Divisibility and Prime numbers

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box ☐, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes ☐/☐, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. For how many positive integer values of x is $\frac{65}{x}$ an integer?

☐

2. If x is a number such that $0 < x \leq 20$, for how many values of x is $\frac{20}{x}$ an integer?

(A) 4
(B) 6
(C) 8
(D) 10
(E) More than 10

3.

Quantity A	**Quantity B**
The number of even factors of 27	The number of even factors of 81

4.

Quantity A	**Quantity B**
The number of distinct factors of 10	The number of distinct prime factors of 210

5.

Quantity A	**Quantity B**
The least common multiple of 22 and 6	The greatest common factor of 66 and 99

6. The number of students who attend a school could be divided among 10, 12, or 16 buses, such that each bus transports an equal number of students. What is the minimum number of students that could attend the school?

(A) 120
(B) 160
(C) 240
(D) 320
(E) 480

7.

Quantity A	**Quantity B**
The number of distinct prime factors of 27	The number of distinct prime factors of 18

8.

Quantity A	**Quantity B**
The number of distinct prime factors of 31	The number of distinct prime factors of 32

9. How many factors greater than 1 do 120, 210, and 270 have in common?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 30

10. Company H distributed $4,000 and 180 pencils evenly among its employees, with each employee getting an equal integer number of dollars and an equal integer number of pencils. What is the greatest number of employees that could work for Company H?

(A) 9
(B) 10
(C) 20
(D) 40
(E) 180

11. n is divisible by 14 and 3. Which of the following statements must be true?

Indicate all such statements.

- ☐ 12 is a factor of n
- ☐ 21 is a factor of n
- ☐ n is a multiple of 42

12. Positive integers a and b each have exactly four factors. If a is a one-digit number and $b = a + 9$, what is the value of a?

13. Ramon wants to cut a rectangular board into identical square pieces. If the board is 18 inches by 30 inches, what is the least number of square pieces he can cut without wasting any of the board?

(A) 4
(B) 6
(C) 9
(D) 12
(E) 15

14. If n is the product of 2, 3, and a two-digit prime number, how many of its factors are greater than 6?

15.

m is a positive integer that has a factor of 8.

Quantity A	**Quantity B**
The remainder when m is divided by 6	The remainder when m is divided by 12

16. When the positive integer x is divided by 6, the remainder is 4. Each of the following could also be an integer EXCEPT

(A) $\frac{x}{2}$

(B) $\frac{x}{3}$

(C) $\frac{x}{7}$

(D) $\frac{x}{11}$

(E) $\frac{x}{17}$

17. If $x^y = 64$ and x and y are positive integers, which of the following could be the value of $x + y$?

Indicate all such values.

- ☐ 2
- ☐ 6
- ☐ 7
- ☐ 8
- ☐ 10
- ☐ 12

18. If k is a multiple of 24 but not a multiple of 16, which of the following cannot be an integer?

(A) $\frac{k}{8}$

(B) $\frac{k}{9}$

(C) $\frac{k}{32}$

(D) $\frac{k}{36}$

(E) $\frac{k}{81}$

19. If $a = 16b$ and b is a prime number greater than 2, how many positive distinct factors does a have?

20. If a and c are positive integers and $4a + 3 = b$ and $4c + 1 = d$, which of the following could be the value of $b + d$?

(A) 46
(B) 58
(C) 68
(D) 74
(E) 82

21. Each factor of 210 is inscribed on its own plastic ball, and all of the balls are placed in a jar. If a ball is randomly selected from the jar, what is the probability that the ball is inscribed with a multiple of 42?

(A) $\frac{1}{16}$
(B) $\frac{5}{42}$
(C) $\frac{1}{8}$
(D) $\frac{3}{16}$
(E) $\frac{1}{4}$

22. At the Canterbury Dog Fair, 1/4 of the poodles are also show dogs and 1/7 of the show dogs are poodles. What is the least possible number of dogs at the fair?

23. A "prime power" is an integer that has only one prime factor. For example, $5 = 5$, $25 = 5 \times 5$, and $27 = 3 \times 3 \times 3$ are all prime powers, while $6 = 2 \times 3$ and $12 = 2 \times 2 \times 3$ are not. Which of the following numbers is not a prime power?

(A) 49
(B) 81
(C) 100
(D) 121
(E) 243

24. If a and b are integers such that $a > b > 1$, which of the following cannot be a multiple of either a or b?

(A) $a - 1$

(B) $b + 1$
(C) $b - 1$
(D) $a + b$
(E) ab

25. 616 divided by 6 yields remainder p, and 525 divided by 11 yields remainder q. What is $p + q$?

26. If x is divisible by 18 and y is divisible by 12, which of the following statements must be true?

Indicate all such statements.

☐ $x + y$ is divisible by 6
☐ xy is divisible by 48
☐ x/y is divisible by 6

27. If p is divisible by 7 and q is divisible by 6, pq must have at least how many factors greater than 1?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 8

28. If r is divisible by 10 and s is divisible by 9, rs must have at least how many factors?

(A) 2
(B) 4
(C) 12
(D) 14
(E) 16

29. If t is divisible by 12, what is the least possible integer value of a for which $\frac{t^2}{2^a}$ might not be an integer?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

30. If a, b, and c are multiples of 3 such that $a > b > c > 0$, which of the following values must be divisible by 3?

Indicate all such values.

☐ $a + b + c$
☐ $a - b + c$
☐ $abc/9$

31. New cars leave a car factory in a repeating pattern of red, blue, black, and gray cars. If the first car to exit the factory was red, what color is the 463rd car to exit the factory?

(A) red
(B) blue
(C) black
(D) gray
(E) It cannot be determined from the information given.

32. Jason deposits money at a bank on a Tuesday and returns to the bank 100 days later to withdraw the money. On what day of the week did Jason withdraw the money from the bank?

(A) Monday
(B) Tuesday
(C) Wednesday
(D) Thursday
(E) Friday

33. x and h are both positive integers. When x is divided by 7, the quotient is h with a remainder of 3. Which of the following could be the value of x?

(A) 7
(B) 21
(C) 50
(D) 52
(E) 57

34. a, b, c, and d are all positive integers. If $\frac{ab}{c+d} = 3.7$, which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ ab is divisible by 5.
☐ $c + d$ is divisible by 5.
☐ If c is even, then d must be even.

35. When x is divided by 10, the quotient is y with a remainder of 4. If x and y are both positive integers, what is the remainder when x is divided by 5?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

36. What is the remainder when $13^{17} + 17^{13}$ is divided by 10?

37. a, b, c, and d are positive integers. If $\frac{a}{b}$ has a remainder of 9 and $\frac{c}{d}$ has a remainder of 10, what is the minimum possible value for bd?

38. If n is an integer and n^3 is divisible by 24, what is the largest number that must be a factor of n?

(A) 1
(B) 2
(C) 6
(D) 8
(E) 12

39.

10! is divisible by 3^x5^y, where x and y are positive integers.

Quantity A	**Quantity B**
The greatest possible value for x	Twice the greatest possible value for y

40.

Quantity A	**Quantity B**
The number of distinct prime factors of 100,000	The number of distinct prime factors of 99,000