Forest Growth Model

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Background

Consider the following model of forest growth (where forest size is measured in units of carbon (C)):

$$dC/dt = r * C$$

for forest where **C is below a threshold canopy closure** and

$$dC/dt = g * (1 = C/K)$$

for forests where **carbon is at or above the threshold canopy closure**

and K is a carrying capacity in units of Carbon. The size of the forest (C), canopy closure threshold and carrying capacity are all in units of carbon. Think of the canopy closure threshold as the size of the forest at which growth rates change from exponential to linear. Think of r as the early exponential growth rate and g as the linear growth rate once canopy closure has been reached.

C = forest size, units of carbon T = time, years r = early exponential growth rate (before canopy closure has been reached) g = the linear growth rate once canopy closure has been reached K = carrying capacity, units of carbon

canopy closure threshold = the size of the forest at which growth rates change from exponential to linear

Implement Model

```
source(here("R/forest_growth.R"))
forest_growth
```

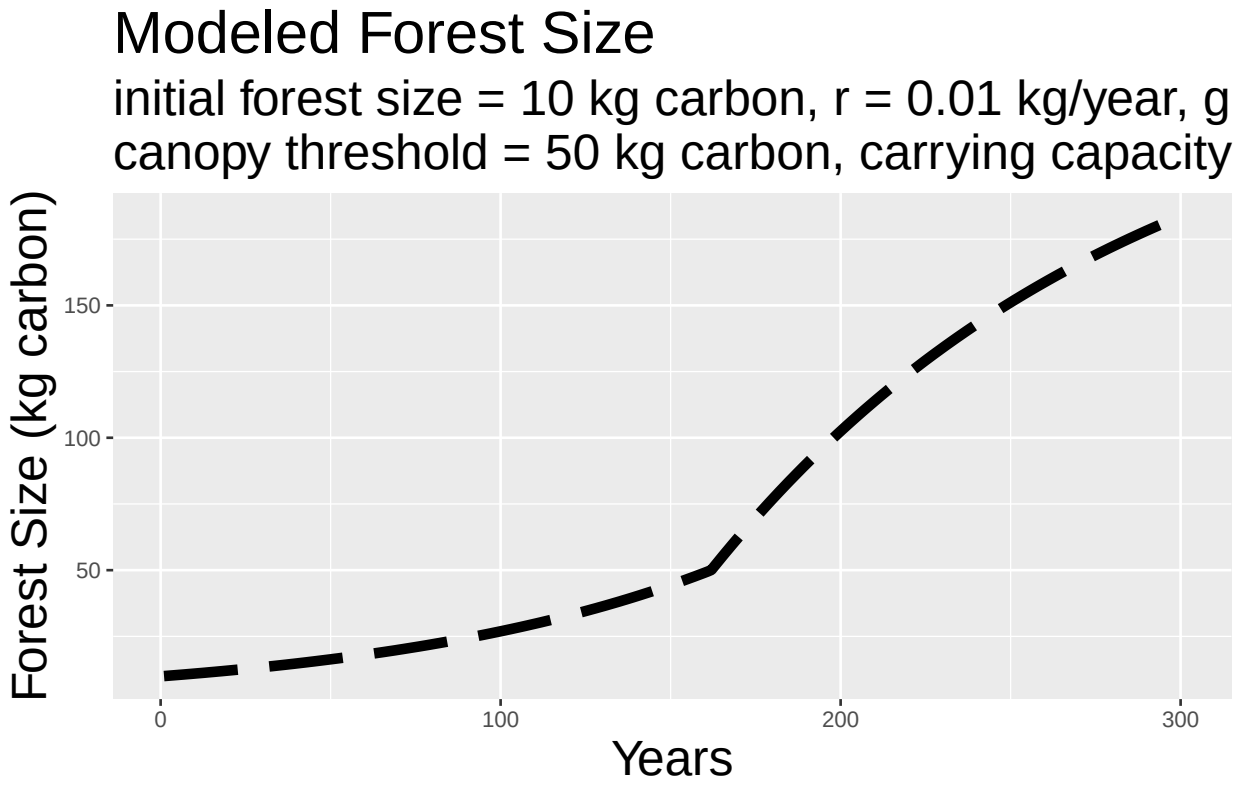
```
## function (time, C, parms)
## {
##     dC = ifelse(C < parms$canopy_threshold, parms$r * C, parms$g *
##         (1 - (C/parms$K)))
##     return(list(dC))
## }
```

