltages at the collector, base, and emitter terminals. (b) Determine the small-signal voltage gain Av = v_o/v_s . (c) Find the input resistance Ri .

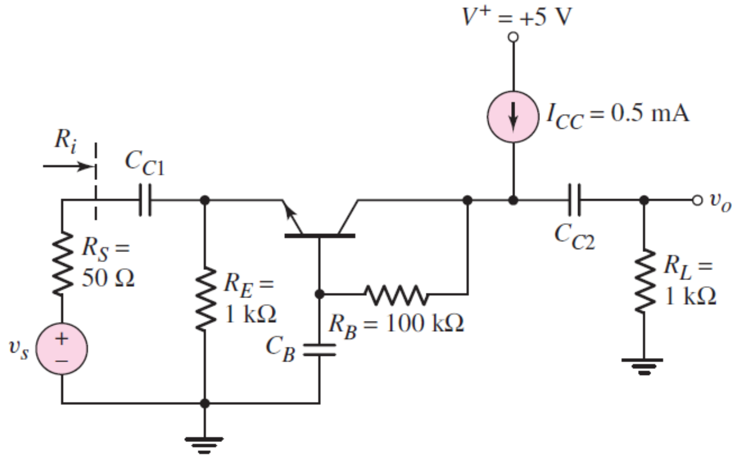
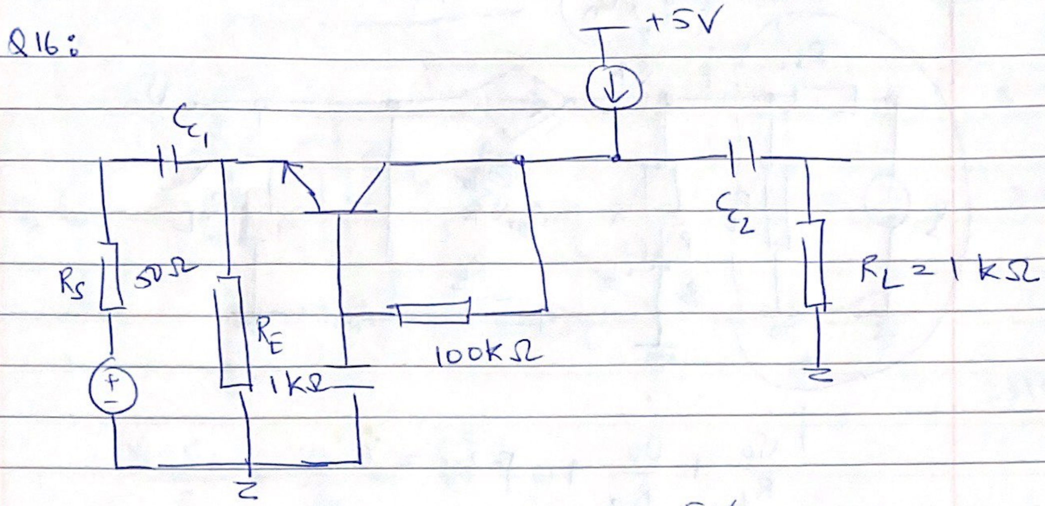
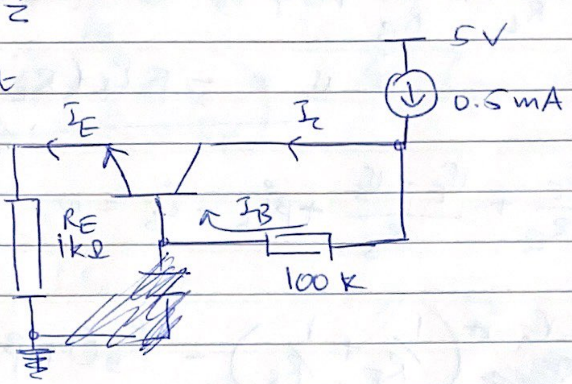


Figure 13: Circuit for Q15

Q16:



