DIGITAL ASSIGNMENT

Reg no: 18BCE0975

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1)

a) $L = \{ a^n b^m c^n / n,m > 1 \} \cup \{ a^n b^n c^m / n,m > 1 \}$

$S \to Acc / aaB$

$B \to aB / bCc$

$C \to bc$

$A \to Ac / aDb$

$D \to aDb / ab$

b) $L = \{ a^n b^m c^n d^n / n \geq 1, m \geq 3 \}$

$S \to PQY$

$P \to aPc$

$P \to aC$

$Y \to BBZdd$

$Z \to BZd$

$Z \to Bd$

$CB \to BC$

$aB \to ab$

$bB \to bb$

$Cd \to cd$

$cC \to cc$

c) $L = \{ a^i b^j c^k d^l / i,j,k,l \geq 0, i < l \text{ and } j \neq k \}$

$S \to PQ$

$P \to QPd / R$

$Q \to Qd / d$

$R \to MN / NB$

$N \to bNK / \varepsilon$

$M \to bM$

$M \to b$

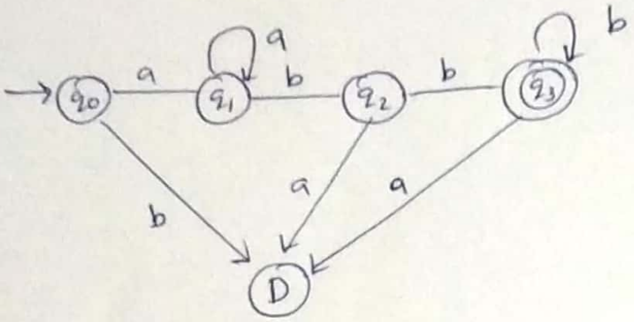
$B \to BC$

$B \to C$

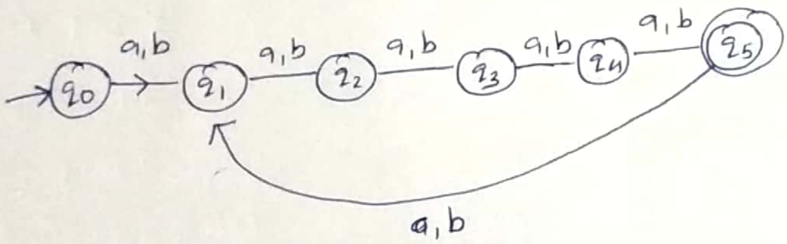
2.

a) $L = \{a^n b^m / n \geq 1, m \geq 2\}$

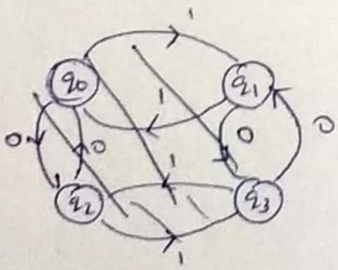
$L = \{ abb, aabb, abbb, aaabbb, \ldots \}$

