

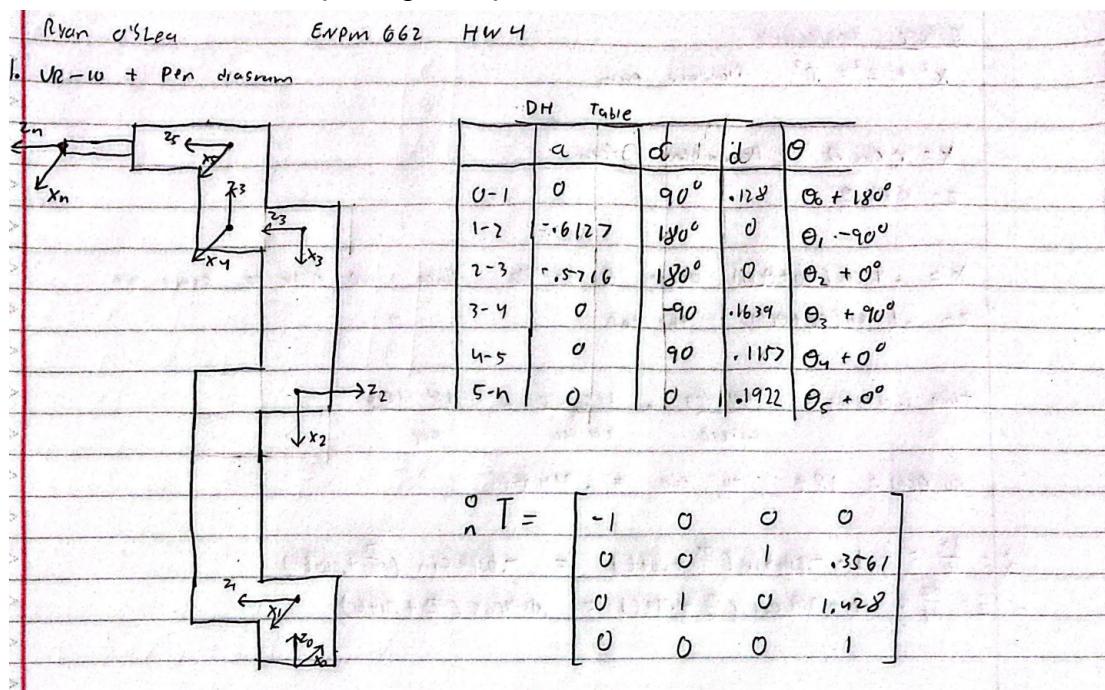
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Homework 4 Report

Problem 1

Manipulator Setup

The frame assignments and DH table were copied from the previous homework assignment with only the final frame transformation being changed. The d value of the 5-n transformation was updated by adding .1 meters to it to account for the desired position now being at the end of the pen attached to the final joint. If the end effector was anything other than a pen then the rotation would likely need to be taken into account and we'd have to keep the 6th frame at the attachment point and add a 7th frame for the end effector. In this setup though we were able to just use 6 transformations to keep things simple and save on matrix calculations.



DH Table

Frame	A(meters)	Alpha (degrees)	D (meters)	Theta (degrees)
0-1	0	90	0.128	180
1-2	-0.6127	180	0	-90

2-3	-0.5716	180	0	0
3-4	0	-90	0.1639	90
4-5	0	90	0.1157	0
5-n	0	0	0.1922	0

Jacobian Calculation

The first method was used to calculate the general form of the jacobian since it was easier to do matrix manipulations in code instead of partial derivatives. A sample of finding the J1 vector in the home position is shown below to validate the method before implementing it in code.

Handwritten notes for Jacobian calculation:

- Jacobian calculations (using first method)
- Individual transformation computed during Iter 3. Using code outputs to multiply them together w/ the updated d value for \hat{n}^T
- $i = 1$
- $Z_0 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & .128 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
- $O_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$
- $O_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ .3561 \end{bmatrix}$
- $J_1 = \begin{bmatrix} Z_0 \times (O_n - O_0) \\ Z_0 \end{bmatrix} = \begin{bmatrix} 0 \cdot .128 - 1 \cdot .3561 \\ 1 \cdot 0 - 0 \cdot .128 \\ 0 \cdot .3561 - 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} -.3561 \\ 0 \\ 0 \end{bmatrix}$
- $J_1 = \begin{bmatrix} -.3561 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