DC circuit

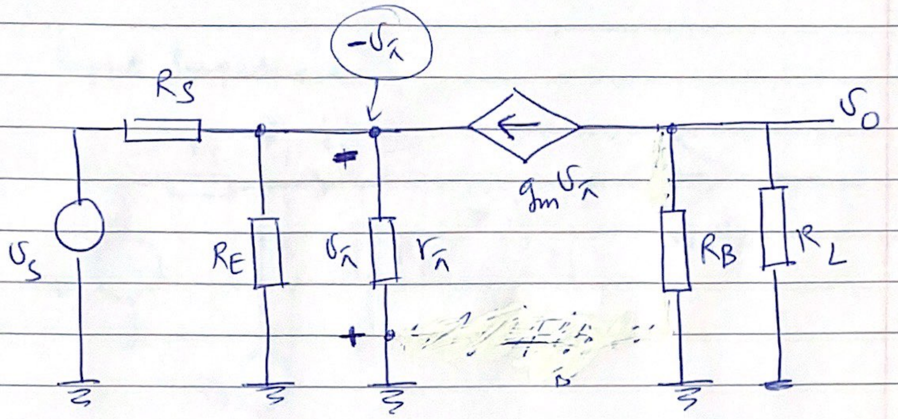


$I_E = 0.5\ mA \qquad I_B = \frac{0.5}{1+\beta} = \frac{0.5}{101}\ mA$

$V_E = I_E \times R_E = 0.5 \times 1 = 0.5\,V$

$V_B = V_E + V_{BE(ON)} = 0.5 + 0.7 = 1.2\ V$

$V_C = V_B + I_B \times R_B = 1.2 + \frac{0.5}{101} \times 100 = \underline{\underline{1.7\ V}}$



$$v_o = -g_m v_\pi (R_L \,//\, R_B) \quad \text{— (1)}$$

$$\frac{-v_\pi}{r_\pi} + \frac{-v_\pi}{R_E} + \frac{-v_\pi - v_s}{R_s} = g_m v_\pi$$

$$-v_\pi \left[\frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} + g_m \right] = \frac{v_s}{R_S}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 0.026}{0.5 \times \frac{100}{101}} = 5.25\ k\Omega$$

$$g_m = \frac{\beta I_c}{V_T} = \frac{100 \times 0.5/101}{0.026} = \underline{19.0}\ mA/V$$

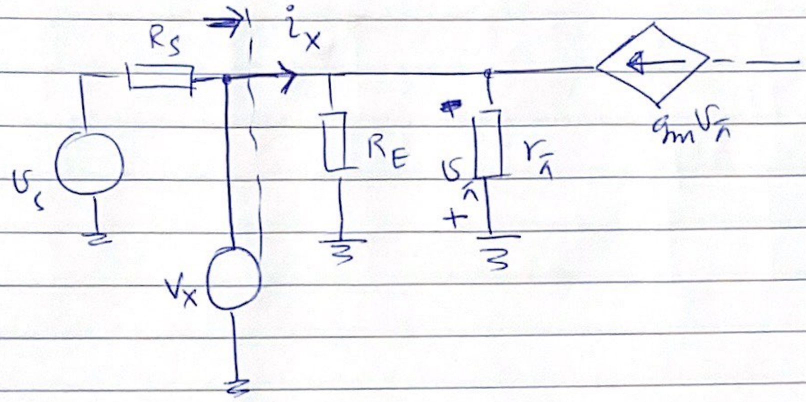
$$v_\pi \left[\frac{1}{5.25k} + \frac{1}{1k} + \frac{1}{50} + 19m \right] = \frac{-v_s}{50}$$

$$v_\pi = -0.497\, v_s$$

$$v_o = -19 \times 10^{-3} \times (-0.497)\, v_s \left[\frac{\overset{R_L}{1k} \times \overset{R_B}{100k}}{1k + 100k} \right]$$

$$\underline{\underline{\frac{v_o}{v_s} = +9.34}}$$

input impedance



$$i_x = \frac{v_x}{R_E} + \frac{v_x}{r_\pi} - g_m v_\pi$$

$$v_x = -v_\pi$$

$$i_x = \frac{v_x}{R_E} + \frac{v_x}{r_\pi} + g_m v_x$$

$$i_x = v_x\left[\frac{1}{1k} + \frac{1}{5.25k} + 19\times10^{-3}\right]$$

$$R_i = \frac{v_x}{i_x} = \frac{1}{\left[1/1k + 1/5.25k + 19\times10^{-3}\right]}$$

