Date

GRE

Ankit Singhai...

Date

Maths.

Geometry

* Types of Angles -

1) Acute 0° to 90°

2) Right 90°

3) Obtuse 90° to 180°

4) Straight 180°

5) Reflex 180° to 360°

6) Complete 360°

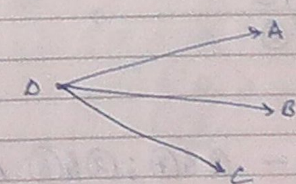
* Complementary Angles → Sums up to 90° {∠A + ∠B = 90°}

* Supplementary Angles → Sums up to 180° {∠A + ∠B = 180°}

* Adjacent Angles -

1) Share common side & a common vertex.

2) Do not share any interior points (or) one angle not contained in other angle.



[∠AOB & ∠BOC are adjacent angles.]

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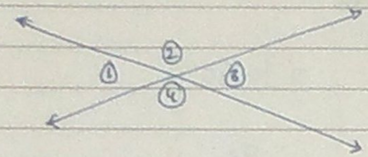
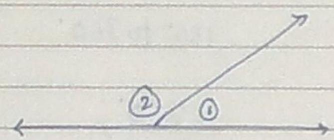
* Vertical Angles — Angles opposite to each other. They are always equal in measure.



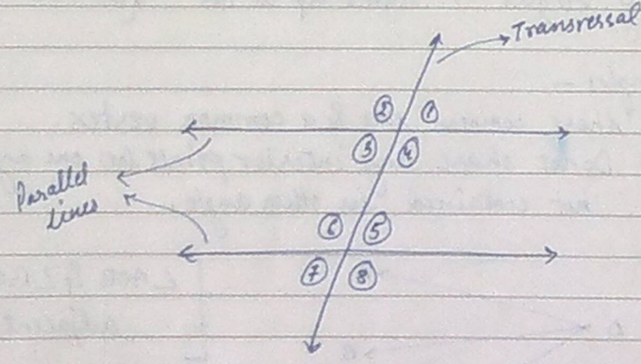
1 & 3, 2 & 4 } Vertical Angles.; Also, 1 = 3 & 2 = 4

* Linear Pairs <on a line>



1 & 2 are linear pairs.
Also, $\angle 1 + \angle 2 = 180°$

*



Alternate Exterior Angles — 1 & 7 ; 2 & 8. Also, 1 = 7 ; 2 = 8.

Alternate Interior Angles — 3 & 5 ; 4 & 6. Also, 3 = 5 ; 4 = 6

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Corresponding Angles — 2 & 6 ; 1 & 5

Also, 2 = 6 & 1 = 5.

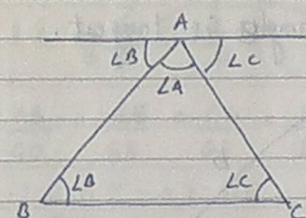
* Transversal is a line intersecting two or more line {lines may or may not be parallel}

POLYGONS :—

Sum of Interior Angles = $(n-2) \times 180°$ $\{n \geq 3\}$

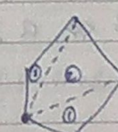
Sum of Exterior Angles = 360° → Always {when taken in one direction}

* Why sum of Interior Angles of a Δ equals 180°?



So, $\angle A + \angle B + \angle C = 180°$

* <Kisi bhi polygon me (n-2) triangles hote hain>



Pentagon → 3 triangles.

* Regular Polygon → 1) All sides equal in length.
2) All angles equal in measure.

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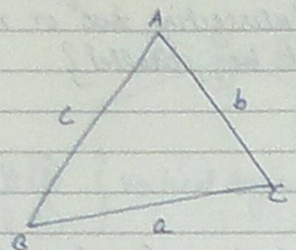
* < A circle is a polygon with infinite sides { length of each side is infinitesimally small } !

TRIANGLES —



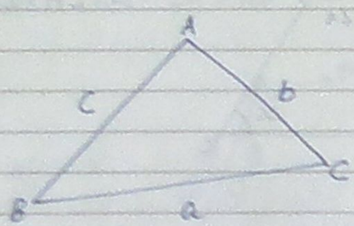
[At least two angles of a Δ must be acute.]

$a+b > c$ | $a-b \not< c$

$b+c > a$ | $b-c \not< a$

$a+c > b$ | $c-a \not< b$

* Side opposite to largest angle is longest.



$\angle A > \angle C > \angle B \Leftrightarrow a > c > b$.

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Categories of Δ's

Based on Sides

1.) Scalene (All sides diff.)
2.) Isosceles (Two sides equal)
3.) Equilateral (All sides equal)

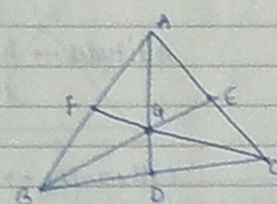
Based on Angles.

1.) Acute Angled (All angles acute)
2.) Right Angled (One Angle 90°)
3.) Obtuse Angled (One Angle obtuse)

* Points and lines in a triangle —

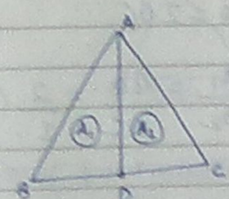
1.) Centroid & Median.

BD = DC
BF = FA
AE = EC



- Median → divides the side of Δ into halves.
- Centroid → POI of medians.
- Median also divides the area of Δ into two equal parts.

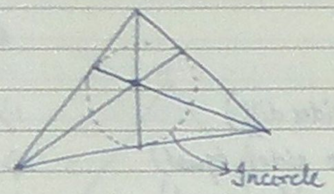
$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}.$$



$$\frac{A_1}{A_2} = \frac{BD}{DC} = \frac{1}{1}.$$

* Appolonius Theorem for Medians → $AB^2 + AC^2 = 2[AD^2 + BD^2]$

= Angle Bisectors and Incentre →

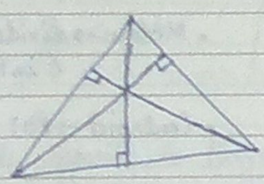


Angle Bisector → divides the angles into 2 equal parts.

Incentre → POI of AB

Radius of Incircle = r. ; $r = \frac{\text{Area of } \Delta}{s}$ $\left\{ s = \frac{a+b+c}{2} \to \text{Semiperimeter} \right\}$

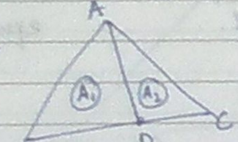
= Altitude & Orthocentre –



Altitudes → height of Δ; perpendicular dropped from an Apex to opposite side.

Orthocentre → POI of Altitudes.

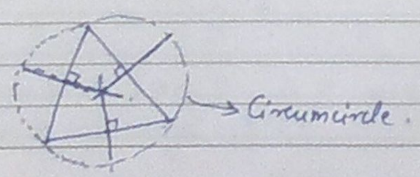
⇒



Let, BD = 2DC

$$\frac{A_1}{A_2} = \frac{BD}{DC} = \frac{2}{1}.$$

= Perpendicular Bisector & Circumcentre →



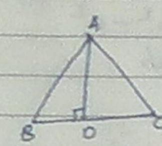
Radius of Circumcircle = R. ; $R = \frac{a \cdot b c}{4 \text{ Area of } \Delta}$

⇕

$$\text{Area of } \Delta = \frac{a \cdot bc}{4R}$$

= AREA OF TRIANGLE –

1.)

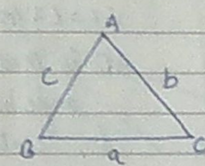


AD = h

BC = b

$$\text{Area} = \frac{1}{2} bh \quad \langle \text{half base} \times \text{height} \rangle$$

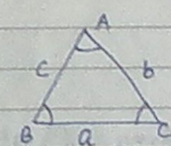
2.)



$$S = \frac{a+b+c}{2}$$

$$\text{Area of } \Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

3.)



$$\text{Area of } \Delta = \frac{1}{2} ab \sin C = \frac{1}{2} bc \cdot \sin A = \frac{1}{2} ac \cdot \sin B$$

Sine formula / Lami's Theorem ⇒ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$

where, R = radius of circumcircle.