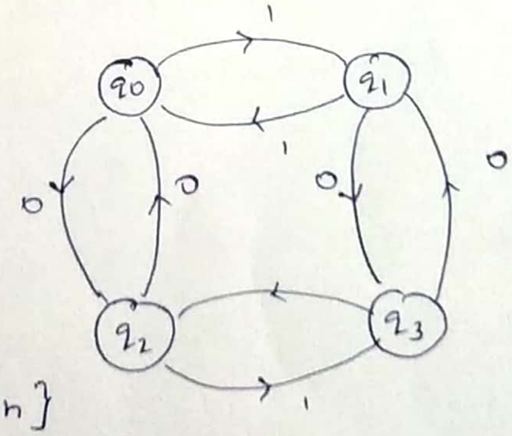


b) $L = \{w \in \{a,b\}^*, / |w| \bmod 5 = 0\}$



c) $(01+10)(01+10)(01+10)$



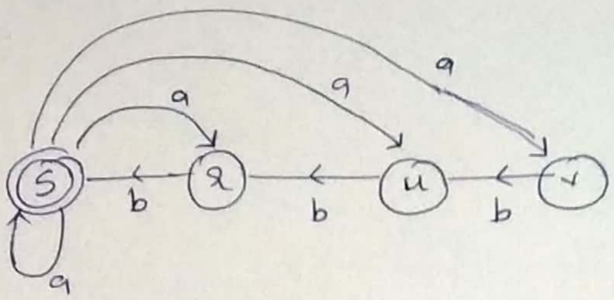


3. $\{a^n b^m / (n+m) \text{ is even}\}$

n+m is even when n and m is even or n and m is odd

$(aa)^*(bb)^* + (aa)^*a(bb)^*b$

4) Construct a DFA for the NFA given below



$M = (\{s, r, u, v\}, \{a, b\}, \{s\}, \delta, \{s\})$

δ	a	b
$\rightarrow$ s	$\{r, u, v, s\}$	ϕ
r	ϕ	s
u	ϕ	r
v	ϕ	u

$\delta(\{s\}, a) = \delta(s, a) = \{r, u, v, s\}$

$\delta(\{r, u, v, s\}, a) = \delta(r, a) \cup \delta(u, a) \cup \delta(v, a) \cup \delta(s, a)$

$= \{\phi\} \cup \{\phi\} \cup \{\phi\} \cup \{r, u, v, s\}$

$= \{r, u, v, s\}$

$\delta(\{r, u, v, s\}, b) = \delta(r, b) \cup \delta(u, b) \cup \delta(v, b) \cup \delta(s, b)$

$= \{s\} \cup \{r\} \cup \{u\} \cup \{\phi\}$

$= \{s, r, u\}$

$\delta(\{s, r, u\}, b) = \delta(s, b) \cup \delta(r, b) \cup \delta(u, b)$

$= \{\phi\} \cup \{s\} \cup \{r\}$

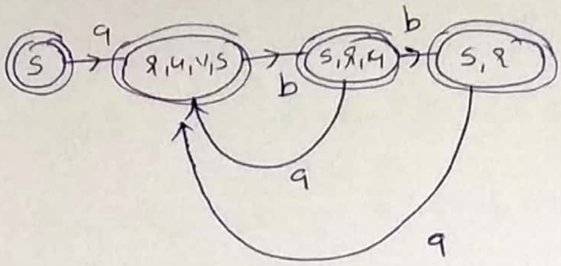
$= \{s, r\}$

$\delta(\{s, r\}, b) = \delta(s, b) \cup \delta(r, b) = \{\phi\} \cup \{s\} = \{s\}$

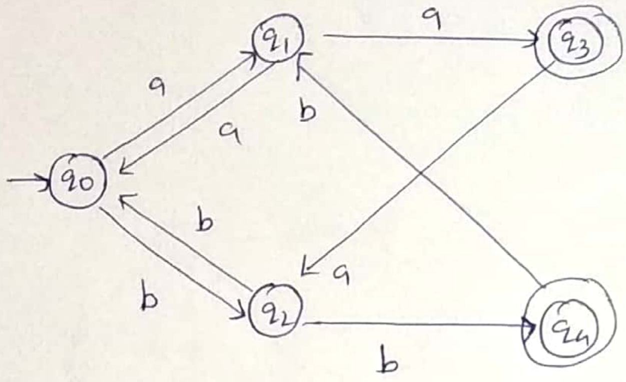
$\delta(\{s, r, u\}, a) = \delta(s, a) \cup \delta(r, a) \cup \delta(u, a)$

$= \{r, u, v, s\} \cup \{\phi\} \cup \{\phi\} = \{r, u, v, s\}$

$\delta(\{s, r\}, a) = \delta(s, a) \cup \delta(r, a) = \{r, u, v, s\} \cup \{\phi\} = \{r, u, v, s\}$



ii)



	a	b
$\rightarrow q_0$	q_1	q_2
q_1	$q_3 q_0$	–
q_2	–	$q_0 q_4$
q_3	q_2	–
q_4	–	q_1

$\delta(\{q_0\}, a) = \delta(q_0, a) = \{q_1\}$

$\delta(\{q_1\}, a) = \delta(q_1, a) = \{q_0, q_3\}$

$\delta(\{q_0, q_3\}, a) = \delta(q_0, a) \cup \delta(q_3, a) = \{q_1\} \cup \{q_2\} = \{q_1, q_2\}$

$\delta(\{q_1, q_2\}, a) = \delta(q_1, a) \cup \delta(q_2, a) = \{q_3\} \cup \{q_0\} = \{q_3, q_0\}$

$\delta(\{q_1, q_2\}, b) = \delta(q_1, b) \cup \delta(q_2, b) = \{\phi\} \cup \{q_0, q_4\} = \{q_0, q_4\}$

$\delta(\{q_0, q_4\}, a) = \delta(q_0, a) \cup \delta(q_4, a) = \{q_1\} \cup \{\phi\} = \{q_1\}$

