Chapter 13

of

5 lb. Book of GRE® Practice Problems

Divisibility and Primes

In This Chapter...

Divisibility and Primes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box ☐, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{☐}{☐}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. For how many positive integer values of x is $\frac{65}{x}$ an integer?

☐

2. If x is a number such that $0 < x \leq 20$, for how many values of x is $\frac{20}{x}$ an integer?

(A) 4
(B) 6
(C) 8
(D) 10
(E) More than 10

3.

Quantity A	**Quantity B**
The number of even factors of 27	The number of even factors of 81

4.

Quantity A	**Quantity B**
The number of distinct factors of 10	The number of distinct prime factors of 210

5.

Quantity A	Quantity B
The least common multiple of 22 and 6	The greatest common factor of 66 and 99

6. The number of students who attend a school could be divided among 10, 12, or 16 buses, such that each bus transports an equal number of students. What is the minimum number of students that could attend the school?

(A) 120
(B) 160
(C) 240
(D) 320
(E) 480

7.

Quantity A	Quantity B
The number of distinct prime factors of 27	The number of distinct prime factors of 18

8.

Quantity A	Quantity B
The number of distinct prime factors of 31	The number of distinct prime factors of 32

9. How many factors greater than 1 do 120, 210, and 270 have in common?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 30

10. Company H distributed $4,000 and 180 pencils evenly among its employees, with each employee getting an equal integer number of dollars and an equal integer number of pencils. What is the greatest number of employees that could work for Company H?

(A) 9
(B) 10
(C) 20
(D) 40
(E) 180

11. n is divisible by 14 and 3. Which of the following statements must be true?

    Indicate all such statements.

    - ☐ 12 is a factor of n
    - ☐ 21 is a factor of n
    - ☐ n is a multiple of 42

12. Positive integers a and b each have exactly four factors. If a is a one-digit number and $b = a + 9$, what is the value of a?

    ☐

13. Ramon wants to cut a rectangular board into identical square pieces. If the board is 18 inches by 30 inches, what is the least number of square pieces he can cut without wasting any of the board?

    (A) 4
    (B) 6
    (C) 9
    (D) 12
    (E) 15

14. If n is the product of 2, 3, and a two-digit prime number, how many of its factors are greater than 6?

    ☐

15.

m is a positive integer that has a factor of 8.

Quantity A	Quantity B
The remainder when m is divided by 6	The remainder when m is divided by 12

16. When the positive integer x is divided by 6, the remainder is 4. Each of the following could also be an integer EXCEPT

(A) $\frac{x}{2}$

(B) $\frac{x}{3}$

(C) $\frac{x}{7}$

(D) $\frac{x}{11}$

(E) $\frac{x}{17}$

17. If $x^y = 64$ and x and y are positive integers, which of the following could be the value of $x + y$?

Indicate <u>all</u> such values.

- ☐ 2
- ☐ 6
- ☐ 7
- ☐ 8
- ☐ 10
- ☐ 12

18. If k is a multiple of 24 but not a multiple of 16, which of the following cannot be an integer?

(A) $\frac{k}{8}$

(B) $\frac{k}{9}$

(C) $\frac{k}{32}$

(D) $\frac{k}{36}$

(E) $\frac{k}{81}$

19. If $a = 16b$ and b is a prime number greater than 2, how many positive distinct factors does a have?

☐

20. If a and c are positive integers and $4a + 3 = b$ and $4c + 1 = d$, which of the following could be the value of $b + d$?

(A) 46
(B) 58
(C) 68
(D) 74
(E) 82

21. Each factor of 210 is inscribed on its own plastic ball, and all of the balls are placed in a jar. If a ball is randomly selected from the jar, what is the probability that the ball is inscribed with a multiple of 42?

(A) $\frac{1}{16}$
(B) $\frac{5}{42}$
(C) $\frac{1}{8}$
(D) $\frac{3}{16}$
(E) $\frac{1}{4}$

22. At the Canterbury Dog Fair, 1/4 of the poodles are also show dogs and 1/7 of the show dogs are poodles. What is the least possible number of dogs at the fair?

$\boxed{}$

23. A "prime power" is an integer that has only one prime factor. For example, $5 = 5$, $25 = 5 \times 5$, and $27 = 3 \times 3 \times 3$ are all prime powers, while $6 = 2 \times 3$ and $12 = 2 \times 2 \times 3$ are not. Which of the following numbers is not a prime power?