Run model over 300 years using the ODE solver (`deSolve` package) starting with initial forest size of 10 kg carbon with the following parameters:

- canopy_threshold = 50 kg (canopy closure threshold)
- K = 250 kg C (carrying capacity)
- r = 0.01 (exponential growth rate before canopy closure)
- g = 2 kg/year (linear growth rate after canopy closure)

```
years = seq(from = 1, to = 300)

Cinitial = 10

metrics = list(K=250, r=0.01, g=2, canopy_threshold=50)

carbon_growth = ode(y = Cinitial,
                    times = years,
                    func = forest_growth,
                    parms = metrics)

colnames(carbon_growth)=c("year","carbon")
```

```
ggplot(as.data.frame(carbon_growth),aes(year, carbon))+
  geom_line(size = 2, linetype = "longdash") +
  labs(title = "Modeled Forest Size",
       subtitle = paste0("initial forest size = 10 kg carbon, r = 0.01 kg/year, g = 2 kg/year, \ncanopy
       x = "Years",
       y = "Forest Size (kg carbon)") +
  theme(title = element_text(size = 20))
```



Run a Sobel sensitivity analysis and explore how the estimated max and mean forest size varies with the pre-canopy closure growth rate (r), post-canopy closure rate (g), canopy closure threshold, and carrying capacity (K).

Assumption is all parameters are normally distributed with a standard deviation of 10% of mean values.

```
# set number of parameters
n = 300

K = rnorm(mean=250, sd=25, n=n)
r = rnorm(mean=0.01, sd=0.001, n=n)
g = rnorm(mean=2, sd=0.2, n=n)
canopy_threshold = rnorm(mean=50, sd=0.1*50, n=n)

X1 = cbind.data.frame(r=r, K=K, g=g, canopy_threshold=canopy_threshold)

# repeat to get our second set of samples
K = rnorm(mean=250, sd=25, n=n)
r = rnorm(mean=0.01, sd=0.001, n=n)
g = rnorm(mean=2, sd=0.2, n=n)
canopy_threshold = rnorm(mean=50, sd=0.1*50, n=n)

X2 = cbind.data.frame(r=r, K=K, g=g, canopy_threshold=canopy_threshold)


# create our sobel object and get sets of parameters for running the model
sens_C = sobolSalt(model = NULL, X1,X2, nboot = 300)

# rename columns
colnames(sens_C$X) = c("r", "K", "g", "canopy_threshold")
head(sens_C$X)
```

```
##                r        K        g canopy_threshold
## [1,] 0.010359401 221.8921 1.725335         42.97691
## [2,] 0.010340308 245.3222 1.978029         49.29052
## [3,] 0.009351790 265.5821 1.786866         39.73651
## [4,] 0.009530740 290.9793 1.979978         59.41202
## [5,] 0.008162389 215.1124 2.157133         51.57235
## [6,] 0.009540541 234.5274 1.686417         47.28016
```