$\delta(\{q_0\}, b) = \delta(q_0, b) = \{q_2\}$

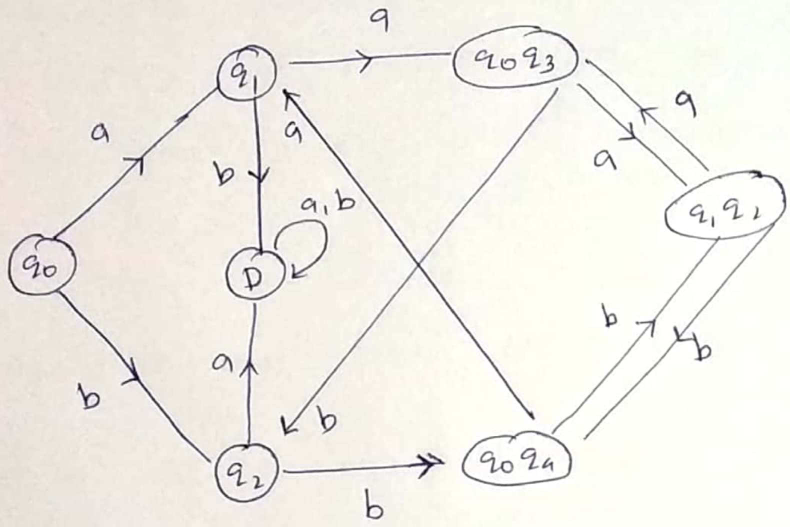
$\delta(\{q_2\}, a) = \delta(q_2, a) = \{\phi\}$

$\delta(\{q_2\}, b) = \delta(q_2, b) = \{q_0, q_4\}$

$\delta(\{q_1\}, b) = \delta(q_1, b) = \{\Phi\}$

$\delta(\{q_0, q_3\}, b) = \delta(q_0, b) \cup \delta(q_3, b) = \{q_2\} \cup \{\Phi\} = \{q_2\}$

$\delta(\{q_0, q_4\}, b) = \delta(q_0, b) \cup \delta(q_4, b) = \{q_2\} \cup \{q_1\} = \{q_1, q_2\}$



5. For any regular language L there exists an integer n such that for all $x \in L$ with $|x| \geq n$, there exists $u, v, w \in \Sigma^*$, such that $x = uvw$ and

1) $|uv| \leq n$

2) $|v| \geq 1$

3) for all $i \geq 0$: $uv^i w \in L$

In simple terms, this means that if a string v is pumped, i.e, if v is inserted any number of times, the resultant string still remains in L

i) $L = \{ a^{n!} / n \geq 1 \}$

Assume that L is regular

Let n be a positive integer greater than 2

$w = a^{n!}$ $\quad |w| = n! \geq 2$

By pumping lemma w can be decomposed as

$w = xyz$ $\quad |xy| \leq n$ $\quad |y| \geq 1$

$|y| \geq 1 \Rightarrow |y| = k$, $\quad 1 < k \leq n$

consider $w_0 = xy^0z = xz$

$|xyz| = n!$ $\quad |y| = k$

The string $xz \in L$ if and only if

$\exists$ some j such that

$n! - k = j!$

But this is impossible, since $n > 2$ and $k \leq n$,

we have

$n! - k > (n-1)!$

$n! > n! - k$, $n! - k > (n-1)!$

$\Rightarrow (n-1)! < n! - k < n!$

Hence $|xz|$ strictly lies between $(n-1)!$ and $n!$ but it is not equal to any one of them

i.e $|xz|$ can not be written as factorial of some integer.

$w_0 = xy^0z \notin L$ a contradiction

$\Rightarrow$ L is not regular

ii) $L = \{0^n 1^n / n \geq 1\}$

Assume that L is regular

Let n be a positive integer

By pumping lemma w can be decomposed as $w = xyz$ $|xy| \leq n$ $|y| \geq 1$

$|y| \geq 1 \Rightarrow |y| = k,$ $1 < k \leq n$

$w = 0^n 1^n = xyz$

$x = 0^s$ $y = 0^t$, $w = 0^p 1^n$ with

$s + t \leq n$, $t \geq 1$, $p \geq 0$, $s + t + p = n$

But

$w_0 = xy^0z = xz = 0^s 0^p 1^n = 0^{s+p} 1^n \notin L$ since $s+p \neq n$ a contradiction

$\Rightarrow$ L is not regular

iii) $L = \{ a^p / p$ is prime $\}$

Assume that L is regular.