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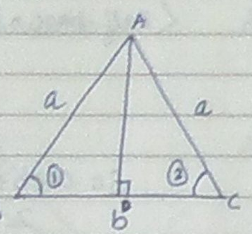
* Cosine formula — $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{b^2+a^2-c^2}{2ab}$$

ISOSCELES TRIANGLES —



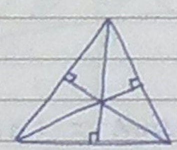
$\angle 1 = \angle 2 \Leftrightarrow AB = AC$

AD is Median, Altitude, angle bisector & $\perp$ bisector.

So, centroid, orthocentre, incentre & circumcentre ~~coincide~~ are colinear. <lie on same line>

$$\text{Area of } \Delta = \frac{b}{4}\sqrt{4a^2-b^2}$$

* Equilateral Δ —



$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

$$\text{Height} = \frac{\sqrt{3}}{2}a$$

Median, Altitude, Angles Bisectors & $\perp$ bisectors are all same, so, centroid, orthocentre, incentre & circumcentre coincide <are same>

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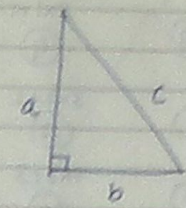
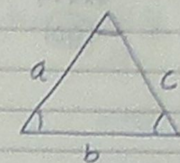
PYTHAGORUS THEOREM —

1)



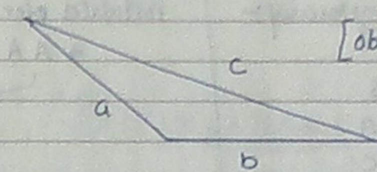
$a^2+b^2=c^2$ [Right Δ]

2)



[Acute Angled Δ] $a^2+b^2>c^2$

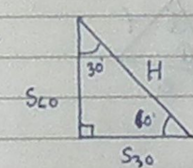
3)



[obtuse Angled Δ] $a^2+b^2<c^2$

SPECIAL Δ'S —

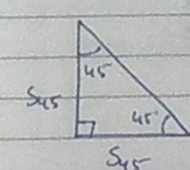
1)



[30°-60° Δ]

$S_{30} = \dfrac{H}{2}$; $S_{60} = \dfrac{\sqrt{3}}{2}H$

2)



[45°-45° Δ]

$$S_{45} = \frac{H}{\sqrt{2}}$$

other Pythogorean Triplets → 8, 15, 17 & 7, 24, 25

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Pythogorean Triplets — {Also called odd triplets}

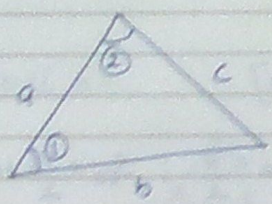
3, 4, 5	— ① first odd →	③	3×1+1 ④	4+1 ⑤
5, 12, 13	— ② second odd →	⑤	5×2+2 ⑫	12+1 ⑬
7, 24, 25	— ③ 3rd odd →	⑦	7×3+3 ㉔	24+1 ㉕
9, 40, 41	— ④ 4th odd →	⑨	9×4+4 ㊵	40+1 ㊶

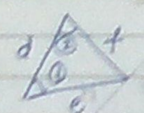
$\boxed{8, 15, 17}$

* Congruent & Similar Δ's —

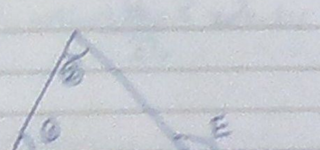
Postulates for Congruency :-
1) SSS
2) SAS
3) ASA
4) RHS

Postulates for Similar Δ's —
→ AA
→ AAA





so, By Similar Δ's → $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$



E = ① + ②

{Exterior angle is equal to sum of opposite interior angles}

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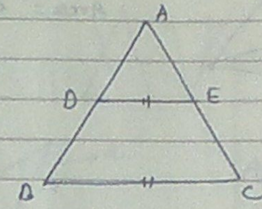
Basic Proportionality Theorem —



$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

{∵ ΔABC & ΔADE are similar}

* Sin, Cos & tan in diff. Quadrants —

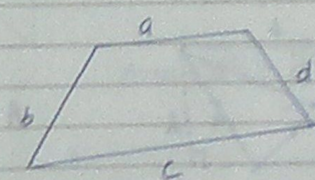
II sin⊕	I All⊕
tan⊕ III	cos⊕ IV

{Add Sugar To Coffee}

Quadrilaterals

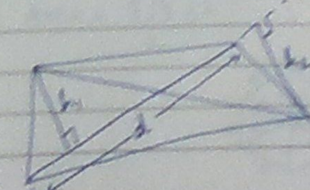
Polygon with 4 sides. → Sum of Interior Angles = 360°
Sum of Ext. Angles = 360°

1)



Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, $s = \frac{a+b+c+d}{2}$

2)



Area = $\frac{d}{2}(h_1 + h_2)$

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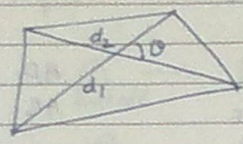
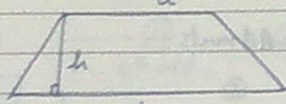
③



Area = $\dfrac{d_1 \cdot d_2 \cdot \sin\theta}{2}$

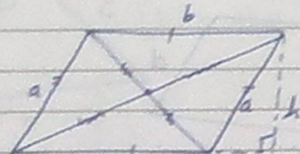
* Trapezium —



Area = $\frac{1}{2}(a+b)h$

Isosceles Trapezium → 1> Non-parallel sides have equal length.
2> length of diagonals is same
3> Diagonals intersect each other in same ratio.

* Parallelogram —



Area = bh

1> length of Diagonals is not same.
2> But, diagonals ~~do~~ bisect each other.

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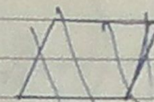
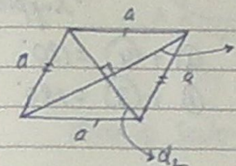
* Rhombus -

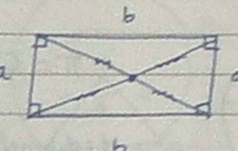


1> Diagonals ⇒ ⊥ bisectors.
2> Not of Equal length.



Area = $\dfrac{d_1 d_2}{2}$

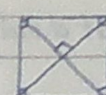
* Rectangle -



1> Diagonals bisect.
2> Equal in length (diagonals)

Area = ab ; Perimeter = 2(a+b)

* Square -

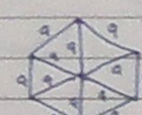


1> Diagonals ⇒ ⊥ bisectors.
2> Equal in length (diagonals)

Area = $a^2 = \dfrac{d^2}{2}$

* Regular Hexagon - 1> Polygon with 6 sides
2> All sides & angles are equal.

Area = $\dfrac{3\sqrt{3}}{2}a^2$



[6-equilateral Δ of side a formed]

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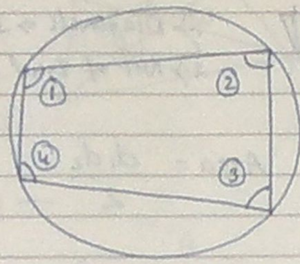
Cyclic Quadrilateral -

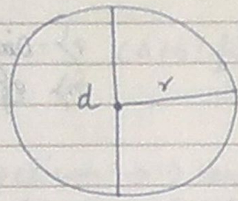


opposite angles are supplementary.