The code for calculating the Jacobian and its inverse is shown below. For the first vector in the Jacobian matrix the Zi-1 vector is assumed to be [0, 0, 1] and the Oi-1 vector is assumed to be [0, 0, 0]. This fits with the use of the 4x4 identity matrix as the starting matrix since the Z and O correspond with the first 3 values in the 3rd and 4th column respectively. For all Jacobian vectors after the first, the 0 to -1 transformation vector is grabbed and has the relevant values extracted from its 3rd and 4th columns for the Zi-1 and Oi-1 vectors. From there the Jacobian vector is calculated using the equations provided in the slides. These Jacobian vectors are appended to a list and then

transposed after all have been added to form the final Jacobian matrix. In order to calculate the inverse the determinant is first checked to see if we need to calculate the pseudo inverse instead. If the determinant is 0 within a certain threshold (5 decimal places) then the pseudo inverse is used. The inverse jacobian is then saved for later for inverse velocity kinematics calculations.

```
# Uses the most up to date transformation matrices and joint angles to calculate the jacobian matrix and its inverse
def calculateInvJacobian(self):
    self.calculateTransMats()
    # Calculate the jacobian vectors J1 through n
    jacobian_vecs = []
    # Define the Z and O vectors for the 0th frame to simplify calculations for J1
    z_0 = np.array([0, 0, 1])
    o0 = np.array([0, 0, 0])
    # Grab the translation vector from the final transformation matrix for On
    On = [self.final_trans_mat.row(i)[3] for i in range(3)]

    # sympy.pprint(self.successive_trans_mats)

    # Calculate the individual Ji components
    for idx in range(len(self.successive_trans_mats)):
        # Special case for J1 since the identity matrix isn't stored as the first matrix in the list
        if idx == 0:
            z_base = z_0
            o_base = o0
        # Otherwise grab Z and O from the 3rd and 4th columns of the i-1 transformation matrix
        else:
            trans_mat = self.successive_trans_mats[idx-1]
            z_base = np.array([trans_mat.row(i)[2] for i in range(3)])
            o_base = np.array([trans_mat.row(i)[3] for i in range(3)])

        # Calculate L (On - Oi-1)
        L = On - o_base

        # Calculate the cross product of Zi-1 and L. Technically this would be different if we had prismatic joints as well but
        # Since it's all revolute joints we're gonna keep it simple
        j_vec_1st_half = np.cross(z_base, L)

        j_vec = np.append(j_vec_1st_half, z_base)

        jacobian_vecs.append(j_vec)

    # Combine the individual vectors into a matrix to form the jacobian
    jacobian = sympy.Matrix(np.array(jacobian_vecs).transpose())

    # Calculate the pseudo inverse of the jacobian so we can use it to calculate joint velocities
    # Check the determinant to see if we can use the normal inverse or if we need to use the pseudo inverse instead
    det = round(jacobian.det(), 5)
    if det == 0:
        psuedo_inv = jacobian.pinv() #((jacobian.T*jacobian).inv())*jacobian.T
    else:
        psuedo_inv = jacobian.inv()
    # psuedo_inv = roundExpr(psuedo_inv, 5)

    self.pseudo_inv_j = psuedo_inv
```

The Z and O vectors along with the Jacobian matrix for the initial home position are shown below both with and without the epsilon added to the initial joint angles. The vectors and the Jacobian matrix evaluated with symbols are included in the appendix because they are incredibly long and hard to read. They can be printed out in the code

by setting the use_symbols and display variables to True on line 178. This will print out the general vectors and jacobian for the home position evaluated with variables and then exit.

With Epsilon

```
On
[-2.0e-5, 0.3561, 1.428]

Z0
[0 0 1]

00
[0 0 0]

Z1
[3.e-5 1.00000000000000 0]

01
[0 0 0.12800]

Z2
[-3.e-5 -1.00000000000000 0]

02
[-2.e-5 0 0.74070]

Z3
[3.e-5 1.00000000000000 0]

03
[-3.e-5 0 1.31230]

Z4
[3.e-5 0 1.00000000000000]

04
[-3.e-5 0.16390 1.31230]

Z5
[5.e-5 1.00000000000000 0]

05
[-3.e-5 0.16390 1.42800]
```