(A) 49
(B) 81
(C) 100
(D) 121
(E) 243

24. If a and b are integers such that $a > b > 1$, which of the following cannot be a multiple of either a or b?

(A) $a - 1$
(B) $b + 1$
(C) $b - 1$
(D) $a + b$
(E) ab

25. 616 divided by 6 yields remainder p, and 525 divided by 11 yields remainder q. What is $p + q$?

26. If x is divisible by 18 and y is divisible by 12, which of the following statements must be true?

Indicate all such statements.

☐ $x + y$ is divisible by 6
☐ xy is divisible by 48
☐ x/y is divisible by 6

27. If p is divisible by 7 and q is divisible by 6, pq must have at least how many factors greater than 1?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 8

28. If r is divisible by 10 and s is divisible by 9, rs must have at least how many factors?

(A) 2
(B) 4
(C) 12
(D) 14
(E) 16

29. If t is divisible by 12, what is the least possible integer value of a for which $\frac{t^2}{2^a}$ might not be an integer?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

30. If a, b, and c are multiples of 3 such that $a > b > c > 0$, which of the following values must be divisible by 3?

Indicate all such values.

☐ $a + b + c$
☐ $a - b + c$
☐ $abc/9$

31. New cars leave a car factory in a repeating pattern of red, blue, black, and gray cars. If the first car to exit the factory was red, what color is the 463rd car to exit the factory?

(A) red
(B) blue
(C) black
(D) gray
(E) It cannot be determined from the information given.

32. Jason deposits money at a bank on a Tuesday and returns to the bank 100 days later to withdraw the money. On what day of the week did Jason withdraw the money from the bank?

(A) Monday
(B) Tuesday
(C) Wednesday
(D) Thursday
(E) Friday

33. x and h are both positive integers. When x is divided by 7, the quotient is h with a remainder of 3. Which of the following could be the value of x?

(A) 7
(B) 21
(C) 50
(D) 52
(E) 57

34. a, b, c, and d are all positive integers. If $\frac{ab}{c+d} = 3.7$, which of the following statements must be true?

Indicate all such statements.

- ☐ ab is divisible by 5.
- ☐ $c + d$ is divisible by 5.
- ☐ If c is even, then d must be even.

35. When x is divided by 10, the quotient is y with a remainder of 4. If x and y are both positive integers, what is the remainder when x is divided by 5?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

36. What is the remainder when $13^{17} + 17^{13}$ is divided by 10?

☐

37. a, b, c, and d are positive integers. If $\frac{a}{b}$ has a remainder of 9 and $\frac{c}{d}$ has a remainder of 10, what is the minimum possible value for bd?

☐

38. If n is an integer and n^3 is divisible by 24, what is the largest number that must be a factor of n?

(A) 1
(B) 2
(C) 6
(D) 8
(E) 12

39.

10! is divisible by 3^x5^y, where x and y are positive integers.

Quantity A	**Quantity B**
The greatest possible value for x	Twice the greatest possible value for y

40.

Quantity A	Quantity B
The number of distinct prime factors of 100,000	The number of distinct prime factors of 99,000

41. For which two of the following values is the product a multiple of 27?

Indicate two such values.

☐ 1
☐ 7
☐ 20
☐ 28
☐ 63
☐ 217
☐ 600
☐ 700

42. Which of the following values times 12 is not a multiple of 64?

Indicate all such values.

☐ 6^6
☐ 12^2
☐ 18^3
☐ 30^3
☐ 222

43. If $3^x(5^2)$ is divided by $3^5(5^3)$, the quotient terminates with one decimal digit. If $x > 0$, which of the following statements must be true?

(A) x is even
(B) x is odd
(C) $x < 5$
(D) $x \geq 5$
(E) $x = 5$

44. <u>abc</u> is a three-digit number in which a is the hundreds digit, b is the tens digit, and c is the units digit. Let &(<u>abc</u>)& = $(2^a)(3^b)(5^c)$. For example, &(203)& = $(2^2)(3^0)(5^3) = 500$. For how many three-digit numbers <u>abc</u> will the function &(<u>abc</u>)& yield a prime number?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 9

Divisibility and Primes Answers

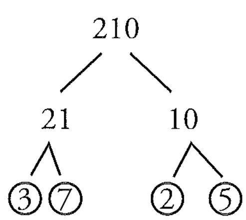
1. **4.** If x is a positive integer such that $\frac{65}{x}$ is also an integer, then x must be a factor of 65. The factors of 65 are 1, 5, 13, and 65. Thus, there are 4 positive integer values of x such that $\frac{65}{x}$ is an integer.