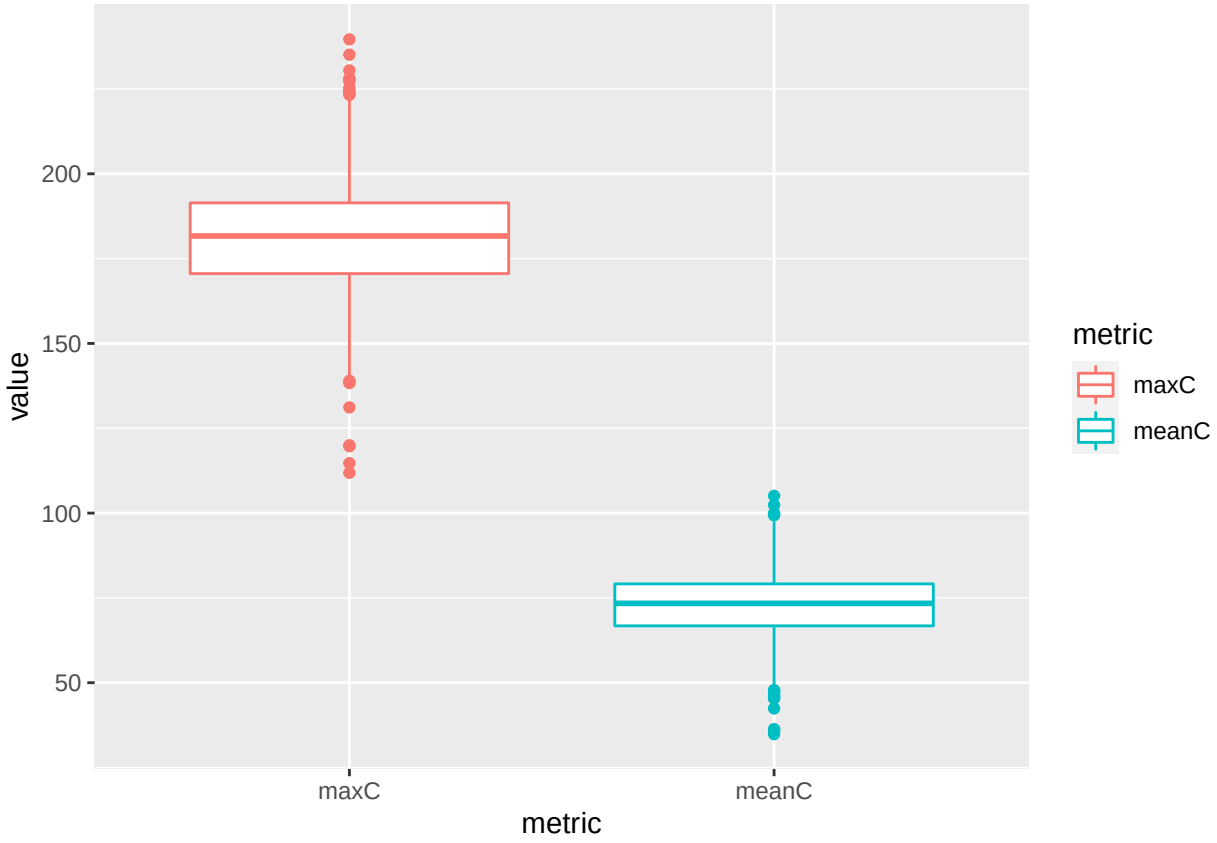
Create two functions to compute metrics. Then, run the ODE solver to compute metrics with wrapper function.

```
# turn computing our metrics into a function
compute_metrics = function(result) {
  maxC = max(result$carbon)
  idx1 = which.max(result$carbon)
  maxyear = result$year[idx1]

  meanC = mean(result$carbon)
```

```
return(list(maxC=maxC,
            maxyear=maxyear,
            meanC=meanC))}

# wrapper function
p_wrapper <- function(r, g, canopy_threshold, K, Cinitial, years, func) {

  parms <- list(r = r, g = g, canopy_threshold = canopy_threshold, K = K)
  result <- ode(y = Cinitial,
                times = years,
                func = func,
                parms = parms,
                method = "lsode")
  colnames(result) <- c("year", "carbon")

  # get metrics
  metrics <- compute_metrics(as.data.frame(result))

  return(metrics)

}
```

```
# now use pmap as we did before
allresults <- as.data.frame(sens_C$X) %>%
  pmap(p_wrapper, Cinitial=Cinitial, years=years, func=forest_growth)

# extract out results from pmap into a data frame
allres <-  allresults %>%
  map_dfr(`[`,c("maxC","meanC"))
```