Jacobian

$$\begin{bmatrix} -0.3561 & 1.29999995231628 & -0.687300205230713 & 0.115700006484985 & -0.192200183868408 & 0 \\ -1.0e-5 & -3.9053e-5 & 2.0647e-5 & -3.47571e-6 & 1.23307227113401e-5 & 0 \\ 0 & 2.07110861083493e-5 & -2.07110861083493e-5 & 6.83925463818014e-7 & -3.8492e-6 & -2.31513695325702e-6 \\ 0 & 3.0e-5 & -3.0e-5 & 3.0e-5 & -2.0e-5 & 4.0e-5 \\ 0 & 1.0 & -1.0 & 1.0 & 0 & 1.0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Without Epsilon

```
On
[0, 0.3561, 1.428]

Z0
[0 0 1]

00
[0 0 0]

Z1
[0 1.00000000000000 0]

01
[0 0 0.12800]

Z2
[0 -1.00000000000000 0]

02
[0 0 0.74070]

Z3
[0 1.00000000000000 0]

03
[0 0 1.31230]

Z4
[0 0 1.00000000000000]

04
[0 0.16390 1.31230]

Z5
[0 1.00000000000000 0]

05
[0 0.16390 1.42800]
```

Jacobian

$$\begin{bmatrix} -0.3561 & 1.29999995231628 & -0.687300205230713 & 0.115700006484985 & -0.192200183868408 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & -1.0 & 1.0 & 0 & 1.0 \\ 1 & 0 & 0 & 0 & 1.0 & 0 \end{bmatrix}$$

Trajectory Calculation

To calculate the ideal trajectory of the end effector the parametric form of a circle was used with z substituted for y because we were working exclusively in the xz plane. A 90 degree offset was added to the theta value because the robot was starting its drawing at the top of the circle instead of on the right hand side. Theta was set up as a function of time so we could take into account the 20 second completion time and 0.1 meters was used as the value of r. Using the 20 second completion time it was found that the circle would need to be drawn at a rate of 18 degrees per second which was then converted to radians for a rate of 0.314 radians per second. This was plugged back into the equations which then had their derivatives taken so we could get end effector velocity at each timestamp.

Circle trajectory

$x^2 + z^2 = r^2$ Standard form

$x = r \cos \theta$ Parametric form

$z = r \sin \theta$

$x = .1 \cos(90 + \theta(t))$ offset θ by 90° since we need to start at the top

$z = .1 \sin(90 + \theta(t))$ the top

time increment = $\frac{360 \text{ deg}}{\text{circle}} \cdot \frac{1 \text{ circle}}{20 \text{ sec}} = \frac{18 \text{ deg}}{\text{sec}}$

so $\theta(t) = 18t$ to rad = $.314t$

$\dot{x} = \frac{dx}{dt} = -.1 \cdot -.314 \sin\left(\frac{\pi}{2} + .314t\right) = -.6314 \sin\left(\frac{\pi}{2} + .314t\right)$

$\dot{z} = \frac{dz}{dt} = .1 \cdot .314 \cos\left(\frac{\pi}{2} + .314t\right) = .0314 \cos\left(\frac{\pi}{2} + .314t\right)$

In the code the 20 second completion time was broken down into n timestamps for the code to calculate the end effector velocity and joint angle velocity at. This n value ended up being incredibly important because if it was too low the joint angle calculations over time would be incredibly inaccurate due to numerical integration taking large steps with instantaneous velocities. A value of 2000 timestamps was found to produce good results. This works out to a rate of 100hz which isn't abnormal for a sensor publishing rate.

