Annoucements

1. Gradiance 5 released
   - Due next Thesday midnight

Random Variable

X — Domain $\mathcal{X}$ (Support)

Discrete

$\boxed{P(X=x)}$ $x \in \mathcal{X}$

PMF

Continuous

$p(X=x)$

min $\leq x \leq$ m

PDF $p(x)$

$\boxed{P_A(X=x)}$

$P_B(X=x)$

A probability distribution

A PDF or a PMF

1. Domain ✓
2. PMF or PDF ✓

Bernoulli

Domain — $\{0, 1\}$

$\{Yes, No\}$

$\{Heads, Tails\}$

PMF $\boxed{\text{Parameter} - \theta}$

Bernoulli parameter — p.

$0 \leq p \leq 1$

$$\boxed{\begin{aligned} P(X = heads) &= p \\ P(X = tails) &= 1 - p \end{aligned}} \rightarrow \text{PMF}$$

Binomial Distribution

$\underline{n, \theta}$ (Same as p)

Domain? $\{0, 1, 2, \ldots, n\}$

PMF $p(X = k) = \text{Bin}(k \mid n, \theta)$

$$= \binom{n}{k} \theta^{k} (1-\theta)^{n-k}$$

$$\boxed{{}^{n}C_{k} = \frac{n!}{k!\,(n-k)!}}$$

Multinoulli $\rightarrow$ Generalization of Bernoulli

$$\theta = [\overbrace{p_1, p_2, p_3 \ldots p_k}^{k}] \rightarrow \boxed{\begin{array}{l} 0 \le p_i \le 1 \\ \sum p_i = 1 \end{array}}$$

PMF $p(X=1) = p_1$

$\vdots$

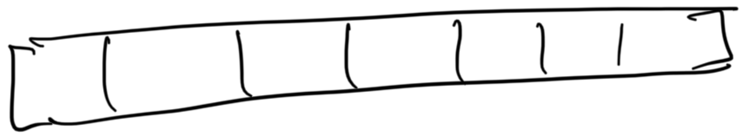
$p(X=k) = p_k$

$[\,0 \mid 0 \mid 0 \mid 0 \mid 1 \mid 0 \mid 0 \mid 0\,]$

Multinomial — Generalization of Binomial

n, $\theta \rightarrow$ vector

X



Poisson

$$\boxed{P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}}$$

$$\mathbb{E}[X] = \sum_{k=0}^{\infty} k \frac{\lambda^k e^{-\lambda}}{k!}$$

$$= \lambda$$

Gaussian (or Normal) Distribution

Domain: $-\infty, \infty$

$$\boxed{\text{pdf: } p(X=x) = \mathcal{N}(x \mid \mu, \sigma^2)}$$

$$\boxed{= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}}$$

$$\mathbb{E}[X] = \int^{\infty} x p(X=x)\, dx$$

$-\infty$

$= \mu$

first moment.

$$\boxed{E[X - E[X]]^2 = \sigma^2}$$

If $\cancel{X} \in \mathbb{R}^D$

$-\infty, \infty$

MVN → Multivariate Gaussian or Normal

1D	
μ	$\underline{\mu}$ → a vector with D values
σ^2	$\underline{\Sigma}$ → a matrix $(D \times D)$

X → a vector with D values

pdf(X)

$$= \mathcal{N}(X|\mu, \Sigma) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(X-\mu)^T \Sigma^{-1}(X-\mu)\right]$$

Σ – Σ^{-1} should exist

CSE474/574 Machine Learning

1. Gradiance Quiz - Due Tuesday
2. PA2 - released
3. Need to work in groups
4. Office hours

$P(\text{heads})$ $\quad 0 \leq P(1) \leq 1$

$P(\text{tails})$ $\quad \underline{P(\text{heads}) + P(\text{Tails}) = 1}$

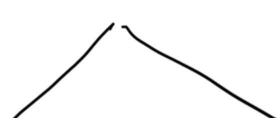
<u>Random Variable</u>

<u>Domain</u> $\quad \{\text{Heads}, \text{Tails}\}$

$\{1, 2, 3, 4, 5, 6\}$

$\{1, 2, 3\}$

$\{1, 2, 3, 4 \ldots, 12\}$



Discrete or Categorical

Continuous random variables

Domain - finite

X

x

$P(X = x) \equiv P(x)$

$\boxed{p \text{ — density}}$

Joint probability

$P(A \wedge B) \equiv P(A, B)$

$$\boxed{\begin{aligned} P(A, B) &= P(A \mid B)\, P(B) \\ &= P(B \mid A)\, P(A) \end{aligned}}$$