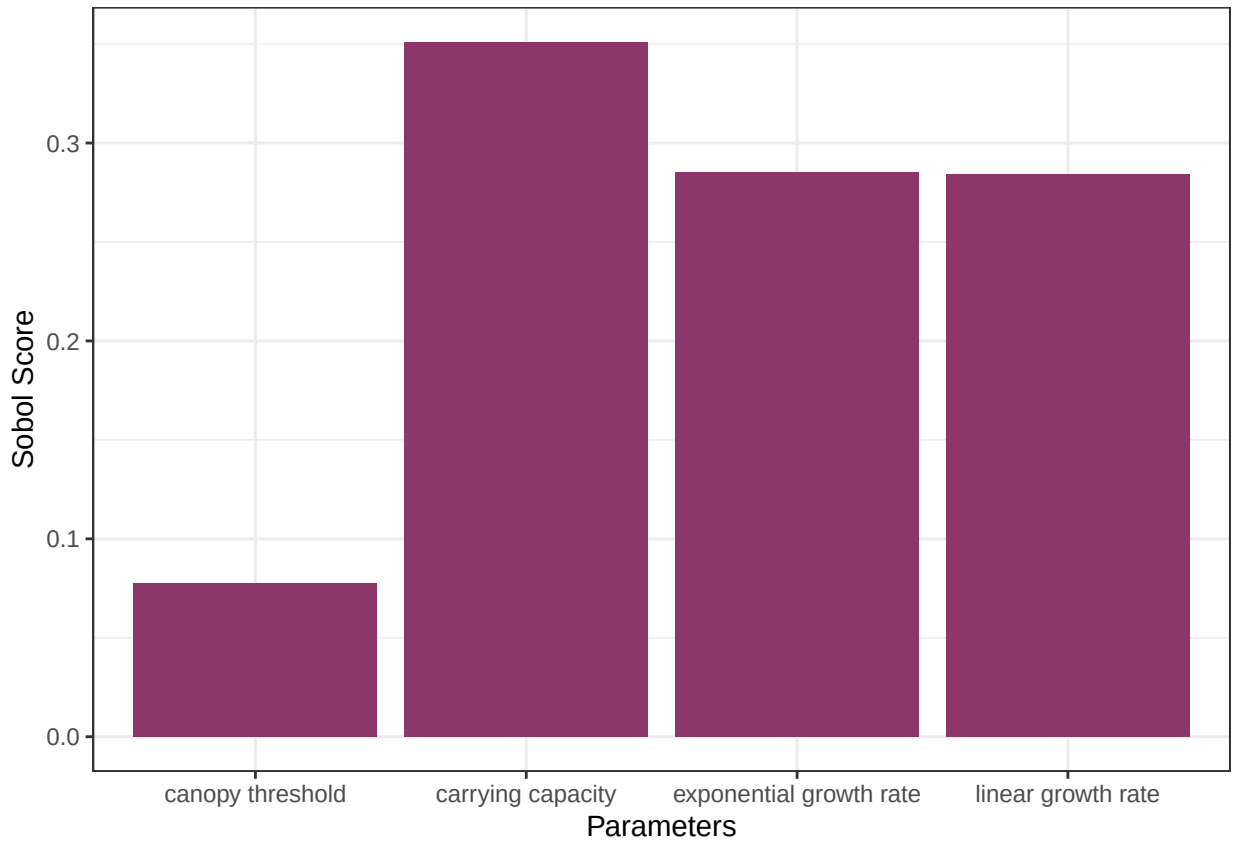
```
# create boxplots
tmp <- allres %>%
  pivot_longer(cols = 1:2,
               names_to = "metric",
               values_to = "value")


ggplot(tmp, aes(metric, value, col=metric)) +
  geom_boxplot()
```