The code below shows the process for going from end effector velocity to joint angle velocity to joint angle position to end effector position. As mentioned previously, joint angles are calculated over time using numerical integration. End effector position is extracted from the 0 to n transformation matrix that models the forward kinematics of the manipulator as a function of its joint angles which are calculated at each timestamp.

```
# Loop through the timestamps to find the end effector velocity at each timestamp
# Use the end effector velocity to calculate the joint angle velocities
for stamp_num, stamp in enumerate(timestamps):
    time_diff = (stamp - last_stamp)

    # Grab the latest position of the end effector with respect to the base frame from the full i to n homogenous transformation matrix
    latest_trans = j_utils.final_trans_mat
    x_pos = latest_trans.row(0)[3]
    y_pos = latest_trans.row(1)[3]
    z_pos = latest_trans.row(2)[3]

    x_list.append(x_pos)
    y_list.append(y_pos)
    z_list.append(z_pos)

    # Calculate the end effector x and z velocities from the parametric circle equation derivatives
    x_dot = -0.0314*np.sin(math.pi/2 + .314*stamp)
    z_dot = 0.0314*np.cos(math.pi/2 + .314*stamp)

    if (stamp_num + 1) % print_every == 0:
        print("Idx: {} \t X: {} \t Y: {} \t Z: {}".format(stamp_num + 1, x_pos, y_pos, z_pos))

    # Build the 6x1 end effector state vector
    ee_vel_state = np.array([x_dot, 0, z_dot, 0, 0, 0]).transpose()
    # print(j_utils.pseudo_inv_j.shape)

    # Find the joint angles based on the previous state and vel
    for idx, angle in enumerate(joint_angles):
        joint_angles[idx] += joint_angle.vels[idx]*time_diff

    # Update the jacobian based on the new angles
    j_utils.updateThetas(joint_angles)
    j_utils.calculateInvJacobian()

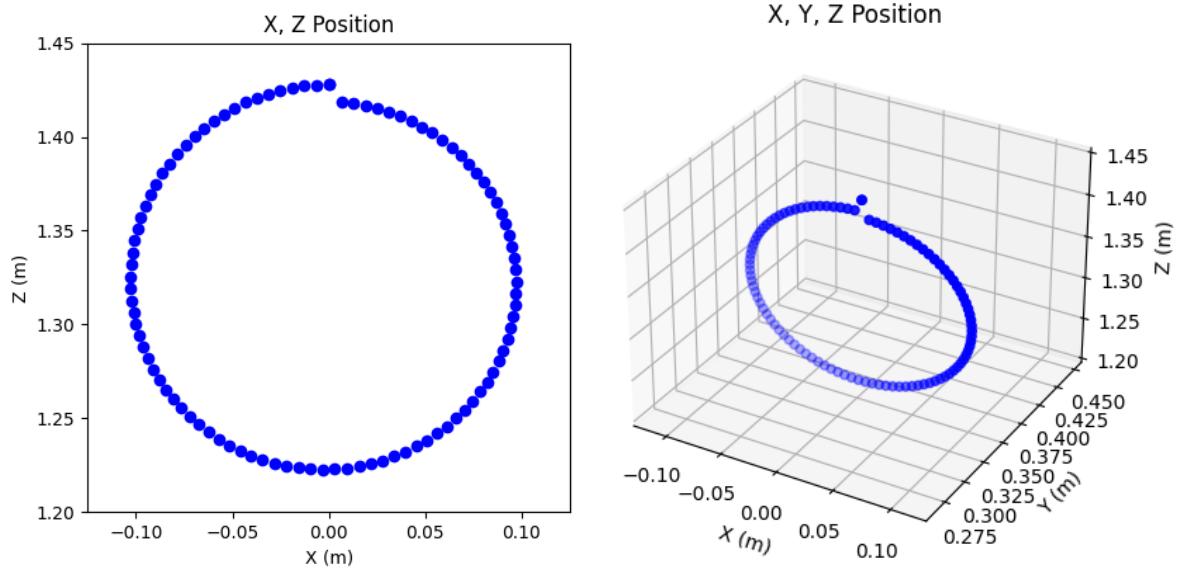
    # Calculate the new joint vels based on the end effector vel
    joint_angle.vels = np.matmul(j_utils.pseudo_inv_j, ee_vel_state)

last_stamp = stamp
```