$$= \underline{\underline{49.5\ \Omega}}$$

Q17. The parameters of the op-amp in the circuit shown in Figure 14 are $A_v = 10^5$, $R_i = 30k\Omega$, and $R_o = 500\Omega$. The transistor parameters are hFE = 140 and VA = ∞. Assume that $v_O = 0$ at the quiescent point. Determine (a) A_{vf} , (b) R_{if} , and (c) R_{of}.

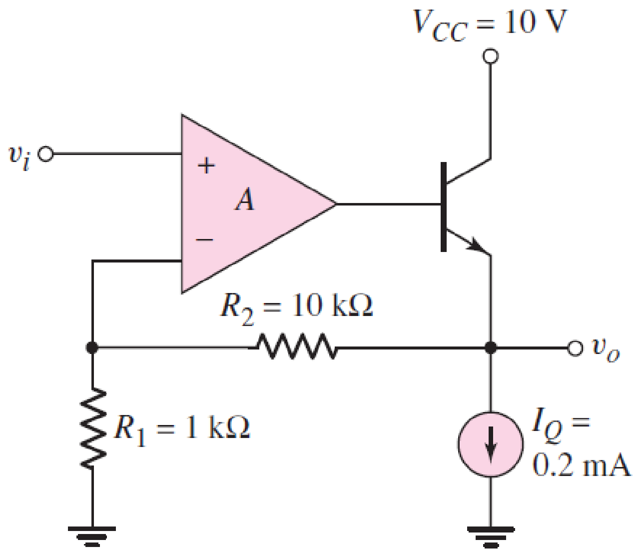


Figure 14: Q17

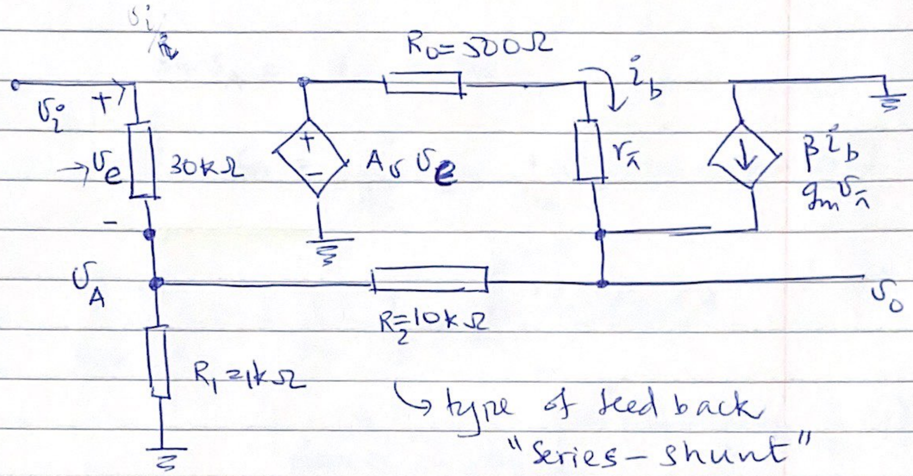
Q17:

$I_{CQ} \simeq 0.2\ mA$

$$r_\pi = \frac{140 \times 0.026}{0.2} = 18.2\ k\Omega$$

$$g_m = \frac{0.2}{0.026} = 7.692\ mA/V$$

(a)



↳ type of feed back "Series − shunt"

$$i_b = \frac{(A_v v_e - v_o)}{R_o + r_\pi}$$

$$\frac{v_o - v_A}{R_2} = (\beta+1) i_b \quad \text{—②}$$

$$i_b = \frac{A_v (v_i - v_A) - v_o}{R_o + r_\pi} \quad \text{—①}$$

$$\frac{v_A - v_o}{R_2} + \frac{v_A}{R_1} + \frac{v_A - v_i}{R_i} = 0 \quad \text{—③}$$

$$v_A \left[\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right] = \frac{v_o}{R_2} + \frac{v_i}{R_i} \quad \text{—③}'$$