$② + ④ = 180°$

$① + ③ = 180°$

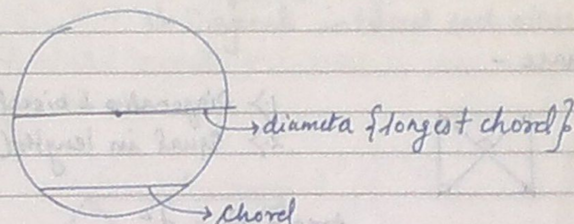
CIRCLES -

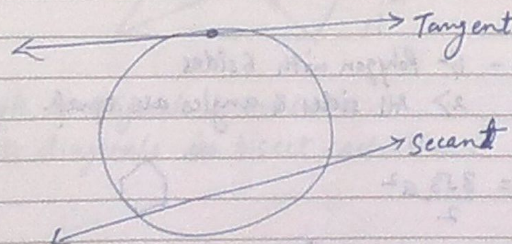


$\pi = \frac{\text{Circumference}}{\text{Diameter}} = \frac{22}{7} \simeq 3.14$

d: diameter ; r: radius

$[d = 2r]$





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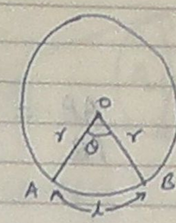


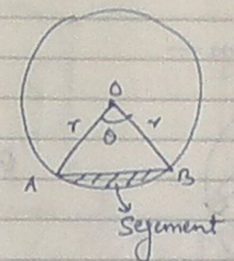
$\overset{\frown}{AB}$ = Arc AB

θ = measure(m) of arc = $m\overset{\frown}{AB}$

l = length (l) of arc = $l\overset{\frown}{AB}$

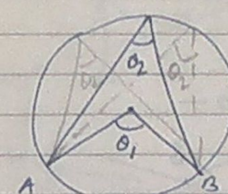
Area enclosed by $\overset{\frown}{AB}$ = Sector.

Area of Sector AB = $\frac{\theta}{360} \times \pi r^2$; $l\overset{\frown}{AB} = \frac{\theta}{360} \times 2\pi r$



Area of Segment = Area of Sector AB − Area of ΔAOB

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

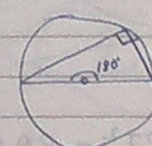


$\theta_1 = 2\theta_2$

θ_1 = Centre Angle

θ_2 = Inscribed Angle.

- Angle inscribed in same segement are equal.
- Angle inscribed in Semicircle is 90°



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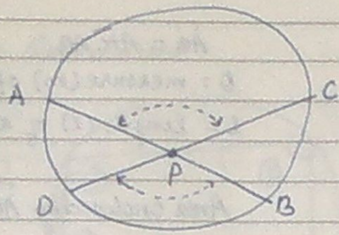
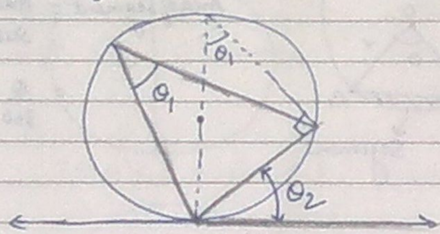


$$\frac{AP}{PB} = \frac{CP}{PD}.$$

$$\cancel{\frac{AP}{AB} = \frac{DP}{CP}}$$

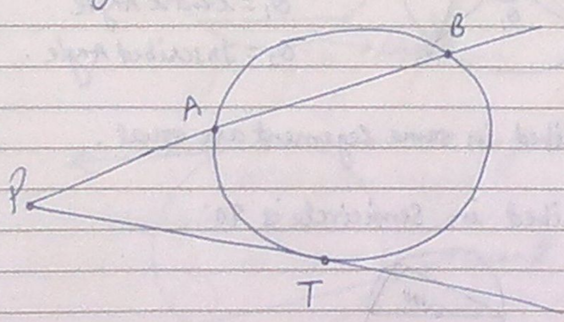
THEOREMS —

1.) Alternate Segement theorem —



$\theta_1 = \theta_2$

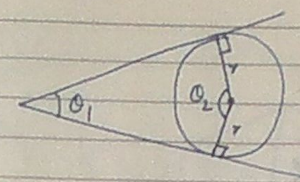
2.) Tangent Secant Theorem —



$$PT^2 = PA \times PB.$$

Date

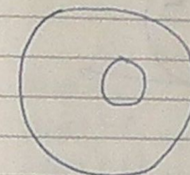
#



$\theta_2 = 2\theta_1$.

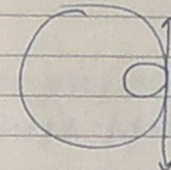
COMMON TANGENTS —

1.)



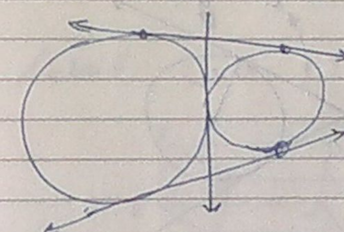
No common tangent.

2.)



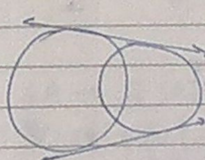
One common tangent

3.)



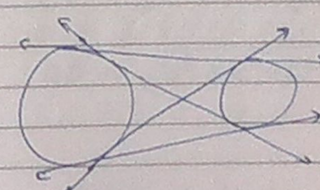
Three Common Tangents.

4.)



Two Common Tangents.

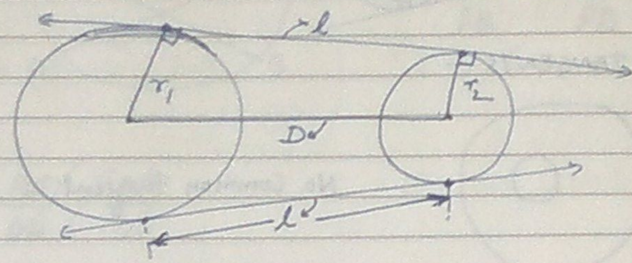
5.)



Four Common Tangents.

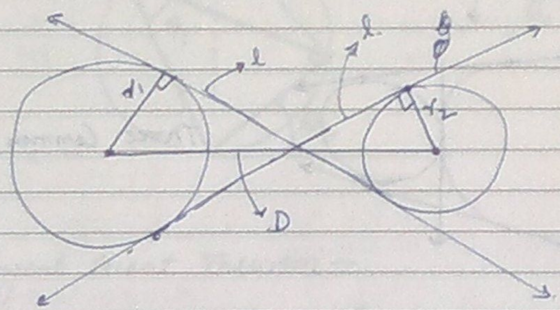
Date

DIRECT COMMON TANGENT—



$$l = \sqrt{D^2 - (r_1 - r_2)^2}$$

TRANSVERSE COMMON TANGENT—

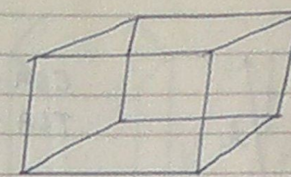


$$l = \sqrt{D^2 - (r_1 + r_2)^2}$$

Date

3D

* Cube

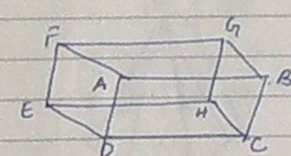


All sides equal in length.
let, length = a.

Faces – 6	SA → $6a^2$
Vertex – 8	V → a^3
Edges – 12	P → $12a$

Length of Face Diagonal (FD) = $a\sqrt{2}$
Length of Body Diagonal (BD) = $a\sqrt{3}$

* Cuboid

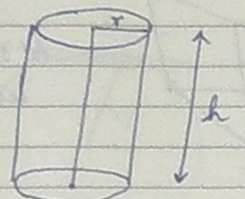


~~ABCDEFGH~~
AB = a
HC = c
BC = b

SA → $2(ab + bc + ac)$
V → $a \cdot b \cdot c$
FD → $\sqrt{a^2+b^2}$; $\sqrt{b^2+c^2}$; $\sqrt{c^2+a^2}$
BD → $\sqrt{a^2+b^2+c^2}$

Date

Cylinder {Right Circular Cylinder}



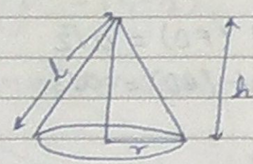
CSA $\rightarrow 2\pi r h$

TSA $\rightarrow 2\pi r h + 2\pi r^2$

$= 2\pi r(r+h)$

V $\rightarrow \pi r^2 h$.