Let n be a positive integer

Let p be a prime number greater than n

ie $p > n$

Let $w = a^p$ $|w| = p > n$

By pumping lemma w can be decomposed as

$w = xyz$

with $|xyz| \leq n$ and $|y| \geq 1$

Let $i = p+1 \Rightarrow p = i-1$

Consider $w_i = xy^iz$

$|xy^iz| = |xyz| + |y^{i-1}|$

$= |xyz| + (i-1)|y|$

$= |xyz| + (i-1)|y|$ - 1

$|y| \geq 1 \Rightarrow |y| = m$, $1 \leq m \leq n$

$|xy^2z| = p + (i-1)m$

$= p + pm$

$= p(1+m)$, is not prime

$w_i = xy^iz \notin L$ a contradiction

$\Rightarrow$ L is not regular

6. Let M be a DFA let $\equiv$ be an equivalence relation on the set of states of M

P is equivalent to q

$P \equiv q$ if for any $x \in \Sigma^*$ such that

$\delta(p, x)$ and $\delta(q, x)$ are both in F or ~~bo~~ both not in F

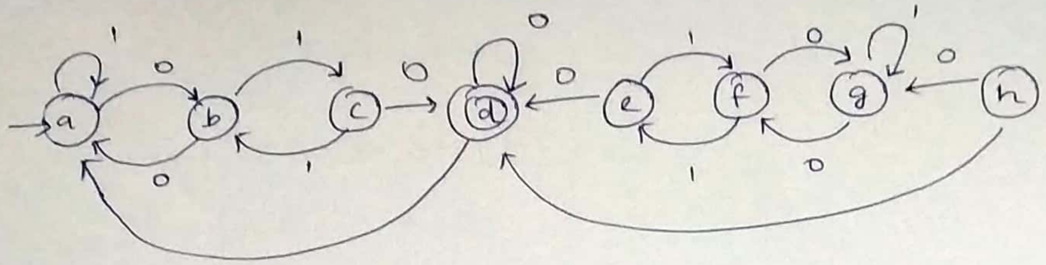
p and q are distinguishable if $\exists\ x \in \Sigma^*$ such that

$$\delta(p, x) \in F \Rightarrow \delta(q, x) \notin F \text{ or}$$

$$\delta(q, x) \in F \Rightarrow \delta(p, x) \notin F$$

x is called distinguishable

Two strings x and y are indistinguishable with respect to language L if for every string z, it holds that $xz \in L$ if and only if $yz \in L$. Otherwise they are distinguishable



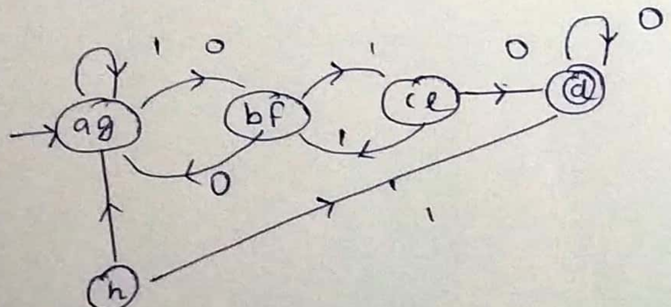
	0	1
→a	b	a
b	a	c
c	d	b
d	d	a
e	d	f
f	g	e
g	f	g
h	g	d

0 Equivalence : {a, b, c, e, f, g, h} {d}

1 Equivalence : {a, b, f, g}, {c, e} {h} {d}

2 Equivalence : {a, g}, {b, f}, {c, e}, {h}, {d}

3 Equivalence : {a, g}, {b, f}, {c, e}, {h}, {d}



a and g, b and f, c and e are equivalent states

h and d are distinguishable states

7. $sh(a(a+a^*aa)+aaa)^*$

$sh(a(a+a^*aa)+aaa)+1$

$max(sh(a(a+a^*aa)), sh(aaa))+1$

$max(max(sh(a), sh(a+a^*aa)), 0))+1$

$max(max(0, max(sh(a), sh(a^*aa)), 0))+1$

$max(max(0, max(0, max(sh(a^*), sh(aa))), 0))+1$

$max(max(0, max(0, max(sh(a)+1, 0)), 0))+1$

$max(max(0, max(0, max(1,0)), 0))+1$

$max(max(0, max(0,1)), 0))+1$

$max(max(0, 1, 0))+1$

$max(1)+1$

$1+1=2$