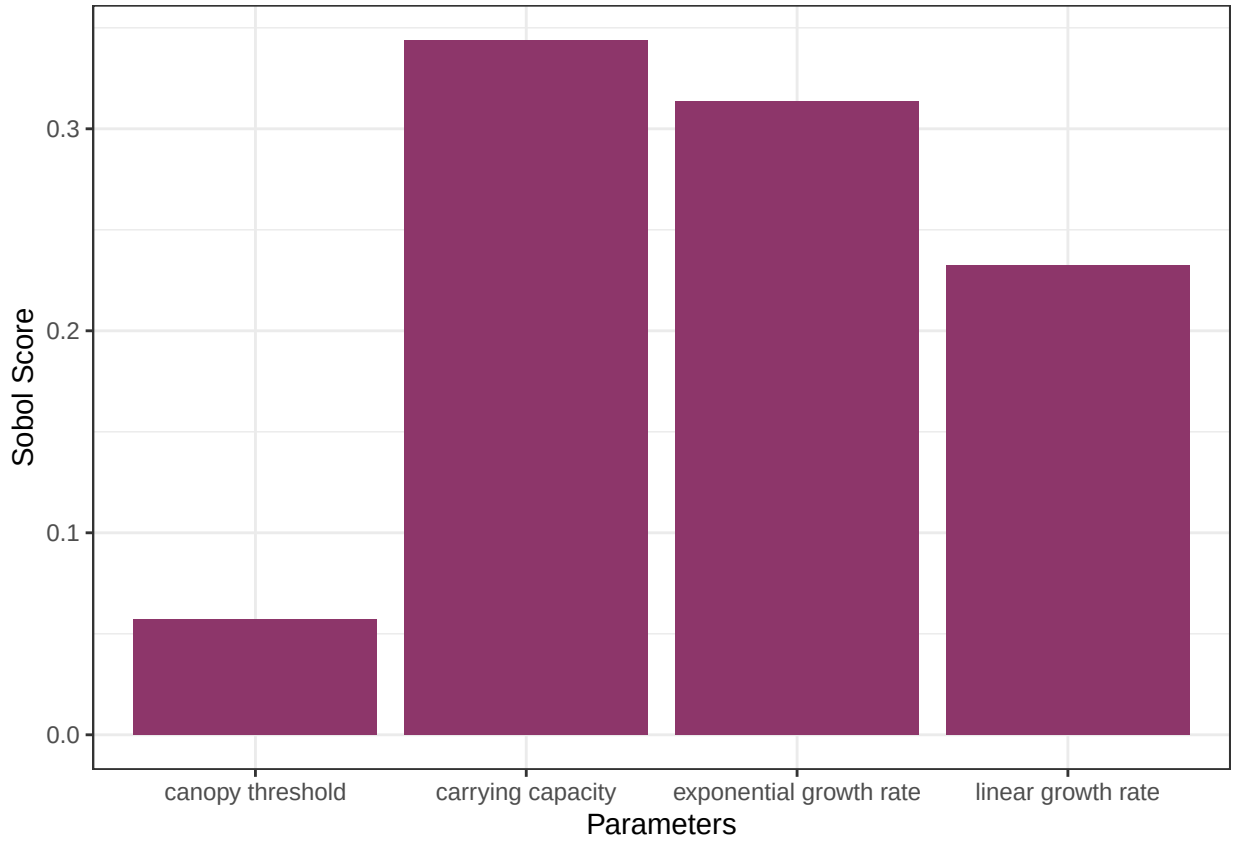
Compute the sobol indicies for each metric

```
# sobol can only handle one output at a time  - so we will need to do them separately

# compute sobol indicies for maxC
sens_C_maxC = sensitivity::tell(sens_C, allres$maxC)

# prepare data frames for plotting maxC first order indices and total indices
# first order indices
sens_C_maxC_S <- sens_C_maxC$S %>%
  rowid_to_column("parameters") %>%
  mutate(parameter = case_when(parameters == "1" ~ "exponential growth rate",
                               parameters == "2" ~ "carrying capacity",
                               parameters == "3" ~ "linear growth rate",
                               parameters == "4" ~ "canopy threshold"))

# total indices
sens_C_maxC_T <- sens_C_maxC$T %>%
  rowid_to_column("parameters") %>%
  mutate(parameter = case_when(parameters == "1" ~ "exponential growth rate",
                               parameters == "2" ~ "carrying capacity",
                               parameters == "3" ~ "linear growth rate",
                               parameters == "4" ~ "canopy threshold"))

# compute sobol indicies for meanC
sens_C_meanC = sensitivity::tell(sens_C, allres$meanC)
```