2. **(E).** Notice that the problem did NOT say that x had to be an integer. Therefore, the factors of 20 will work (1, 2, 4, 5, 10, 20), but so will 0.5, 0.1, 0.25, 2.5, etc. It is possible to divide 20 into fractional parts—for instance, something 20 inches long could be divided evenly into quarter inches (there would be 80 of them, as $\frac{20}{0.25} = 80$). There are an infinite number of x values that would work (it is possible to divide 20 into thousandths, millionths, etc.), so the answer is (E). It is very important on the GRE to notice whether there is an integer constraint on a variable or not! Any answer like "More than 10" should be a clue that this problem may be less straightforward than it seems.

3. **(C).** When counting factors, it helps to list them in pairs so you don't miss any. The factors of 27 are: 1 & 27, 3 & 9. The factors of 81 are: 1 & 81, 3 & 27, 9 & 9. Neither number has any even factors, so Quantity A and Quantity B are each 0 and therefore equal.

4. **(C).** The *factors* of 10 are 1 & 10, and 2 & 5. Since there are 4 factors, Quantity A is 4.

The *prime factors* of 210 are 2, 3, 5, and 7.



210 has 4 prime factors, so Quantity B is 4. Thus, the two quantities are equal.

5. **(A).** The least common multiple of 22 and 6 is 66. One way to find the least common multiple is to list the larger number's multiples (it is more efficient to begin with the larger number) until you reach a multiple that the other number goes into. The multiples of 22 are 22, 44, 66, 88, etc. The smallest of these that 6 goes into is 66.

The greatest common factor of 66 and 99 is 33. One way to find the greatest common factor is to list all the factors of one of the numbers, and then pick the greatest one that also goes into the other number. For instance, the factors of 66 are 1 & 66, 2 & 33, 3 & 22, and 6 & 11. The greatest of these that also goes into 99 is 33. Thus, Quantity A is greater.

6. **(C).** The number of students must be divisible by 10, 12, and 16. So the question is really asking, "What is the least common multiple of 10, 12, and 16?" Since all of the answer choices end in 0, each is divisible by 10. Just use your calculator to test which choices are also divisible by 12 and 16. Because you are looking for the minimum, start by checking the smallest choices. Since $\frac{120}{16}$ and $\frac{160}{12}$ are not integers, the smallest choice that works is 240.

7. **(B).** *Distinct* means *different from each other*. To find *distinct prime factors*, make a prime tree, and then disregard any repeated prime factors. The integer 27 breaks down into $3 \times 3 \times 3$. Thus, 27 has only 1 *distinct* prime factor. The integer 18 breaks down into $2 \times 3 \times 3$. Thus, 18 has 2 *distinct* prime factors.

8. **(C).** *Distinct* means *different from each other*. To find *distinct prime factors*, you would generally make a prime tree, and then disregard any repeated prime factors. However, 31 *is* prime, so 31 is the only prime factor of 31 and Quantity A is 1.

Any correct prime tree you make for 32 will result in five 2's, so 32 equals 2^5. Since this is the same prime factor repeated five times, 32 has only one *distinct* prime factor. Quantity B is 1, so the quantities are equal.

9. **(D).** Pick one of the numbers and list all of its factors on your paper. The factors of 120 are: 1 & 120, 2 & 60, 3 & 40, 4 & 30, 5 & 24, 6 & 20, 8 & 15, 10 & 12. Since the problem specifically asks for factors "greater than 1," eliminate 1 now. Now cross off any factors that do NOT go into 210:

~~120~~, 2 & ~~60~~, 3 & ~~40~~, ~~4~~ & 30, 5 & ~~24~~, 6 & ~~20~~, ~~8~~ & 15, 10 & ~~12~~

Now cross off any factors remaining that do NOT go into 270. Interestingly, all of the remaining factors (2, 3, 5, 6, 10, 15, 30) *do* go into 270. This is 7 shared factors.

10. **(C).** In order to distribute $4,000 and 180 pencils evenly, the number of employees must be a factor of each of these two numbers. Because you are looking for the greatest number of employees possible, start by checking the greatest choices.

(E) $4000 could not be evenly distributed among 180 employees (although 180 pencils could).
(D) $4,000 could be evenly divided among 40 people, but 180 pencils could not.
(C) is the greatest choice that works—$4,000 and 180 pencils could each be evenly distributed among 20 people.