* Cone {Right Circular Cone}



l = Slant height

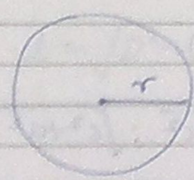
$= \sqrt{h^2+r^2}$

CSA $\rightarrow \pi r l$ $\left\{ \left[\frac{2\pi r + 0}{2}\right] \times l \right\}$

TSA $\rightarrow \pi r(l+r)$

V $\rightarrow \frac{1}{3}\pi r^2 h$

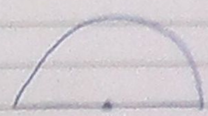
* Sphere –



CSA/TSA/SA $\rightarrow 4\pi r^2$

V $\rightarrow \frac{4}{3}\pi r^3$

* Hemisphere –



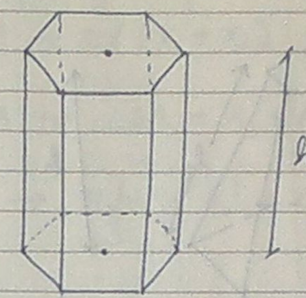
CSA $\rightarrow 2\pi r^2$

TSA $\rightarrow 3\pi r^2$

V $\rightarrow \frac{2}{3}\pi r^3$

Date

PRISM –



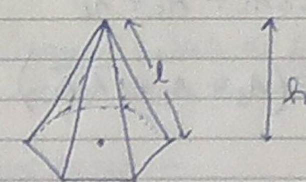
{P = Perimeter
A = Base Area.}

Lateral Surface Area (LSA) $\rightarrow P.h$

TSA $\rightarrow$ LSA + 2A

V $\rightarrow A.h$

PYRAMID –

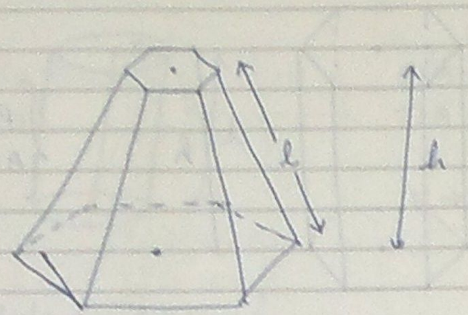


LSA $\rightarrow \frac{P.l}{2}$

TSA $\rightarrow$ LSA + A

V $\rightarrow \frac{1}{3} A.h$

* FRUSTUM OF PYRAMID —



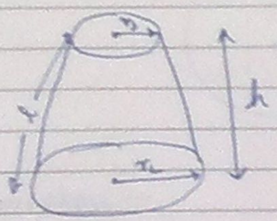
{ P_s → Perimeter of small base | A_s → Area of small base
P_L → Perimeter of large base | A_L → Area of large base. }

$$LSA \rightarrow \left(\frac{P_s + P_L}{2}\right) l$$

$$TSA \rightarrow LSA + A_s + A_L$$

$$V \rightarrow \frac{h}{3} \cdot \left(A_s + A_L + \sqrt{A_s A_L}\right)$$

* FRUSTUM OF CONE —



$$LSA \rightarrow \left(\frac{P_s + P_L}{2}\right) \cdot l$$

$$TSA \rightarrow LSA + A_s + A_L$$

$$V \rightarrow \frac{h}{3}\left(A_s + A_L + \sqrt{A_s A_L}\right)$$

formulas in Reduce forms of Frustum of Cone —

$$LSA \rightarrow \pi l (r_1 + r_2)$$

$$TSA \rightarrow LSA + A_s + A_L \rightarrow LSA + \pi r_s^2 + \pi r_L^2$$

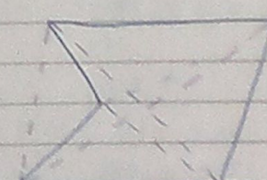
$$V \rightarrow \frac{\pi h}{3} \cdot \left(r_L^2 + r_s^2 + r_s r_L\right)$$

CONVEX & CONCAVE POLYGONS:—

1) CONVEX POLYGON:-

a) All diagonals within the figure.
b) All interior angles less than 180°

2) CONCAVE POLYGON:-
↳ (Caves in)



a) At least one diagonal lies outside the figure.
b) At least one angle is greater than 180°.

Date

No. of sides	Name of Polygon.
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon.

Date

WORD PROBLEMS

1) Profit & Loss
2) Simple Interest & Compound Interest
3) Mixtures
4) Work & Time
5) Time, Speed & Distance.

PROFIT & LOSS –

* $SP = CP + P$ | $SP = CP - L$
$CP = SP - P$ | $CP = SP + L$
$P = SP - CP$ | $L = CP - SP$

* $P\% = \frac{P}{CP} \times 100$; $L\% = \frac{L}{CP} \times 100$

* $SP = CP\left[1 \pm \frac{x}{100}\right]$ { +ve for profit, −ve for loss }

* Concept of same Selling Price –

	S_1	S_2
SP →	200	200
P/L →	10% ↑	10% ↓
CP →	$\frac{200}{1.1} = 181.81$	$\frac{200}{0.9} = 222.22$

Total CP = 404.04
Total SP = 400
} ⇒ Always a loss is incurred

$\left[\text{Loss}\% = \left(\frac{\text{Common Loss \& Gain}\%}{10}\right)^2 = \frac{x^2}{100}\right]$

Date

= Concept of false weighing :-

Assume SP = CP & profit gained by using false weights only

$$\text{Gain\%} = \frac{\text{Error}}{\text{True Value} - \text{Error}} \times 100$$

= Marked | Tagged | Listed | Asking Price :- <MP>

$SP = MP - \text{Discount} = MP - x\% \text{ of } MP$.

$$SP = MP\left[1 - \frac{x}{100}\right]$$

= Successive Discounts :-

* first x% discount & than y% discount.

$$SP = MP\left(1 - \frac{x}{100}\right)\left(1 - \frac{y}{100}\right)$$

Net Discount after successive discounts of x% & y% -

$$\Rightarrow x + y - \frac{xy}{100} \quad \left\{ 100 - \left[100\left(1 - \frac{x}{100}\right)\left(1 - \frac{y}{100}\right)\right]\right\}$$

* SIMPLE INTEREST & COMPOUND INTEREST :-

* Simple Interest —

$$SI = \frac{PRT}{100}$$

Date

$$\text{Amount (A)} = P + SI = P\left(1 + \frac{RT}{100}\right)$$

* Compound Interest :-

1.) Compounded Anually -

$$\text{Amount (A)} = P\left(1 + \frac{r}{100}\right)^T$$

$$CI = A - P$$

2.) Compounded half yearly -

$$A = P\left(1 + \frac{r/2}{100}\right)^{2T}$$

3.) Compounded Quaterly -

$$A = P\left(1 + \frac{r/4}{100}\right)^{4T}$$

4.) Compounded Monthly -

$$A = P\left(1 + \frac{r/12}{100}\right)^{12T}$$

* Difference in CI & SI for different years on same Principal -

1.) One Year $\Rightarrow$ $CI_1 - SI_1 = 0$

2.) Two Years $\Rightarrow$ $CI_2 - SI_2 = P\left(\frac{R}{100}\right)^2$

Date

3.) Three Years

$$CI_3 - SI_3 = \frac{PR^2(300+R)}{100^3}$$

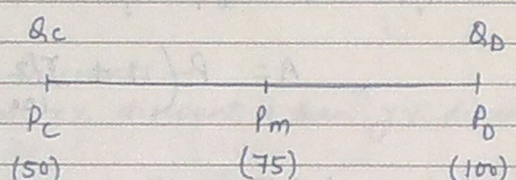
4.) Four Years

$$CI_4 - SI_4 = P\left[6\left(\frac{R}{100}\right)^2 + 4\left(\frac{R}{100}\right)^3 + \left(\frac{R}{100}\right)^4\right]$$