$$P(A \mid B) = \frac{P(A, B)}{P(B)}$$

$$P(B|A) = \frac{P(A, B)}{P(A)}$$

$$\boxed{\begin{aligned} &P(X_1 = x_1, X_2 = x_2, \ldots, X_d = x_d) \\ &= P(X_1 = x_1)\, P(X_2 = x_2 \mid X_1 = x_1) \\ &\quad \ldots \end{aligned}}$$

Let the domain of $X = \{1, 2, 3\}$

Let domain of $Y = \{a, b\}$

$Z = (X, Y)$ $\quad \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$P(X=x) = \sum P(X=x) | Y=y$

marginal dist. of

Bayes Rule or theorem

$$P(X=x \mid Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)}$$

$$P(Y=y) = \sum_{y'} P(X=x, Y=y')$$

$$\boxed{P(X=x, Y=y) = P(Y=y \mid X=x) * P(X=x)}$$

Example:

$$\boxed{P(Y=1 \mid X=1)}$$

X – test is +ve or not

Y – have cancer or not

$$\boxed{P(X=1 \mid Y=1)}$$

$$P(Y=1 \mid X=1) = \frac{\overbrace{P(X=1 \mid Y=1)}^{0.8}\ \overbrace{P(Y=1)}^{0.004}}{\underbrace{P(X=1 \mid Y=1)}_{0.8}\ \overbrace{P(Y=1)}^{0.004} + P(X=1 \mid Y=0)\ \underbrace{P(Y=0)}_{1-0.004\,=\,0.996}}$$

$$\boxed{P(X=1 \mid Y=0)} \rightarrow \text{false alarm rate}$$

0.1

X

If X is a categorical or discrete r.v

$\text{Domain}(x) = \{1, 2, 3, 4\}$

$P(X)$ $\quad$ $P(X=1)$

$P(X=2)$

$P(X=3)$

$P(X=4)$

Continuous r.v

Let $\text{Dom}(x) = (-\infty, \infty)$

$P(X=0)$

$P(X=0.3)$

Probability density. (PDF)

$p(X=x)$

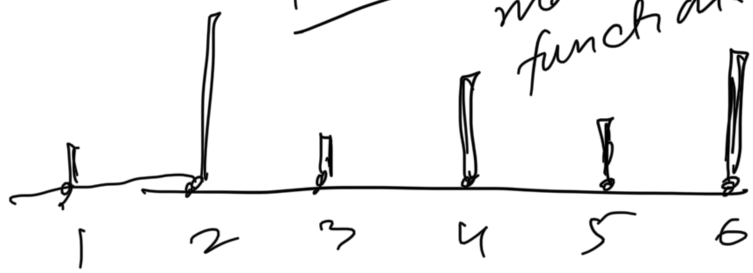
$$P(a < X \le b)$$
$$= P(X \le b) - P(X < a)$$

CDF - cumulative distribution function.

$$F(x) = P(X \le x)$$

$$p(a < X \le b) = \int_a^b p(x)\,dx$$

pmf probability mass function



$\boxed{p(X=x)}$

$\boxed{p(x)}$

$$\boxed{\int_{-\infty}^{\infty} p(x)\,dx = 1}$$

$$\mathbb{E}[X]$$

DRV: $\mathbb{E}[X] = \sum_{x \in \mathcal{X}} x P(X=x)$

$\mathcal{X}$ ← domain of X

CRV $\quad \mathbb{E}[X] = \int_{\mathcal{X}} x\, p(x)\, dx$

Also known as the mean (μ)

$$f(X) = \begin{cases} -100 & \text{if } x = \text{tails} \\ +900 & \text{if } x = \text{heads} \end{cases}$$

$p(X = h) = 0.5$

$p(X = t) = 0.5$

$$\mathbb{E}[f(X)] \sum_{x \in \mathcal{X}} f(x)\, p(X = x)$$

$$= 0.5 \times (-100) + 0.5 \times 900$$

$$= \underline{400}$$