Alternatively, find the greatest common factor (GCF) of the two numbers. Factor: $4{,}000 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^5 \times 5^3$ and $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$. These numbers have $2 \times 2 \times 5$ in common, so 20 is the GCF. The correct answer is (C).

11. **II and III only.** Since n is divisible by 14 and 3, n contains the prime factors of both 14 and 3, which are 2, 7, and 3. Thus, any numbers that can be constructed using only these prime factors (no additional factors) are factors of n. Since $12 = 2 \times 2 \times 3$, you CANNOT make 12 by multiplying the prime factors of n (you would need one more 2). However, you CAN construct 21 by multiplying two of the known prime factors of n ($7 \times 3 = 21$), so the second statement is true. Finally, n must be at least 42 ($= 2 \times 7 \times 3$, the *least common multiple* of 14 and 3), so n is definitely a multiple of 42. That is, n can only be 42, 84, 126, etc...

12. **6.** Start by considering integer a, which is the most constrained variable. It is a positive one-digit number (between 1 and 9, inclusive) and it has four factors. Prime numbers have exactly two factors: themselves and one, so you only need to look at non-prime one-digit positive integers. That's a short enough list:

1 has just one factor!
4 has 3 factors: 1, 2, and 4
6 has 4 factors: 1, 2, 3, and 6
8 has 4 factors: 1, 2, 4, and 8
9 has 3 factors: 1, 3, and 9

So the two possibilities for a are 6 and 8. Now apply the two constraints for b. It is 9 greater than a, and it has exactly four factors. Check the possibilities:

If $a = 6$, then $b = 15$, which has 4 factors: 1, 3, 5, and 15.
If $a = 8$, then $b = 17$, which is prime, so it has only has 2 factors: 1 and 17.
Only $b = 15$ works, so a must be 6.

13. **(E).** Cutting a rectangular board into square pieces means that Ramon needs to cut pieces that are equal in length and width. "Without wasting any of the board" means that he needs to choose a side length that divides evenly into both 18 and 30. "The least number of square pieces" means that he needs to choose the largest possible squares. With these three stipulations, choose the largest integer that divides evenly into 18 and 30, or the greatest common factor, which is 6. This would give Ramon 3 pieces going one way and 5 pieces going the other. He would cut $3 \times 5 = 15$ squares of dimension 6" × 6". Note that this solution ignored squares with non-integer side length for the sake of convenience, a potentially dangerous thing to do. (After all, identical squares of 1.5" by 1.5" could be cut without wasting any of the board.) However, to cut fewer squares that are larger than 6" × 6", Ramon could only cut 2 squares of 9" or 1 square of 18" from the 18" dimension of the rectangle, neither of which would evenly divide the 30" dimension of the rectangle. The computed answer is correct.

14. **4.** Because this is a numeric entry question, you can infer that the answer will be the same regardless of which two-digit prime you pick. So for the sake of simplicity, pick the smallest and most familiar two-digit prime: 11.

If n is the product of 2, 3, and 11, n equals 66 and its factors are:

Small	Large
1	66
2	33
3	22
6	11

There are four factors greater than 6: 11, 22, 33, and 66.

Notice that because the other given prime factors of n (2 and 3) multiply to get exactly 6, you can only produce a factor greater than 6 by multiplying by the third factor, the "two-digit prime number." The right-hand column represents that third factor multiplied by all of the other factors: 11×6, 11×3, 11×2, and 11×1. If you replace 11 with any other two-digit prime, you will get the same result. (If you're not sure, try it!)

15. **(D).** Test values for m with the goal of proving (D). Because m has a factor of 8, m could equal 8, 16, 24, 32, 40, etc. If m is 24, both quantities are equal to 0. But if m is 32, Quantity A is 2 and Quantity B is 8.

16. **(B).** When dealing with remainder questions on the GRE, the best thing to do is test a few real numbers.

Multiples of 6 are 0, 6, 12, 18, 24, 30, 36, etc.

Numbers with a remainder of 4 when divided by 6 are those 4 greater than the multiples of 6:

x could be 4, 10, 16, 22, 28, 34, 40, etc.

You could keep listing numbers, but this is probably enough to establish a pattern.

(A) $x/2$ $\longrightarrow$ ALL of the listed x values are divisible by 2. Eliminate (A).
(B) $x/3$ $\longrightarrow$ NONE of the listed x values are divisible by 3, but continue checking.
(C) $x/7$ $\longrightarrow$ 28 is divisible by 7.
(D) $x/11$ $\longrightarrow$ 22 is divisible by 11.
(E) $x/17$ $\longrightarrow$ 34 is divisible by 17.