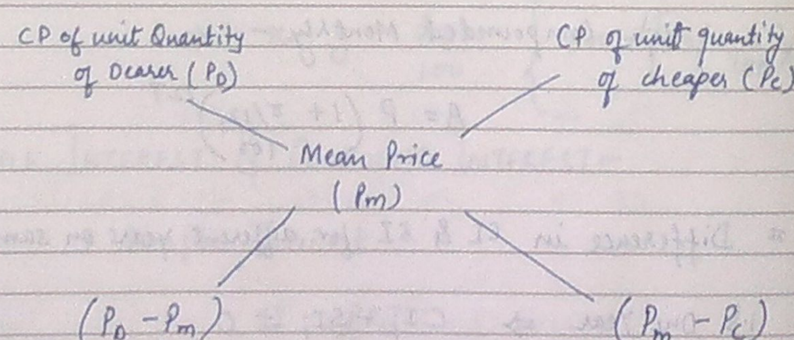
* MIXTURES :-

* Price of mixtures always lies between cheaper & dearer.

* Mixture's Price is never less than cheaper & never more than dearer.

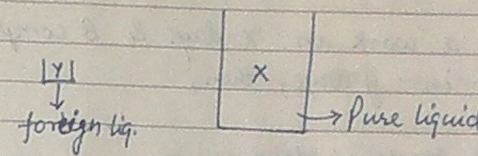


$$\frac{Q_C}{Q_D} = \frac{P_D - P_m}{P_m - P_C}$$



Date

* Mixtures of liquids :-



Suppose a container contains X units of liquid from which Y units are taken out & replaced by another liquid. After n-operations, the quantity of pure liquid can be given as-

Quantity of Pure liquid (Q_F) after n-operations $= Q_I\ X\left(1 - \frac{Y}{X}\right)^n$

Q_F = final Quantity of liquid

Q_I = Initial Quantity of liquid

Also, $\frac{Q_F}{Q_I} = \left(\frac{x-y}{x}\right)^n$ { Q_I may be x or other }

Also, If replacement is first done a-times by y lt vessel, than b-times by w lt vessel & c-times by z lt vessel,

$$\frac{Q_F}{Q_I} = \left(\frac{x-y}{x}\right)^a\left(\frac{x-w}{x}\right)^b\left(\frac{x-z}{x}\right)^c$$

* WORK & TIME :-

Rate of doing Work $= \frac{\text{Work Done}}{\text{Time}} \Rightarrow R = \frac{W}{T}$

Date

When $w=1 \Rightarrow R = \frac{1}{T}$ & $T = \frac{1}{R}$.

* A completes a work in x-days & B completes a work in y-days, then,

1.) A & B together done in –

$$\text{Work in one day} = \frac{1}{x} + \frac{1}{y}$$

$$= \frac{x+y}{xy}$$

$$\text{So, Total Days} = \frac{xy}{x+y}.$$

2.) A & B together ~~simultaneously~~ alternately – [Let; A → 8 days, B → 12 days]

a.) A starts the work -

$$\text{Work done @ end of first day} = \frac{1}{8} \rightarrow A$$

$$\text{Work done @ end of second day} = \frac{1}{8} + \frac{1}{12} \rightarrow B$$

$$= \frac{5}{24}.$$

So, work done in two days $= \frac{5}{24}$

Now,	2 days	4 days	6 days	8 days	10 days.
	$\frac{5}{24}$	$\frac{10}{24}$	$\frac{15}{24}$	$\frac{20}{24}$	$\frac{25}{24} \rightarrow >1$

Date

So, work is completed after 8 days but before 10-days.

@ 8 days → $W = \frac{20}{24}$.

@ 9th day A will do the work → ~~$W = \frac{20}{24} + \frac{1}{8}$~~

$$\text{Work left to be done} = 1 - \frac{20}{24}$$

$$= \frac{4}{24} = \frac{1}{6}.$$

On 9th day A will do $\frac{1}{8}$ work.

$$\text{So, work left for 10th day} = \frac{1}{6} - \frac{1}{8} = \frac{1}{24}.$$

@ 10th day B will come & will do the remaining $1/24$ work.

$$\text{Time reqd. by B to do } \frac{1}{24} \text{ work} = \frac{1/24}{1/12} = 0.5 \text{ days.}$$

So, work will be finished in 9.5 days if A starts the work & A & B work on alternate days.

b.) B starts the work – Again work done in 8-days $= \frac{20}{24}$

$$\text{Work left to be done} = \frac{1}{6}$$

@ 9th day B comes and does $1/12$ work, so, work left to be done by A on 10th day $= \frac{1}{6} - \frac{1}{12} = \frac{1}{12}$.

Time taken by A to do $1/12$ work $= \frac{1/12}{1/8} = 0.67$ days. So, total time reqd $= 9.67$ days.

Date

* WORK EQUIVALENCE CONCEPT –

$A \rightarrow x \uparrow$; $B \rightarrow y \uparrow$; $C \rightarrow z \downarrow$

Rate of filling $= \frac{1}{x} + \frac{1}{y} - \frac{1}{z}$

Rate of Emptying $= \frac{1}{z} - \left[\frac{1}{x} + \frac{1}{y}\right]$

* TIME, SPEED & DISTANCE :-

$S = \frac{D}{T}$; $T = \frac{D}{S}$; $D = S \times T$.

for $T =$ constant $\rightarrow D \propto S \Rightarrow \frac{D_1}{S_1} = \frac{D_2}{S_2} = K$

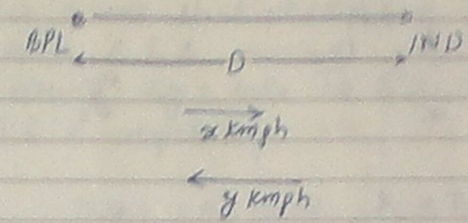
for $S =$ constant $\rightarrow D \propto T \Rightarrow \frac{D_1}{T_1} = \frac{D_2}{T_2} = K$

for $D =$ Constant $\rightarrow S \propto \frac{1}{T} \Rightarrow S_1 T_1 = S_2 T_2 = K$.

* 1 km/hr $= \frac{5}{18}$ m/s ; 1 m/s $= \frac{18}{5}$ km/hr

* Average Speed $= \frac{\text{Total Distance travelled}}{\text{Total time taken}}$

Date



Avg. Speed $= \frac{d + d}{\frac{d}{x} + \frac{d}{y}} = \frac{2xy}{x+y}$.

* RELATIVE SPEED :-

1) In same direction – $\xrightarrow{u}$ $\xrightarrow{v}$

Relative Speed $= |u - v|$

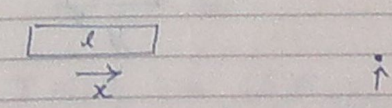
2) In opposite direction –

a) $\xrightarrow{u}$ $\xleftarrow{v}$

b) $\xleftarrow{u}$ $\xrightarrow{v}$

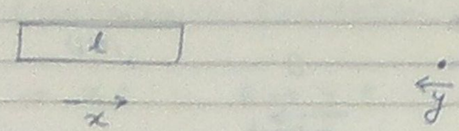
Relative speed $= u + v$.

