Mapping Non-Linear SEIR Dynamics to Quantum Hilbert Spaces

Comparative Analysis of Classical Ensembles vs. QSVM for Dengue Prediction

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1. The Biophysical Framework: Non-Linear Dynamics

The Coupled SEIR-Vector System

We model the disease transmission not as a statistical trend, but as a dynamical system governed by coupled **Non-Linear Ordinary Differential Equations (ODEs)**.

Human Population Dynamics Describes the flow from Susceptible (S_h) to Infected (I_h):

$$\frac{dS_h}{dt} = -\beta_h S_h I_v$$
$$\frac{dI_h}{dt} = \sigma_h E_h - \gamma I_h$$

Vector (Mosquito) Thermodynamics Mosquito lifecycle is constrained by temperature-dependent parameters (μ_v, σ_v):

$$\frac{dE_v}{dt} = \beta_v S_v I_h - (\sigma_v(T) + \mu_v(T))E_v \tag{1}$$

Physics Insight: The system exhibits *hysteresis* (time-lag) due to the incubation period (σ^{-1}), which we capture using lag-features in the machine learning models.

2. Classical Computational Framework

Ensemble Learning as Statistical Mechanics

To establish a baseline, we utilize the **Random Forest Regressor**, a bagging ensemble method.

Mathematical Formulation

The prediction $\hat{f}(x)$ is the averaged sum of B uncorrelated decision trees, reducing variance (noise):

$$\hat{f}(x) = \frac{1}{B} \sum_{b=1}^{B} f_b(x) \tag{2}$$

- **Strength:** robust against overfitting in high-dimensional thermodynamic data (Temperature, Humidity, Precipitation).
- **Limitation:** operates in classical Euclidean space, where complex non-linear decision boundaries can be computationally expensive to map.

We propose mapping the classical feature vector $\vec{x}$ into a quantum state $|\Phi(\vec{x})\rangle$ in a complex Hilbert Space $(\mathcal{H})$.

A. Superposition & Qubits

Unlike classical bits, a Qubit exists in a linear combination of states:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

This allows parallel processing of 2^n feature states.

B. Entanglement

Variables become correlated such that the state cannot be factored:

$$|\Phi^+\rangle \neq |\psi\rangle_A \otimes |\phi\rangle_B$$

Captures the coupled nature of the SEIR equations.

The Quantum Kernel (Our Metric)

We evaluate the similarity of epidemiological states via the inner product (overlap) in Hilbert Space:

$$K(\vec{x}_i, \vec{x}_j) = |\langle\Phi(\vec{x}_i)|\Phi(\vec{x}_j)\rangle|^2 \tag{3}$$