The question is "Each of the following could also be an integer EXCEPT." Since four of the choices could be integers, (B) must be the answer.

17. **III, IV, and V only.** If $x^y = 64$ and x and y are positive integers, perhaps the most obvious possibility is that $x = 8$ and $y = 2$. However, "<u>all</u> such values" implies that other solutions are possible. One shortcut is noting that only an even base, when raised to a power, could equal 64. So you only have to worry about even possibilities for x. Here are all the possibilities:

$2^6 = 64 \longrightarrow x + y = 8$
$4^3 = 64 \longrightarrow x + y = 7$
$8^2 = 64 \longrightarrow x + y = 10$
$64^1 = 64 \longrightarrow x + y = 65$

The only possible values of $x + y$ listed among the choices are 7, 8, and 10.

18. **(C).** If k is a multiple of 24, it contains the prime factors of 24: 2, 2, 2, and 3. (It could also contain other prime factors, but you can only be sure of the prime factors contained in 24.)

If k were a multiple of 16, it would contain the prime factors of 16: 2, 2, 2, and 2.

Thus, if k is a multiple of 24 but NOT of 16, k must contain 2, 2, and 2, but NOT a fourth 2 (otherwise, it would be a multiple of 16).

Thus: k definitely has 2, 2, 2, and 3. It could have any other prime factors (including more 3's) EXCEPT for more 2's.

An answer choice in which the denominator contains more than three 2's would guarantee a non-integer result. Only choice (C) works. Since k has fewer 2's than 32, $k/32$ can never be an integer.

Alternatively, list multiples of 24 for k: 24, 48, 72, 96, 120, 144, 168, etc.

Then, eliminate multiples of 16 from this list: 24, ~~48~~, 72, ~~96~~, 120, ~~144~~, 168, etc.

A pattern emerges: k = (an odd integer) $\times$ 24.

(A) $k/8$ can be an integer, for example when $k = 24$.
(B) $k/9$ can be an integer, for example when $k = 72$.
(C) $k/32$ is correct by process of elimination.
(D) $k/36$ can be an integer, for example when $k = 72$.
(E) $k/81$ can be an integer, for example when $k = 81 \times 24$.

19. **10.** Because this is a numeric entry question, there can be only one correct answer. So, plugging in any prime number greater than 2 for b must yield the same result. Try $b = 3$.

If $a = 16b$ and $b = 3$, then a is 48. The factors (NOT prime factors) of 48 are: 1 & 48, 2 & 24, 3 & 16, 4 & 12, and 6 & 8. There are 10 distinct factors.

20. **(C).** The two equations are already solved for b and d, and the question is about the value of $b + d$. So, stack the equations and add:

$$\begin{array}{l} 4a + 3 = b \\ \underline{4c + 1 = d} \\ 4a + 4c + 4 = b + d \end{array}$$

Because a and c are integers, $4a + 4c + 4$ is the sum of three multiples of 4, which is a multiple of 4 itself. Therefore, the other side of the equation, $b + d$, must also equal a multiple of 4.

You could also factor out the 4:

$$\begin{array}{l} 4a + 4c + 4 \\ 4(a + c + 1) \end{array}$$

Since a and c are integers, $a + c + 1$ is an integer, so $4(a + c + 1)$ is definitely a multiple of 4, and $b + d$ is also a multiple of 4. Only choice (C) is a multiple of 4.

21. **(C).** The factors of 210 are as follows:

1 & 210
2 & 105
3 & 70
5 & 42
6 & 35
7 & 30
10 & 21
14 & 15

Out of the list of 16 factors, there are two multiples of 42 (42 and 210).

Thus, the answer is 2/16 or 1/8.

22. **10.** If 1/4 of the poodles are also show dogs, the number of poodles must be divisible by 4. (The number of dogs is necessarily an integer.) Since the least possible number is the goal, try an example with 4 poodles.

If 1/7 of the show dogs are poodles, the number of show dogs must be divisible by 7. Since the least possible number is the goal, try an example with 7 show dogs.

So far there are:

4 poodles, 1 of which is a show dog
7 show dogs, 1 of which is a poodle

Note that the one poodle that is also a show dog *is the same dog* as the one show dog that is also a poodle! To get the total number of dogs, only count that dog *once,* not twice. In total:

3 poodles (non-show dogs)
1 dog that is both poodle and show dog
6 show dogs (non-poodles)

This equals 10 dogs in total. This example met all the constraints of the question while using minimum values at each step, so this is the least possible number of dogs at the fair.