* TRAINS :- {6 cases possible}

I) 

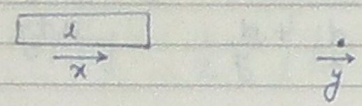
$T = \frac{l}{x}$

II)



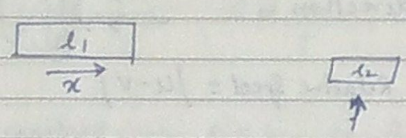
$$T = \frac{l}{x+y}$$

III)



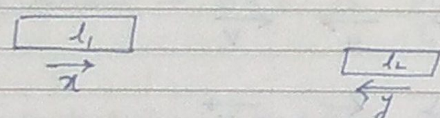
$$T = \frac{l}{x-y}$$

IV)



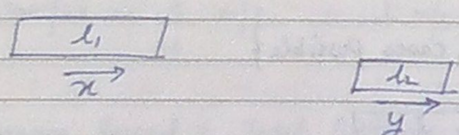
$$T = \frac{l_1 + l_2}{x}$$

V)



$$T = \frac{l_1 + l_2}{x+y}$$

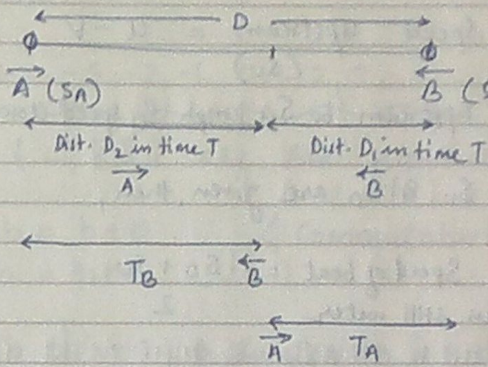
VI)



$$T = \frac{l_1 + l_2}{x+y}$$

4. If two trains start at the same time from two points A & B towards each other and after crossing they take T_a & T_b hours in reaching B & A respectively.

Then, A's speed : B's speed $= \sqrt{T_b} : \sqrt{T_a}$



$$T = \frac{D_1}{S_B} = \frac{D_2}{S_A}$$

$$T_A = \frac{D - D_2}{S_A} = \frac{D_1}{S_A}$$

$$T_B = \frac{D - D_1}{S_B} = \frac{D_2}{S_B}.$$

So, $\frac{T_B}{T_A} = \left(\frac{S_A}{S_B}\right)^2 \Rightarrow \frac{S_A}{S_B} = \sqrt{\frac{T_b}{T_a}}$

$\therefore$, $T \propto \frac{1}{S^2} \Rightarrow S \propto \frac{1}{\sqrt{T}}$

Date

* **BOATS & STREAMS :-**

Let, speed of boat = u kmph
Speed of stream = v kmph

So, Speed downstream $= u + v$
(S_D)

Speed upstream $= u - v$
(S_U)

~~If speed upstream be Sa kmph & speed downstream~~

If S_U & S_D are given, then,

Speed of boat in still water $= \frac{S_D + S_U}{2}$

Speed of stream $= \frac{S_D - S_U}{2}$

Date

Arithematic & Fractions -

* Real number is any number that can be represented on number line.

→ All number are either positive or negative except zero.

→ Integers :- ..., -3, -2, -1, 0, 1, 2, 3, ...

→ Properties {a, b & c are real numbers}.

1) $a + b = b + a$ < Commutative Property >
$a \times b = b \times a$

2) * $a - b \neq b - a$ ⇒ Substraction is not commutative.

Only, If $a = b$, then $a - b = b - a$

Also, $a \div b \neq b \div a$

2) Associative Properties -

$(a+b) + c = a + (b+c)$

$(a \times b) \times c = a \times (b \times c)$

* $(a-b) - c \neq a - (b-c)$
& $(a \div b) \div c \neq a \div (b \div c)$

3) Distributive Property -
$a(b+c) = ab + ac$
$a(b-c) = ab - ac$

Date

4) $1 \times a = a$

$a \div 1 = a$

$a \times 0 = 0$

$a + 0 = a$

$a \div a = 1 \quad \{a \neq 0\}$

$a + (-b) = a - b$

$a - (-b) = a + b$

* Adding Positive Number → Move right along the number line.
* Subtracting Positive Number → Move left along no. line.
* Multiplying & Dividing Signed Numbers :-

+ + = +

+ − = −

− + = −

− − = −

→ 2 like signs produce a positive number.

→ 2 unlike sign produce a negative number.

Date

COMBINATION OPERATIONS –

→ Order of Operation = PEMDAS | BEDMAS

P		B
E		E
MD	(left to right)	DM
AS		A

< Evaluate expression from left to right.

→ Absolute value of x :-

$|x|$

* It is the numbers distance from zero on the number line.

* Absolute value of a number is the simply the positive version of that number. Absolute value is always positive.

$|-4| = 4 \quad ; \quad |5| = 5$

DECIMALS :-

$743 = 7 \times 100 + 4 \times 10 + 3 \times 1$

1's

10's

100's

$82.743 = 8\times10 + 2\times1 + 7\times\frac{1}{10} + 4\times\frac{1}{100} + 3\times\frac{1}{1000}$

* Rounding Decimals –

If next digit is 0, 1, 2, 3 or 4 → round down.

If next digit is 5, 6, 7, 8, or 9 → round up.

Eg:–

1) 7.38241 rounded to nearest tenth

7.38241 = 7.4 (Round Up)

2) 15.02318 rounded to nearest thousandth –

15.02318 = 15.023 (Round Down)

FRACTIONS:–

$\frac{3}{4}$ 3 → Numerator, 4 → Denominator.

* Equivalent fraction –

$\frac{1}{2} = \frac{5}{10} = \frac{15}{30} = \ldots$; $\frac{7}{9} = \frac{14}{18} = \frac{21}{27}$ ...

To create equivalent fraction – multiply or divide the numerator or denominator by same number.

* Lowest Number:–

$\frac{2}{3} = \frac{10}{15} = \frac{22}{33} = \frac{14}{21} = \frac{90}{135}$

$\frac{2}{3}$ ↓ Lowest term.

* Converting entire fraction/improper fraction to mixed number:–

[* Num > Dino] $\frac{7}{2} = 3\frac{1}{2} \rightarrow \frac{2\times3+1}{2}$

$3\frac{1}{2}$ ↓ mixed no.

* Fractions to Decimal Convertion –

* $\frac{1}{4} = 0.25$ (Treat fraction as division)
* $\frac{5}{6} = 0.8\overline{3}$

☆ $\frac{1}{2} = 0.5$	$1/6 = 0.166$	$1/10 = 0.1$
$1/3 = 0.33$	$1/7 = 0.1428$	$1/11 = 0.0909$
$1/4 = 0.25$	$1/8 = 0.125$	
$1/5 = 0.20$	$1/9 = 0.11$	

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* Decimal to fraction conversation –

$0.35 = \frac{35}{100} = \frac{7}{20}$.

$0.004 = \frac{4}{1000} = \frac{1}{250}$

* Properties of fraction –

$n = \frac{n}{1}$; $\frac{n}{0}$ = Not defined

$\frac{n}{n} = 1$ $(n \neq 0)$; $\frac{1}{a/b} = \frac{b}{a}$ $(a \neq 0, b \neq 0)$

$\frac{a}{b} \times \frac{b}{a} = 1$; $\frac{a}{b} \div \left(\frac{a}{b}\right) = 1$ $(a \neq 0, b \neq 0)$

* Bigger Numirator ⇒ Bigger value.

$\frac{3}{11}$ & $\frac{4}{11}$ ⇒ $\frac{4}{11} > \frac{3}{11}$.

↳ Smaller Numirator ⇒ Smaller Number/Value.

* Bigger Dinominator ⇒ Smaller Value.

$\frac{2}{3}$ & $\frac{2}{4}$ ⇒ $\frac{2}{4} < \frac{2}{3}$

↳ Smaller Dinominator ⇒ Bigger Value.