~~from ① & ②~~ $\cancel{\left[\frac{A_v (v_i - v_A) - v_o}{R_o + r_\pi} \right] (1+\beta) = \frac{v_o - v_A}{R_2}}$

From (1) & (2)

$$\frac{v_o - v_A}{R_2} = (1+\beta)\left[\frac{A_v(v_i - v_A) - v_o}{R_o + r_\pi}\right] \quad \text{—— (4)}$$

$$(3)' \Rightarrow \quad v_A\left[\frac{34}{30}\right] = \frac{v_o}{10} + \frac{v_i}{30}$$

$$34 v_A = 3v_o + v_i$$

$$v_A = \frac{3}{34}v_o + \frac{1}{34}v_i \quad \text{—— (5)}$$

$$\frac{v_o}{30k} - \frac{1}{30k}\left[\frac{3}{34}v_o + \frac{1}{34}v_i\right] = \cancel{147}^{\,1+\beta}\left[\cancel{10^5}\cancel{(v_i - \beta}\right.$$

$$= \frac{147}{18.2k + 500}\left[10^5\left(v_i - \frac{3}{34}v_o - \frac{1}{34}v_i\right) - v_o\right]$$

$$\frac{v_o}{30} - \frac{1}{30}\left[\frac{3v_o}{34} + \frac{v_i}{34}\right] = \frac{147}{18.7}\left[10^5\left(\frac{33}{34}v_i - \frac{3}{34}v_o\right) - v_o\right]$$

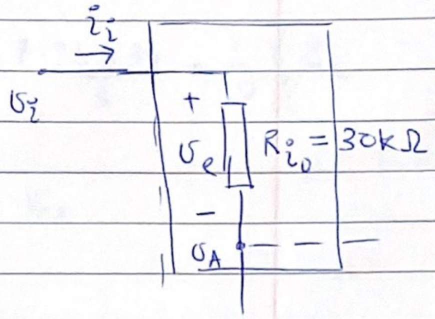
$$v_o\left[\frac{1}{30} - \frac{3}{30\times34}\right] - \frac{v_i}{30\times34} = \frac{147\times10^5}{18.7}\left[\frac{33}{34}v_i - \frac{3}{34}v_o\right] - \frac{147}{18.7}v_o$$

$$0.03v_o - \frac{v_i}{1020} = 7.63\times10^5 v_i - 0.69^{\times10^5} v_o - 7.86 v_o$$

$$v_o\left[0.03 + 0.69\times10^5 + 7.86\right] = \left[7.63\times10^5 + \frac{1}{1020}\right]v_i$$

$$\frac{v_o}{v_i} = 11.05$$

$$R_{if} = \frac{v_i}{i_i}$$



$$= \frac{v_i}{v_e / R_{i_o}}$$

$$R_{if} = R_{i_o}\left(\frac{v_i}{v_e}\right)$$

$$v_A = v_i - v_e$$

From (5)
$$v_i - v_e = \frac{3}{34} v_o + \frac{1}{34} v_i$$

$$\frac{3}{34} v_o = v_i\left(1 - \frac{1}{34}\right) - v_e$$

$$v_e = \frac{33}{34} v_i - \frac{3}{34} v_o$$

$$= v_i \frac{33}{34} - v_e$$

$$v_o = \frac{1}{3}\left(\frac{33}{\cancel{34}} v_i - 34 v_e\right)$$

$$v_o = 11 v_i - \frac{34}{3} v_e$$

From (4)
$$v_o\left[\frac{1}{R_2} + \frac{(1+\beta)}{R_o + r_\pi}\right] - \frac{v_A}{R_2} = \frac{(1+\beta)A_v}{R_o + r_\pi}\underbrace{(v_i - v_A)}_{\to v_e}$$

$$\left\{11 v_i - \frac{34}{3} v_e\right\}\left[\frac{1}{10K} + \frac{147}{18.7K}\right] - \frac{(v_i - v_e)}{10K} = \frac{147\times10^5}{18.7K} v_e$$

$$7.96\left\{11 v_i - \frac{34}{3} v_e\right\} - \frac{(v_i - v_e)}{10} = \frac{147\times10^5}{18.7} v_e$$

$$v_i\left\{7.96\times 11-\frac{1}{10}\right\}=\left\{\frac{147\times 10^5}{18.7}+\frac{7.96\times 34}{3}-\frac{1}{10}\right\}v_e$$