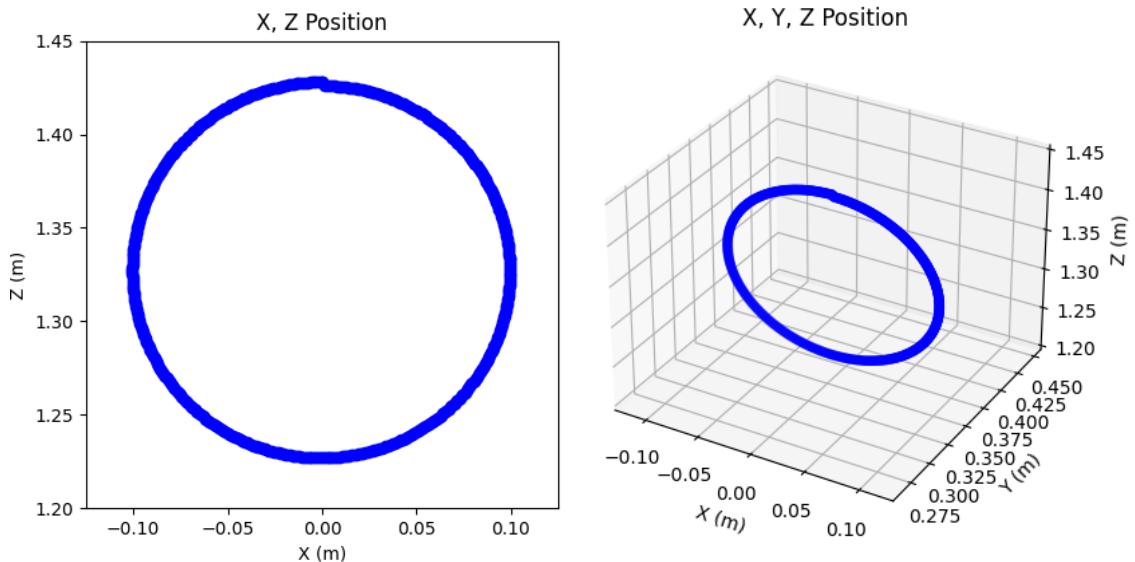
Trajectory Results

As mentioned before the quality of the circular trajectory was dependent on the number of timestamps used over the course of the robot's motion. The images below show the trajectories in 2D and 3D for n=100, 500, and 2000. During the creation of the 3D plots I ran into some issues related to RGBA values and the only way I was able to fix them was to upgrade my matplotlib version. Version 3.7.4 for Python 3.8.10 worked for me.