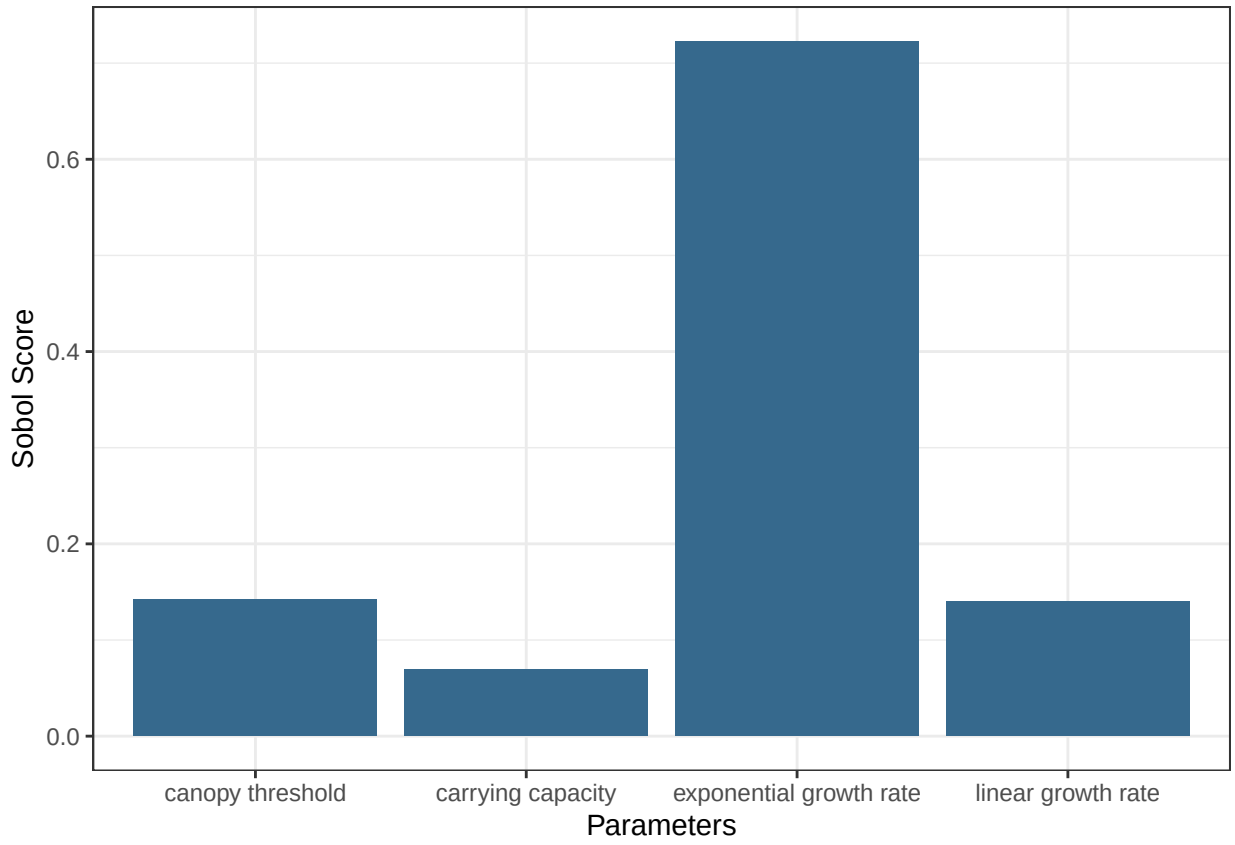
```r
# prepare data frames for plotting meanC first order indices and total indices
# first order indices
sens_C_meanC_S <- sens_C_meanC$S %>%
  rowid_to_column("parameters") %>%
  mutate(parameter = case_when(parameters == "1" ~ "exponential growth rate",
                               parameters == "2" ~ "carrying capacity",
                               parameters == "3" ~ "linear growth rate",
                               parameters == "4" ~ "canopy threshold"))

# total indices
sens_C_meanC_T <- sens_C_meanC$T %>%
  rowid_to_column("parameters") %>%
  mutate(parameter = case_when(parameters == "1" ~ "exponential growth rate",
                               parameters == "2" ~ "carrying capacity",
                               parameters == "3" ~ "linear growth rate",
                               parameters == "4" ~ "canopy threshold"))

# plot maxC first order indices
ggplot(data = sens_C_maxC_S, aes(x = parameter, y = original))+
  geom_col(fill = "#8D366A")+
  theme_bw()+
  labs(x = "Parameters",
       y = "Sobol Score")
```