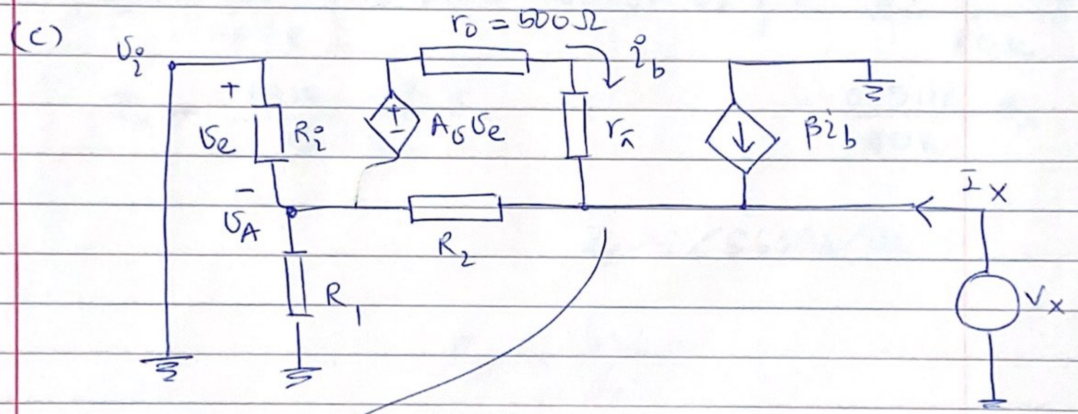
$$87.46\,v_i = 786186\,v_e$$

$$\frac{v_i}{v_e}=8989$$

$$\therefore\ R_{if} = 30\times 10^3\times 8989$$

$$R_{if}\simeq 269\ M\Omega$$

(c)



$$I_x+\beta i_b+i_b=\frac{v_x-v_A}{R_2}$$

Now

$$v_A = v_i - v_e = 0 - v_e = -v_e$$

$$I_x+(1+\beta)i_b=\frac{v_x+v_e}{R_2}$$

$$I_x+(1+\beta)\left[\frac{A_v v_e - V_x}{R_o+r_\pi}\right]=\frac{v_x+v_e}{R_2} \quad \text{———} \ (7)$$

KCL to node v_A gives.

$$\frac{v_A}{R_1} + \frac{v_A - v_X}{R_2} + \frac{v_A}{R_i} = 0$$

$$v_A\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_i}\right] = \frac{v_X}{R_2}$$

$$-v_e\left[\frac{1}{1k}+\frac{1}{10k}+\frac{1}{30k}\right] = \frac{v_X}{\cancel{3}10k}$$

$$v_e = -\frac{30}{340}v_X = -0.0882\, v_X$$

Now, from (7)

$$I_X + \frac{147}{18.7k}\left[10^5 \times (-0.0882 v_X) - v_X\right] = \frac{v_X - 0.0882 v_X}{\cancel{1}0k}$$

$$I_X = \frac{8819 \times 147}{18.7k} v_X \qquad = \frac{0.9118}{\cancel{1}0k} v_X$$

$$\cancel{I_X = 56534\, v_X}$$

$$\cancel{R_o = \frac{v_X}{I_X}}$$

$$I_X = v_X\left\{\frac{0.9118}{10k} + \frac{8819 \times 147}{18.7K}\right\}$$

$$R_o = \frac{v_X}{I_X} = \frac{1}{\left\{\frac{0.9118}{10} + \frac{8819 \times 147}{18.7}\right\}} \times 10^3$$

$$\underline{R_o = 14\ m\Omega}$$

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