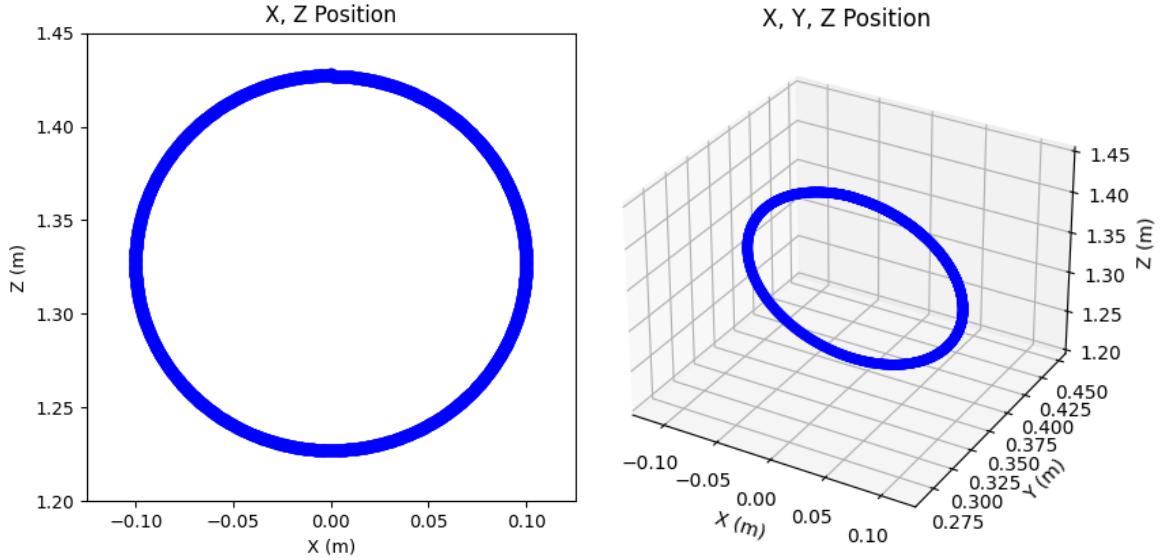
N = 100



N = 500



N = 2000



Appendix

Z and O vectors evaluated symbolic

Jacobian matrix evaluated symbolically

There's simply no way for me to capture this nicely in an image so I'm just pasting the text which isn't great either.

```

[-0.1922·((1.0·sin(θ₀)·sin(θ₁)·sin(θ₂) + 1.0·sin(θ₀)·cos(θ₁)·cos(θ₂))·cos(θ₃) +
(-sin(θ₀)·sin(θ₁)·cos(θ₂) + 1.0·sin(θ₀)·sin(θ₂)·cos(θ₁))·sin(θ₃))·sin(θ₄) +
0.1157·(1.0·sin(θ₀)·sin(θ₁)·sin(θ₂) + 1.0·sin(θ₀)·cos(θ₁)·cos(θ₂))·sin(θ₃) -
0.1157·(-sin(θ₀)·sin(θ₁)·cos(θ₂) + 1.0·s
|
| 0.1922·((1.0·sin(θ₁)·sin(θ₂)·cos(θ₀) + 1.0·cos(θ₀)·cos(θ₁)·cos(θ₂))·cos(θ₃) +
(-sin(θ₁)·cos(θ₀)·cos(θ₂) + 1.0·sin(θ₂)·cos(θ₀)·cos(θ₁))·sin(θ₃))·sin(θ₄) -
0.1157·(1.0·sin(θ₁)·sin(θ₂)·cos(θ₀) + 1.0·cos(θ₀)·cos(θ₁)·cos(θ₂))·sin(θ₃) +
0.1157·(-sin(θ₁)·cos(θ₀)·cos(θ₂) + 1.0·si
|
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0
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1

```

$\sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) + 0.5716 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) +$
 $0.5716 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2) + 0.6127 \cdot \sin(\theta_0) \cdot \cos(\theta_1) + 0.1922 \cdot \cos(\theta_0) \cdot \cos(\theta_4) +$
 $0.1639 \cdot \cos(\theta_0)$

$n(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) + 0.1922 \cdot \sin(\theta_0) \cdot \cos(\theta_4) + 0.1639 \cdot \sin(\theta_0) -$
 $0.5716 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) - 0.5716 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2) - 0.6127 \cdot \cos(\theta_0) \cdot \cos(\theta_1)$

$1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) +$
 $(-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2)$

$$-1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2)$$

$$-1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2)$$

$$) + 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1) \cdot \sin(\theta_3) \cdot \sin(\theta_4) - 0.1157 \cdot (1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1) \cdot \cos(\theta_3) - 0.5716 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) - 0.5716 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2) - 0.61$$

$$- \sin(\theta_2) \cdot \cos(\theta_1) \cdot \cos(\theta_3) \cdot \sin(\theta_4) + 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - 0.5716 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - 0.6127 \cdot \sin(\theta_1) + 0.5716 \cdot \sin(\theta_2) \cdot \cos(\theta_1) \cdot \cos(\theta_0)$$

$$- \sin(\theta_2) \cdot \cos(\theta_1) \cdot \cos(\theta_3) \cdot \sin(\theta_4) + 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - 0.5716 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - 0.6127 \cdot \sin(\theta_1) + 0.5716 \cdot \sin(\theta_2) \cdot \cos(\theta_1) \cdot \sin(\theta_0)$$

$$27 \cdot \sin(\theta_0) \cdot \cos(\theta_1) - 0.1922 \cdot \cos(\theta_0) \cdot \cos(\theta_4) - 0.1639 \cdot \cos(\theta_0) \cdot \sin(\theta_0) + 1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \sin(\theta_3) \cdot \sin(\theta_4) - 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + 1.0 \cdot$$

$$1.0 \cdot \sin(\theta_0)$$

$$-1.0 \cdot \cos(\theta_0)$$

$$0$$

$$\cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2)) + \\ 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_3) + 0.1922 \cdot \sin(\theta_0) \cdot \cos(\theta_4) + 0.1639 \cdot \sin(\theta_0) - \\ 0.5716 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) - 0.5716 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2) - \\ 0.6127 \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_0) - (0.1922 \cdot ((1.0 \cdot \sin(\theta_0) \cdot$$

$$1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot$$

$$1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot$$

$$\sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + \\ 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3) \cdot \sin(\theta_4) - 0.1157 \cdot (1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + \\ 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + \\ 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1))$$

$$) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \\ \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)) \cdot \sin(\theta_4) + 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + \\ 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3) - \\ 0.5716 \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 0.5716 \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos($$

$$) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \\ \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)) \cdot \sin(\theta_4) + 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + \\ 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3) - \\ 0.5716 \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 0.5716 \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\\ \cdot \cos(\theta_3) - 0.5716 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) - 0.5716 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2) - \\ 0.1922 \cdot \cos(\theta_0) \cdot \cos(\theta_4) - 0.1639 \cdot \cos(\theta_0) \cdot \sin(\theta_0) - (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0)$$