23. **(C).** Break down each of the numbers into its prime factors.

(A) $49 = 7 \times 7$
(B) $81 = 3 \times 3 \times 3 \times 3$
(C) $100 = 2 \times 2 \times 5 \times 5$
(D) $121 = 11 \times 11$
(E) $243 = 3 \times 3 \times 3 \times 3 \times 3$

Since 100 has both 2 and 5 as prime factors, it is not a prime power. The correct answer is (C).

24. **(C).** Since a positive multiple must be equal to or larger than the number it is a multiple of, answer choice (C) cannot be a multiple of a or b, as it is smaller than both integers a and b.

You can also try testing numbers such that a is larger than b.

(A) If $a = 3$ and $b = 2$, $a - 1 = 2$, which is a multiple of b.
(B) If $a = 3$ and $b = 2$, $b + 1 = 3$, which is a multiple of a.
(C) Is the correct answer by process of elimination.
(D) If $a = 4$ and $b = 2$, $a + b = 6$, which is a multiple of b.
(E) If $a = 3$ and $b = 2$, $ab = 6$, which is a multiple of both a and b.

25. **12.** Remember, remainders are always whole numbers, so dividing 616 by 6 in your calculator won't quite give you what you need. Rather, find the largest number less than 616 that 6 *does* go into (not 615, not 614, not 613 …). That number is 612. Since $616 - 612 = 4$, the remainder p is equal to 4.

Alternatively, you could divide 616 by 6 in your calculator to get 102.66…. Since 6 goes into 616 precisely 102 whole times, multiply 6×102 to get 612, then subtract from 616 to get remainder 4.

This second method might be best for finding q. Divide 525 by 11 to get 47.7272…. Since $47 \times 11 = 517$, the remainder is $525 - 517 = 8$.

Therefore, $p + q = 4 + 8 = 12$.

26. **I only.** To solve this problem with examples, make a short list of possibilities for each of x and y:

$x = 18, 36, 54\ldots$
$y = 12, 24, 36\ldots$

Now try to *disprove* the statements by trying several combinations of x and y above. In Statement I, $x + y$ could be $18 + 12 = 30$, $54 + 12 = 66$, $36 + 24 = 60$, or many other combinations. Interestingly, all those combinations are multiples of 6. This makes sense, as x and y individually are multiples of 6, so their sum is too. Statement I is true.

To test statement II, xy could be $18(12) = 216$, which is NOT divisible by 48. Eliminate statement II.

As for statement III, x/y could be 18/12, which is not even an integer (and therefore not divisible by 6), so III is not necessarily true.

27. **(D).** This problem is most easily solved with an example. If $p = 7$ and $q = 6$, then $pg = 42$, which has the factors 1 & 42, 2 & 21, 3 & 14, and 6 & 7. That's 8 factors, but read carefully! The question asks how many factors *greater than 1*, so the answer is 7. Note that choosing the smallest possible examples ($p = 7$ and $q = 6$) was the right move here, since the question asks "at least how many factors…?" If testing $p = 70$ and $q = 36$, many, many more factors would have resulted. The question asks for the minimum.

28. **(C).** This problem is most easily solved with an example. If $r = 10$ and $s = 9$, then $rs = 90$. The factors of 90 are 1 & 90, 2 & 45, 3 & 30, 5 & 18, 6 & 15, and 9 & 10. Count to get a minimum of 12 factors.

29. **(D).** If t is divisible by 12, then t^2 must be divisible by 144 or $2 \times 2 \times 2 \times 2 \times 3 \times 3$. Therefore, t^2 can be divided evenly by 2 at least four times, so a must be at least 5 before $\frac{t^2}{2^a}$ might not be an integer.

Alternatively, test values. If $t = 12$, $\frac{t^2}{2^a} = \frac{144}{2^a}$. Plug in the choices as possible a values, starting with the smallest choice and working up.

(A) Since $144/2^2 = 36$, eliminate.
(B) Since $144/2^3 = 18$, eliminate.
(C) Since $144/2^4 = 9$, eliminate.
(D) $144/2^5 = 4.5$. The first choice for which $\frac{t^2}{2^a}$ might not be an integer is (D).