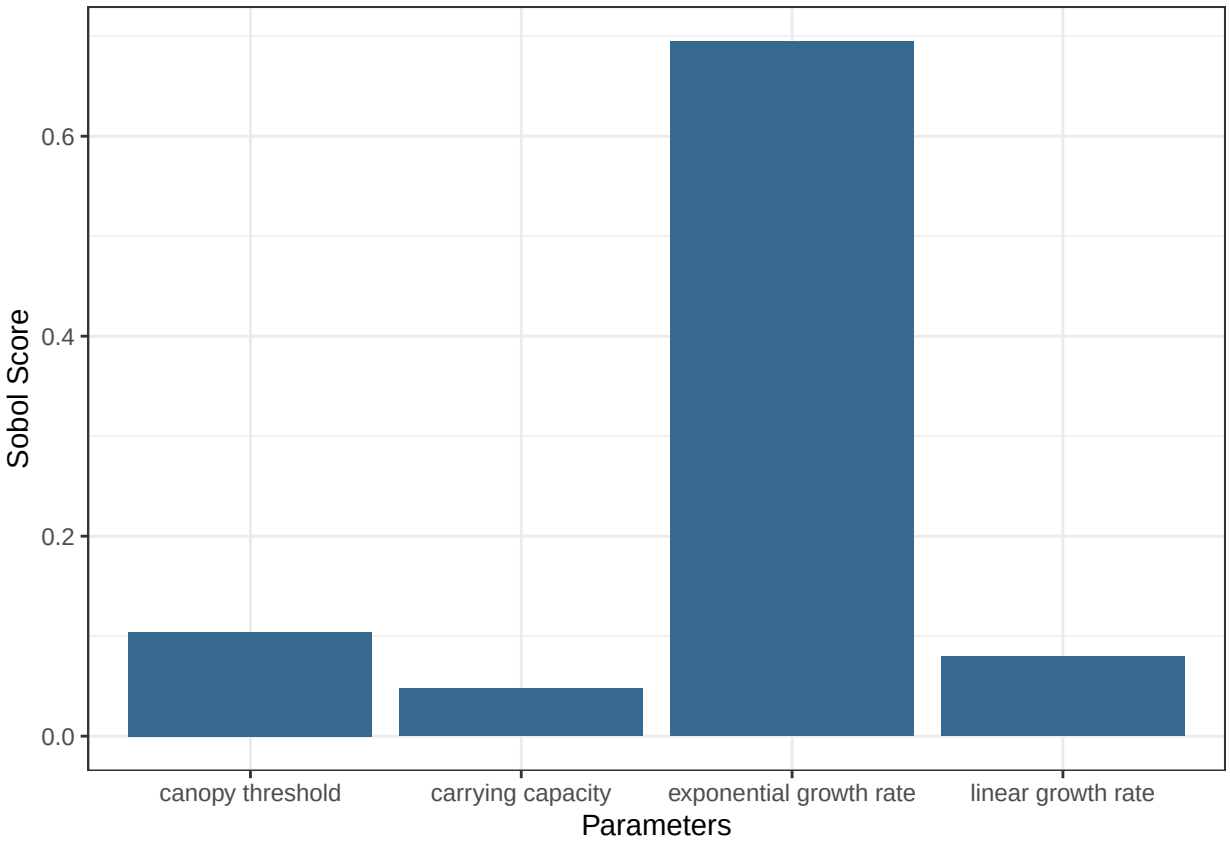
```
# plot maxC total indices
ggplot(data = sens_C_maxC_T, aes(x = parameter, y = original))+
  geom_col(fill = "#8D366A")+
  theme_bw()+
  labs(x = "Parameters",
       y = "Sobol Score")
```




```
# plot meanC first order indices
ggplot(data = sens_C_meanC_S, aes(x = parameter, y = original))+
  geom_col(fill = "#36698D")+
  theme_bw()+
  labs(x = "Parameters",
       y = "Sobol Score")
```




```
# plot meanC total indices
ggplot(data = sens_C_meanC_T, aes(x = parameter, y = original))+
  geom_col(fill = "#36698D")+
  theme_bw()+
  labs(x = "Parameters",
       y = "Sobol Score")
```



In 2-3 sentences, discuss what the results of your simulation might mean for climate change impacts on forest growth (e.g think about what parameters climate change might influence).