$$+ 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot$$

$$-1.0 \cdot \sin(\theta_0)$$

$$1.0 \cdot \cos(\theta_0)$$

$$0$$

$$\theta_0)$$

$$\theta_0)$$

$$\sin(\theta_4) - 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) + 0.1922 \cdot \sin(\theta_0) \cdot \cos(\theta_4) + 0.1639 \cdot \sin(\theta_0) - 0.5716 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) - 0.5716 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta)$$

$$-1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2)$$

$$-1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2)$$

$$0) \cdot 1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_4) - 0.1157 \cdot (1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2))$$

$$+ 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)) \cdot \sin(\theta_4) + \\ 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \\ \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \cos(\theta_0)$$

$$+ 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)) \cdot \sin(\theta_4) + \\ 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \\ \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_0)$$

$$+ 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) - 0.1922 \cdot \cos(\theta_0) \cdot \cos(\theta_4) - 0.1639 \cdot \cos(\theta_0)) \cdot \sin(\theta_0) \\ + 1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + \\ (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_4) - \\ 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_0) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - \\ 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_0)$$

$$1.0 \cdot \sin(\theta_0)$$

$$-1.0 \cdot \cos(\theta_0)$$

$$0$$

$$-(1.0 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0$$

$$(1.0 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0$$

$$\cdot \cos(\theta_2) \cdot \cos(\theta_0) + 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + \\ 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) + 0.1922 \cdot \sin(\theta_0) \cdot \cos(\theta_4) + 0.1639 \cdot \sin(\theta_0)) \cdot \cos(\theta_0) \\ -(-(1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 1.0 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot$$

$$\cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \\ \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + \\ 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + \\ 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_4) - 0.1157 \cdot (1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0$$

$$\cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) - 0.1922 \cdot \cos(\theta_0) \cdot \cos(\theta_4)) + (-1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 1.0 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) - 0.1922 \cdot \cos(\theta_0) \cdot \cos(\theta_4) - 0.1157 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 1.0 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) - 0.1922 \cdot \cos(\theta_0) \cdot \cos(\theta_4))$$

$$\begin{aligned} & \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + \\ & 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) + 0.1922 \cdot \sin(\theta_0) \cdot \cos(\theta_4)) - \\ & (-1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + \\ & 1.0 \cdot (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) \end{aligned}$$

$$n(\theta_3) + 0.1157 \cdot (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_3) + \\ 0.1922 \cdot \sin(\theta_0) \cdot \cos(\theta_4)) + (-1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + \\ 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 1.0 \cdot (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + \\ 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)) \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_0)) \cdot s$$

$$-(1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + \\ 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 1.0 \cdot (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + \\ 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)$$

$$-(1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) \\ + 1.0 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)$$

$$1.0 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2) \cdot \cos(\theta_3)) \\ - (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)$$

$$\theta_3)) \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)) \cdot \sin(\theta_4) + 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)))$$

$$\theta_3)) \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \cos(\theta_3)) \cdot \sin(\theta_4) + 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) - 0.1157 \cdot (1.0 \cdot \sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)))$$

$$\begin{aligned} & \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + \\ & 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_4) - 0.1157 \cdot (1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + \\ & 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \sin(\theta_3) + 0.1157 \cdot (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + \\ & 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1))) \cdot \end{aligned}$$

$$-(0.1922 \cdot ((1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + (-\sin(\theta_0) \cdot \sin(\theta_1) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_0) \cdot \sin(\theta_2) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_4) - 0.1922 \cdot \cos(\theta_0) \cdot \cos(\theta_4)$$

$$1.0 \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) + (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_4) + 0.1922 \cdot \sin(\theta_0) \cdot \cos(\theta_4))$$

$$\begin{aligned} &_0) \cdot \cos(\theta_4)) \cdot (0.1922 \cdot ((1.0 \cdot \sin(\theta_1) \cdot \sin(\theta_2) \cdot \cos(\theta_0) + 1.0 \cdot \cos(\theta_0) \cdot \cos(\theta_1) \cdot \cos(\theta_2)) \cdot \cos(\theta_3) \\ &+ (-\sin(\theta_1) \cdot \cos(\theta_0) \cdot \cos(\theta_2) + 1.0 \cdot \sin(\theta_2) \cdot \cos(\theta_0) \cdot \cos(\theta_1)) \cdot \sin(\theta_3)) \cdot \sin(\theta_4) + \\ &0.1922 \cdot \sin(\theta_0) \cdot \cos(\theta_4)) \Big| \end{aligned}$$

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