30. **I, II, and III.** Since a, b, and c are all multiples of 3, $a = 3x$, $b = 3y$, $c = 3z$, where $x > y > z > 0$ and all are integers. Substitute these new expressions into the statements.

Statement I: $a + b + c = 3x + 3y + 3z = 3(x + y + z)$. Since $(x + y + z)$ is an integer, this number must be divisible by 3.

Statement II: $a - b + c = 3x - 3y + 3z = 3(x - y + z)$. Since $(x + y + z)$ is an integer, this number must be divisible by 3.

Statement III: $abc/9 = (3x3y3z)/9 = (27xyz)/9 = 3xyz$. Since xyz is an integer, this number must be divisible by 3.

31. **(C).** Pattern problems on the GRE often include a very large series of items that would be impossible (or at least unwise) to write out on paper. Instead, this problem requires you to recognize and exploit the pattern. In this case, after every 4th car, the color pattern repeats. By dividing 463 by 4, you find that there will be 115 cycles through the 4 colors of cars—red, blue, black, gray—for a total of 460 cars to exit the factory. The key to solving these problems is the remainder. Because there are $463 - 460 = 3$ cars remaining, the first such car will be red, the second will be blue, and the third will be black.

32. **(D).** This is a pattern problem. An efficient method is to recognize that the 7th day after the initial deposit would be Tuesday, as would the 14th day, the 21st day, etc. Divide 100 by 7 to get 14 full weeks comprising 98 days, plus 2 days left over. For the two leftover days, think about when they would fall. The first day after the deposit would be a Wednesday, as would the first day after waiting 98 days. The second day after the deposit would be a Thursday, and so would the 100th day.

33. **(D).** Division problems can be interpreted as follows: dividend = divisor × quotient + remainder. This problem is dividing x by 7, or distributing x items equally to 7 groups. After the items are distributed among the 7 groups, there are 3 things left over, the remainder. This means that the value of x must be some number that is 3 larger than a multiple of 7, such as 3, 10, 17, 24, etc. The only answer choice that is 3 larger than a multiple of 7 is 52.

34. **II and III only.** Start by rearranging the equation: $\frac{ab}{c+d} = \frac{37}{10}$ is equivalent to $10ab = 37(c + d)$. Remember that all four variables are positive integers. Because 37 and 10 have no shared factors, $(c + d)$ must be a multiple of 10 and ab must be a multiple of 37, in order to make the equation balance.

I. Could be true. All the requirements are met if $ab = (5)(37)$ and $(c + d) = 50$, so ab could be divisible by 5. But all the requirements are met if $ab = 37$ and $(c + d) = 10$, in which case ab is not divisible by 5.

II. MUST be true. Because $(c + d)$ must be divisible by 10, it must be divisible by both 5 and 2.

III. MUST be true. Because $(c + d)$ must be divisible by 10, it must be divisible by both 5 and 2. Thus, $(c + d)$ must be even, so if c were even, d would have to be even, too.

35. **(E).** This is a bit of a trick question—any number that yields remainder 4 when divided by 10 will also yield remainder 4 when divided by 5. This is because the remainder 4 is less than both divisors, and all multiples of 10 are also multiples of 5. For example, 14 yields remainder 4 when divided either by 10 or by 5. This also works for 24, 34, 44, 54, etc.

36. **0.** The remainder when dividing an integer by 10 always equals the units digit. You can also ignore all but the units digits, so the question can be rephrased as: *What is the units digit of* $3^{17} + 7^{13}$*?*

The pattern for the units digits of 3 is [3, 9, 7, 1]. Every fourth term is the same. The 17th power is 1 past the end of the repeat: $17 - 16 = 1$. Thus, 3^{17} must end in 3.

The pattern for the units digits of 7 is [7, 9, 3, 1]. Every fourth term is the same. The 13th power is 1 past the end of the repeat: $13 - 12 = 1$. Thus, 7^{13} must end in 7. The sum of these units digits is $3 + 7 = 10$. Thus, the units digit is 0.

37. **110.** When dividing, the remainder is always less than the divisor. If you divided a by b to get a remainder of 9, then b must have been greater than 9. Similarly, d must be greater than 10. Since b and d are integers, the smallest they could be is 10 and 11, respectively.

Thus, the minimum that bd could be is $10 \times 11 = 110$.

As an example, try $a = 19$, $b = 10$, $c = 21$, and $d = 11$ (generate a by adding remainder 9 to the value of b, and generate c by adding remainder 10 to the value of d.)