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* Increase Numirator & Dinominator by same amount ⟶ fraction approaches to 1.

$\frac{2}{7} \xrightarrow{+10} \frac{12}{17} \xrightarrow{+1000} \frac{1012}{1017}$

closer to 1   more closer to 1.

PROPERTIES OF FRACTIONS –

→ $\frac{abc}{def} = \frac{a}{d} \times \frac{b}{e} \times \frac{c}{f}$

→ $\frac{a+b+c}{d+e+f} \neq \frac{a}{d} + \frac{b}{e} + \frac{c}{f}$.

→ $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

→ $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$

→ $\frac{a+b}{c+d} = \frac{a}{c+d} + \frac{b}{c+d}$

→ $\frac{a-b}{c-d} = \frac{a}{c-d} - \frac{b}{c-d}$

→ If $\frac{a}{b} = \frac{c}{d}$ ⇒ $ad = cb$

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PERCENTAGE & RATIOS :-

+ Percent = per 100

↳ $19\% = \frac{19}{100} = 0.19$

↳ $0.43\% = \frac{0.43}{100} = 0.0043$

↳ $300\% = \frac{300}{100} = 3$

→ Decimal to Percentage ⇒ Move decimal to two places to right.

+ 0.00007 = 0.007% ; + 0.456 = 45.6%

+ 3.5 = 350%

→ Percentage to Decimal ⇒ move decimal to two places to left.

9.63% = 0.0963 ; 125% = 1.25

→ Fraction to Percentage ⇒ Fraction to Decimal to Percentage.

$\frac{3}{8} = 0.375 = 37.5\%$

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Fraction	Decimal	Percentage.
1/2	0.5	50%
1/3	0.3333	33.33%
1/4	0.25	25%
1/5	0.20	20%
1/6	0.1666	16.66%
1/7	0.1428	14.28%
1/8	0.125	12.5%
1/9	0.1111	11.11%
1/10	0.10	10%
1/11	0.0909	9.09%

+ $\frac{3}{8} = 3 \times \frac{1}{8} = 3 \times 0.125 = 0.375 = 37.5\%$

+ $\frac{2}{9} = 2 \times \frac{1}{9} = 2 \times 0.1111 = 22.22\%$

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* $\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}$

* p% of a number 'x = y $\Leftrightarrow \frac{P}{100} \cdot x = y$.

→ Percentage Increase & Decrease –

$$\% \text{ change} = \frac{\text{change}}{\text{original value}} \times 100$$

→ Increase or Decrease :-

If price is increased or decreased by some %age.

$$\text{New Price} = \left(1 \pm \frac{\text{Percent change}}{100}\right) \times \text{original Price}.$$

(+) → increase ; (−) → decrease.

→ Simple Interest & Compound Interest :-

1) $SI = \frac{PRT}{100}$; Amount (A) = P + SI.

2) $A = P\left(1+\frac{r}{100}\right)^n$; CI = A − P.

(or)

$$A = P\left(1 + \frac{r}{100c}\right)^{nc}$$

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P = Principal

r = rate in %age.

C = No. of times the interest is compounded per year (or) no of compounding per year.

n = No. of years.

↳ Compounded half yearly → 2 compounding per year.

$$A = P\left(1 + \frac{r}{200}\right)^{2n}$$

↳ Quaterly → $A = P\left(1 + \frac{r}{400}\right)^{4n}$

↳ Monthly → 12 compounding per year.

$$A = P\left(1 + \frac{r}{1200}\right)^{12n}$$

RATIOS :-

* Proportioning of Ratios :-

15 cookies total ; K : A = 2 : 1 ⇝ 2 + 1 = 3

$K = \frac{2}{3} \times 15$ & $A = \frac{1}{3} \times 15$

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* Combining Ratios –

Boxers : Poodles — Boxers : Terriers.

2 : 5 — 3 : 4

B : P — B : T.

⇓

P : B — B : T

5 : 2 — 3 : 4

(×3) — (×2)

15 : 6 — 6 : 8

⇓

P : B : T

15 : 6 : 8

Power & Roots :–

* Exponents & Bases

$$2^{5}$$

5 → Exponent

Base ← 2

* $1^x = 1$; $0^2 = 0$ $(x \neq 0)$ / 0^0 = not defined)

* $x^0 = 1$; $x^1 = x$

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Even & Odd Exponents :–

→ Negative no. raised to even power results in a positive outcome

$$(-9)^2 = 81$$

→ Negative no. raised to odd power results in a negative outcome.

$$(-3)^3 = -27$$

→ An odd exponent preserves sign of base –

$(-2)^3 = -8$; $(10^5) = 100000$

→ An even exponent always results in +ve no. –

$(-2)^4 = 16$; $10^3 = 1000$

** $\begin{cases} x^3 = 8 \Rightarrow x = 2 \\ x^2 = 4 \Rightarrow x = 2 \text{ or } x = -2 \end{cases}$

* Powers to memorize :–

$2^1 = 2$	$3^1 = 3$	$4^1 = 4$	$5^1 = 5$
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$
$2^5 = 32$			
$2^6 = 64$			
$2^7 = 128$			

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* Exponential Growth →

Positive Bases

base > 1

0 < base < 1

If $x > 1$, then value of x^n increases as value of n ↑s

{Moves away from 1}

If $0 < x < 1$; the value of x^n ↓'s as the value of n ↑'s

{Moves closer to 1}

* Similar pattern is observed in negative bases but sign oscillates from ⊕ & ⊖

even power — odd power

EXPONENT LAW:-

1.) Product Law -

$$x^a \cdot x^b = x^{a+b}$$ {requires equal bases}

2.) Quotient Law -

$$x^a \div x^b = \frac{x^a}{x^b} = x^{a-b}$$ (requires same/equal bases)

3.) Power of power Law:-

$$(x^a)^b = x^{a.b}$$

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4.) $x^{-n} = \frac{1}{x^n}$

5.) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

6.) Power of Product Law:-

$$(x^a \cdot y^b)^n = x^{an} \cdot y^{bn}$$

7.) Power of Quotient Law:-

$$\left(\frac{x^a}{y^b}\right)^n = \frac{x^{an}}{y^{bn}}$$

8.) Combining Base law:-

$$x^n y^n = (xy)^n \; ; \; \frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$$

THE UNIT DIGIT QUESTIONS:-

1.) Look for repeating pattern.

2.) Figure out where that pattern will be at desired power.

* The unit digit of any product will be influenced only by unit digits of two factors.

* Square Roots and Squares:-

$\sqrt{2} = 1.414 \simeq 1.4$

$\sqrt{3} = 1.732 \simeq 1.7$

$\sqrt{5} = 2.236 \simeq 2.2$

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$16^2 = 256$
$7^2 = 49$	$17^2 = 289$
$8^2 = 64$	$18^2 = 324$
$9^2 = 81$	$19^2 = 361$
$10^2 = 100$	$20^2 = 400$

** If $x > 1$, then $\sqrt{x} < x$

If $0 < x < 1$, then $\sqrt{x} > x$

→ ODD ROOTS :-

1) We can find the odd roots of a negative no.

2) The odd root of negative no. will be negative.

3) The odd root of positive no. will be positive.

→ EVEN ROOTS :-

1) We cannot find even roots of a negative no.

2) Even root of +ve no. will be positive.

