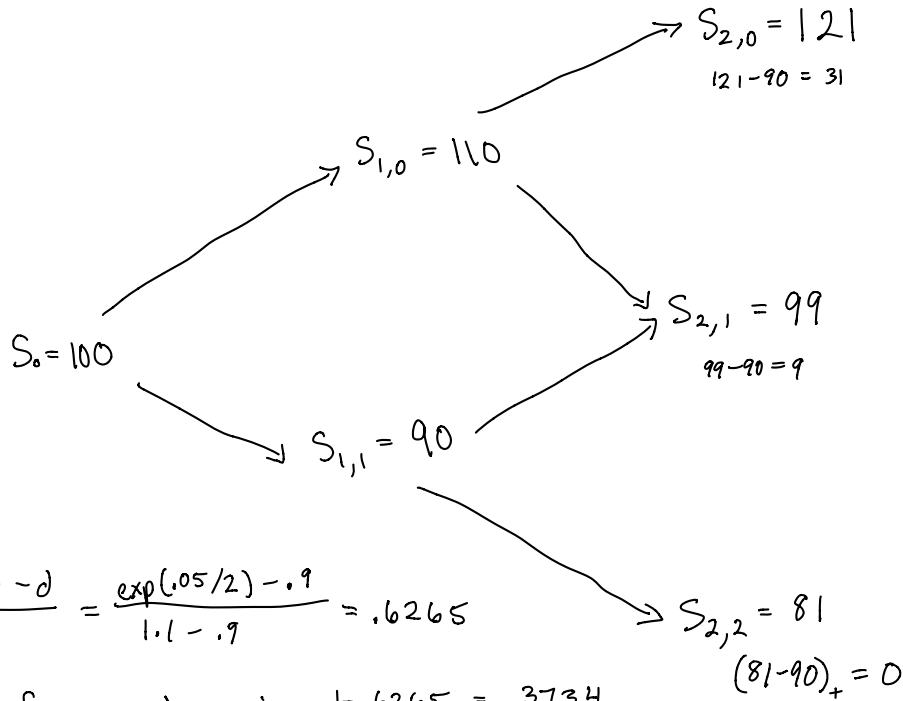


1) a)



b) $q = \frac{\exp(rT) - d}{u - d} = \frac{\exp(.05/2) - .9}{1.1 - .9} = .6265$

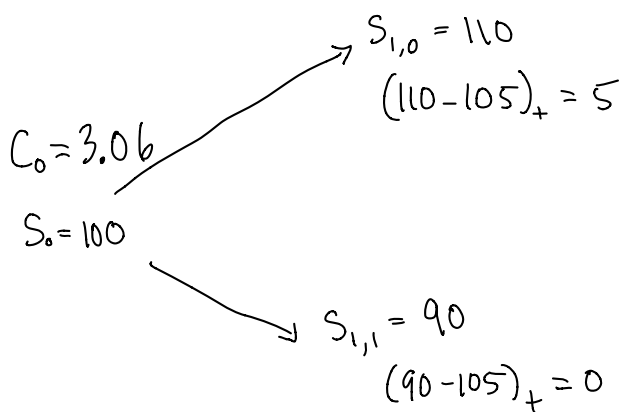
so prob of move down is $1 - .6265 = \underline{\underline{.3734}}$

c) At $S_{1,0}$, the option price is $\exp(-.05/2) \cdot (.6265 \cdot 31 + .3734 \cdot 9) = 22.22$

" " $S_{1,1}$ " $\exp(-.05/2) \cdot (.6265 \cdot 9 + 0) = 5.50$

At S_0 " $\exp(-.05/2) \cdot (.6265 \cdot 22.22 + .3734 \cdot 5.5) = \underline{\underline{\$15.58}}$

d)



$$110\Delta - 5 = 90\Delta \Rightarrow \Delta = .25$$

Action now:

- Sell one call for \$4
- Buy .25 shares for \$25
- Borrow \$21

Action in 6 months:

If $S_t = 110$

- buy .75 shares for \$82.5
- sell 1 share thru call for \$105

If $S_t = 90$

- sell .25 shares for \$22.5
- no action by call owner

$$\bullet \text{ Pay back } \exp(.05/2) \cdot 21$$

$$\bullet \text{ Pay back } \exp(.05/2) \cdot 21$$

$$\left. \begin{aligned} -82.5 + 105 - \exp(.05/2) \cdot 21 &= .97 \\ 22.5 - \exp(.05/2) \cdot 21 &= .97 \end{aligned} \right\} \text{ free lunch}$$

$$2) a) q = \frac{1+r/n - d}{u-d} = \frac{1+r/n - (1+r/n - \sigma/\sqrt{n})}{(1+r/n + \sigma/\sqrt{n}) - (1+r/n - \sigma/\sqrt{n})} = \frac{1+r/n - 1 - r/n + \sigma/\sqrt{n}}{2\sigma/\sqrt{n}} = \frac{\sigma/\sqrt{n}}{2\sigma/\sqrt{n}} = \frac{1}{2}$$

$$\begin{aligned} b) E_Q[S_{k+1}^* | S_k^*, \dots, S_0^*] &= (1+r/n)^{-(k+1)} (q S_k u + (1-q) S_k d) \\ &= (1+r/n)^{-(k+1)} (q S_k (u-d) + S_k d) \\ &= (1+r/n)^{-(k+1)} \left(\frac{1}{2} S_k (u-d) + S_k d \right) \\ &= (1+r/n)^{-k} \cdot S_k = S_k^* \end{aligned}$$

S_0, S_k^* is a martingale

c) For binomial distribution $\text{Bin}(n, q)$, this is simulating taking n trials, each with success probability q , and counting the number of successes. For our situation, each trial is a step (up or down) of the stock price, and the probability of a step up is q , and the number of steps up is X .

X follows the exact description of $\text{Bin}(n, q)$.

$$d) C_0 = (1+r/n)^{-n} E_Q[(S_n - K)_+ | S_0] \quad S_n = S_0 u^j d^{n-j} \quad Q[S_n = S_0 u^j d^{n-j}] = \binom{n}{j} q^j (1-q)^{n-j}$$

$$C_0 = (1+r/n)^{-n} \sum_{j=0}^n [(S_0 u^j d^{n-j}) - K]_+ \cdot \binom{n}{j} q^j (1-q)^{n-j}$$

$$\begin{aligned} e) S_{t_i}^{\wedge} &= S_0 u^x d^{i-x} = S_0 \left(\frac{u}{d}\right)^x d^i = S_0 \left[\frac{\exp(\mu/n + \sigma/\sqrt{n})}{\exp(\mu/n - \sigma/\sqrt{n})} \right]^x [\exp(\mu/n - \sigma/\sqrt{n})]^i = S_0 \exp\left(2 \frac{\sigma}{\sqrt{n}} x\right) \exp\left(\frac{\mu}{n} i - \frac{\sigma^2}{2n} i\right) \\ &= S_0 \exp\left(\frac{\mu}{n} i + \frac{\sigma^2}{2n} (2x - i)\right) \end{aligned}$$

Let W_j be a RV such that $W_j = 1$ if the stock price goes up and $W_j = 0$ if the stock goes down.

$$\sum_{j=1}^i W_j = X$$

$$Y_j = 2W_j - 1$$

$$2X - i = 2 \sum_{j=1}^i W_j - i = 2 \sum_{j=1}^i W_j - \sum_{j=1}^i 1 = \sum_{j=1}^i (2W_j - 1) = \sum_{j=1}^i Y_j$$

$$S_0, \hat{S}_{t_i} = S_0 \exp\left(\mu \frac{i}{n} + \frac{\sigma}{\sqrt{n}} (2 \times -i)\right) = S_0 \exp\left(\mu t_i + \frac{\sigma}{\sqrt{n}} \sum_{j=1}^i Y_j\right) \quad Y = \sum_{j=1}^i Y_j \\ = S_0 \exp\left(\mu t_i + \frac{\sigma}{\sqrt{n}} Y\right)$$

Under prob. measure Q , as $n \rightarrow \infty$ $\frac{\sigma}{\sqrt{n}} Y$ converges in distribution to $(r - \mu - \frac{\sigma^2}{2})t + \sigma W_t$ ($\mu = 0$)

$$S_0, \quad S_n = S_0 \exp(\sigma W_t + [r - \frac{\sigma^2}{2}]t) \quad \checkmark$$

f) We know that stock price is martingale, and since $r=0$, there is no interest (\$100 now is \$100 at any time, no need to discount). Because of this, to know S_1 , we can take any other stock price S_t and "discount back" with $r=0$.

$$\text{So } E_Q[S_1 | S_t] = S_t.$$