It is not possible to generate an example in which *any* of the four numbers are smaller. The least possible value of bd is 110.

38. **(C).** Start by considering the relationship between n and n^3. Because n is an integer, for every prime factor n has, n^3 must have three of them. Thus, n^3 must have prime numbers in multiples of 3. If n^3 has one prime factor of 3, it must actually have two more, because n^3's prime factors can only come in triples.

The question says that n^3 is divisible by 24, so n^3's prime factors must include at least three 2's and a 3. But since n^3 is a cube, it must contain at least three 3's. Therefore n must contain at least one 2 and one 3, or $2 \times 3 = 6$.

39. **(C).** First, expand 10! as $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

(Do NOT multiply all of those numbers together to get 3,628,800—it's true that 3,628,800 is the value of 10!, but analysis of the prime factors of 10! is easier in the current form.)

Note that 10! is divisible by 3^x5^y, and the quantities concern the greatest possible values of x and y, which is equivalent to asking, "What is the maximum number of times you can divide 3 and 5, respectively, out of 10! while still getting an integer answer?"

In the product $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, only the multiples of 3 have 3 in their prime factors, and only the multiples of 5 have 5 in their prime factors. Here are all the primes contained in $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ and therefore in 10!:

$10 = 5 \times 2$
$9 = 3 \times 3$
$8 = 2 \times 2 \times 2$
$7 = 7$
$6 = 2 \times 3$
$5 = 5$
$4 = 2 \times 2$
$3 = 3$
$2 = 2$
$1 =$ no primes

There are four 3's and two 5's total. The maximum values are $x = 4$ and $y = 2$. Therefore, Quantity A and Quantity B are each 4, so the correct answer is (C).

40. **(B).** Since only the number of *distinct* prime factors matter, not what they are or how many times they are present, you can tell on sight that Quantity A has only 2 distinct prime factors, because 100,000 is a power of 10. (Any prime tree for 10, 100, or 1,000, etc. will contain only the prime factors 2 and 5, occurring in pairs.)

In Quantity B, 99,000 breaks down as $99 \times 1{,}000$. Since 1,000 also contains 2's and 5's, and 99 contains even more factors (specifically 3, 3, and 11), Quantity B has more distinct prime factors. It is not necessary to make prime trees for each number.

41. **V and VII only.** For two numbers to have a product that is a multiple of 27, the two numbers need to have at least three 3's among their combined prime factors, since 27 = 33. Only 63 and 600 are multiples of 3, so the other choices could be eliminated very quickly if you see that. There's no need to actually multiply the numbers together. Since 63 is $3 \times 3 \times 7$ and 600 is 3×200, their product will have the three 3's required for a multiple of 27.

42. **III, IV, and V only.** Because $64 = 2^6$, multiples of 64 would have at least six 2's among their prime factors.

Since 12 (which is $2 \times 2 \times 3$) has two 2's already, a number that could be multiplied by 12 to generate a multiple of 64 would need to have, at minimum, the *other* four 2's needed to generate a multiple of 64.

Since you want the choices that don't multiply with 12 to generate a multiple of 64, select only the choices that have *four or fewer 2's* within their prime factors.

6^6	$= (2 \times 3)^6$	six 2's	INCORRECT
12^2	$= (2^2 \times 3)^2$	four 2's	INCORRECT
18^3	$= (2 \times 3^2)^3$	three 2's	CORRECT
30^3	$= (2 \times 3 \times 5)^3$	three 2's	CORRECT
222	$= (2 \times 3 \times 37)$	one 2	CORRECT

43. **(D).** When a non-multiple of 3 is divided by 3, the quotient does not terminate (for instance, $1/3 = 0.333\ldots$).

Since $3^x(5^2)/3^5(5^3)$ does NOT repeat forever, x must be large enough to cancel out the 3^5 in the denominator. Thus, x must be at least 5. Note that the question asks what MUST be true. Choice (D) must be true. Choice (E), $x = 5$, represents one value that would work, but this choice does not *have* to be true.

44. **(B).** Since a prime number has only two factors, 1 and itself, $(2^a)(3^b)(5^c)$ cannot be prime unless the digits a, b and c are such that two of the digits are 0 and the third is 1. For instance, $(2^0)(3^1)(5^0) = (1)(3)(1) = 3$ is prime. Thus, the only three values of $\underline{abc}$ that would result in a prime number &($\underline{abc}$)& are 100, 010, and 001. However, only one of those three numbers (100) is a three digit number.