⟨All roots have atmost 1 value.⟩

* $\sqrt[n]{x} = x^{1/n}$

PROPERTIES OF ROOTS :-

1) $\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{xy} \iff \sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$

2) $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}} \iff \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

3) $x^{a/b} = \sqrt[b]{x^a} \iff x^{a/b} = (\sqrt[b]{x})^a$

4) $a^x = a^y \Rightarrow x = y$ ⟨$a \neq 0$ & $a \neq 1$⟩

Rationalization :- ⟨Eliminating roots from denominator⟩

$$\frac{5}{3+\sqrt{2}} = \frac{5(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{15-5\sqrt{2}}{7}$$

* $(3-\sqrt{2})$ is conjugate of $(3+\sqrt{2})$

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ALGEBRA EQUATIONS & INEQUALITIES:-

1.) $(a+b)^2 = a^2 + 2ab + b^2$

2.) $(a-b)^2 = a^2 - 2ab + b^2$

3.) $a^2 - b^2 = (a+b)(a-b)$

4.) $(x+a)(x+b) = x^2 + (a+b)x + ab$.

* $\underbrace{x^2 + nx + p}_{} = (x+a)(x+b)$

$n \to (a+b)$, $p \to ab$

* {Solutions of equation = Roots of equation}

5.) Quadratic Equation –

$ax^2 + bx + c = 0$

$\Rightarrow x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

* (i) $b^2 - 4ac = 0$ {One Unique Soln / both roots same}

(ii) $b^2 - 4ac > 0$ {two soln / roots}

(iii) $b^2 - 4ac < 0$ {No soln / roots exist}

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NUMBERS of SOLUTIONS OF SYSTEM OF Linear Equations:-

A system of 2 eqn with 2 variables can have zero solutions, one solution or infinately many solutions.

1.) Infinately many solutions –

$3x + 2y = 9$
$6x + 4y = 18$

$\Rightarrow$ $6x + 4y = 18$
$(-)$ $6x + 4y = 18$

$0 \cdot x + 0 \cdot x = 0$

< Infinate no. of solution >

2. Zero Solution:-

$3x + 2y = 9$
$6x + 4y = 11$

$\Rightarrow$ $6x + 4y = 18$
$(-)$ $6x + 4y = 11$

$0 \cdot x + 0 \cdot y = 7$

< Zero Solution >

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(3.) One Solution –

$$3x + y = 17$$
$$2x - 2y = 6$$

$\Rightarrow$ $6x + 2y = 34$
(+) $2x - 2y = 6$

$8x = 40$
$x = 5$

$\rightarrow$ $2x - 2y = 6 \Rightarrow 2 \times 5 - 2y = 6$

$\Rightarrow y = 2.$

* EQUATIONS WITH SQUARE ROOTS :–

* $\sqrt{x-2} = 3 \Rightarrow x - 2 = 9 \Rightarrow x = 11$

* $\sqrt{3x-5} = \sqrt{x+10} \Rightarrow 3x - 5 = x + 10 \Rightarrow x = 7.5$

* EXTRANEOUS ROOTS :–

$\sqrt{x+1} = -2 \Rightarrow x + 1 = 4$
$x = 3.$

Now, put $x=3$ in eqn –

$\sqrt{3+1} = -2 \Rightarrow 2 \neq -2$

So, $x=3$ is an extraneous roots.

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* Extraneous roots can arise when we square numbers an in above case.

$(\sqrt{K})^2 = K$ for $K \geq 0$.

As $(\sqrt{9})^2 = 9$

but, $(\sqrt{-9})^2 \neq -9$ {as $K \not> 0$}

$\rightarrow$ $\sqrt{5-x} = \sqrt{2x-13}$

$5 - x = 2x - 13 \Rightarrow x = 6$ $\rightarrow$ Now check for extraneous roots.

$\sqrt{5-x} = \sqrt{2x-13} \Rightarrow \sqrt{-1} = \sqrt{-1}$
$\hookrightarrow$ This has no real value.

So, $x=6$ is not a solution of the eqn & thus, given pair of eqn have no-solution

$\rightarrow$ $\sqrt{x+14} = x+2 \Rightarrow x + 14 = (x+2)^2$

$x = -5$; $x = 2$.

Now, check for extraneous roots.

$x = -5 \rightarrow \sqrt{9} = -3 \rightarrow$ Not possible.

$x = 2 \rightarrow \sqrt{16} = 4$. So, $x=-5$ is an extraneous root.
$\therefore x=2$ is the only solution of equation

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* EQUATION WITH SQUARE ROOTS :-

1) Eliminate square root by squaring both sides.
2) Solve for the variable
3) Check for extraneous roots.

* Equation with n^{th} root :-

1) Raise both sides by power n.
2) Solve for the variable.
3) If n is even - check for extraneous roots.

EQUATIONS WITH ABSOLUTE VALUE :-

$|x| = x$ if $x \geq 0 \Rightarrow |4| = 4$

$|x| = -x$ if $x < 0 \Rightarrow |-4| = -(-4) = 4$

In general - $|x| = a \Rightarrow \begin{cases} x = a ; & x \geq 0 \\ x = -a ; & x < 0 \end{cases}$

↳ $|x| = 4 \rightarrow x = 4$ or $x = -4$.

↳ $|x| = 3x-4 \Rightarrow x = 3x-4 \Rightarrow x = 2$
$x = -(3x-4) \rightarrow x = -1$

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* when solving equations with Absolute Values, always check for extraneous roots.

So, checking $|x| = 3x-4$ for extraneous roots -

$x = 2 \rightarrow |2| = 3\times2-4 = 2$ (✓)

$x = +1 \rightarrow |+1| = 3\times1-4 = -1$ (×)

So, $x = 1$ is an extraneous root & thus, $x = 2$ is the only solution

∴, for Eqns with Absolute value -
1) Apply rule $|x| = a \Rightarrow \begin{cases} x = a \rightarrow x \geq 0 \\ x = -a \rightarrow x < 0 \end{cases}$
2) Solve eqn for results.
3) Check for extraneous roots.

INEQUALITIES :-

1.) Adding or Substracting ~~to~~ ~~or~~ from both sides does not affect the inequality.

2.) Multiplying & dividing both sides by positive number doesnot affect the inequality.

3.) Multiplying & dividing both sides by negative number reverses the inequality.

* Inequalities can be added.
* Donot ever substract, multiply or divide ~~an~~ inequalities

$$\begin{array}{r} A < B \\ + \; C < D \\ \hline A+B < C+D. \end{array}$$

Inequalities & Absolute value :-

1.) $|x| < 3 \Rightarrow x < 3$
$x > -3$

$$-3 < x < 3$$

* $|x| < a \Rightarrow -a < x < a$ {where a is +ve}

2.) $|x| \geq 2 \Rightarrow x \geq 2$
$x \leq -2$

* $x > a \Rightarrow x > a$ (or) $x < -a$
{where a is +ve}

Trick:-

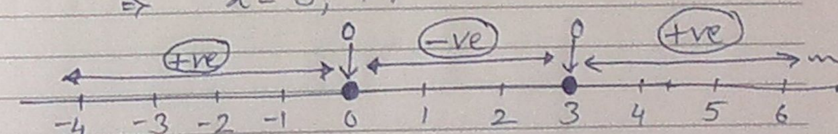
$\|x\| < 5$	$\|x\| > 3$
$-5 < x < 5$	$-3 > x > 3$
	$\Downarrow$
	$x < -3$ (or) $x > 3$.
Trick { Put negative of number to the left of inequality and place inequality sign same as original in b/w them. }	(Trick).

QUADRATIC INEQUALITIES :-

$$x^2 - 2x - 3 > 0$$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x = 3, -1.$



$x = 4 \rightarrow x^2 - 2x - 3$ $= 5 > 0 \Rightarrow +ve$	$x = 2 \rightarrow x^2 - 2x - 3$ $= -3 < 0 \Rightarrow -ve$
$x = -2 \rightarrow x^2 - 2x - 3$ $= 5 > 0 \rightarrow +ve$	P.T.O

So, Solution of Equation $x^2-2x-3<0$ is —

$-1 < x < 3$. { In this region the value is <0 }

Steps :—

1.) Set expression to equal zero.

2.) Find solutions & record them on the number line. The solution will divide the line into 3 regions.

3.) Test a number from each region.

4.) Solve the inequality.