Now since we have an "intuitive" understanding of the double integral learned in section 15.1, this section aims to cover the "computational" process for solving double integrals.

As the previous section mentioned, a double integral representing volume is simply the sum of areas of an infinite amount of rectangles in the x-direction multiplied by the sum of areas of an infinite amount of rectangles in the y-direction. As you may expect, the process is similar to 2d-integrals and anti-partial derivatives–except their is no such thing as a partial integral (even though the intuitive understanding of them seems to point in that direction).

We are iterating over a rectangle R = [a, b] x [c, d] where [a, b] represents the bounds in the x-direction and [c, d] represents the bounds in the y-direction. By definition, either boundary can come first since it is multiplying as noted by Fubini's Theorem:

Fubini's Theorem *R = [a, b] x [c, d] where [a, b] represents the bounds in the x-direction and [c, d] represents the bounds in the y-direction.*

$$\int f(x,y)dA = \int_c^d \int_a^b f(x,y)dydx = \int_a^b \int_c^d f(x,y)dydx$$

To computationally solve, start by boxing the inner integral and its respective differential:

$$\int_a^b \int_c^d f(x,y)dydx --> \int_a^b [\int_c^d f(x,y)dy]dx$$

Now we have a single-variable integral with a double-variable function. To overcome this, we will treat the other variable (the one that is on the outside) as a constant. In this case, x will become constant. Now we the definite single-variable integral which will give a function involving only x's which we can integrate like a single-variable integral.

Essentially, the goal is to get the first integral in terms of only one variable (the other variable). If you do not end up with that, then you are doing something wrong.

This is pretty much all this lesson covers but it is important to note that the problems can be asked in multiple different ways, yet the answer, and the intuitive understanding, is the same. Thus, we will go over those variations here:

- **Standard Variation:**

$$\int_a^b \int_c^d f(x,y)dydx$$

- **Over rectangle:**

$$\int\int f(x,y)dA, R = \{(x,y)|a \le x \le b, c \le y \le d\} --> \int_c^d \int_a^b f(x,y)dxdy$$

  Also noted as:

$$\int\int f(x,y)dA, R = [a,b] * [c,d] --> \int_c^d \int_a^b f(x,y)dxdy$$

Furthermore, it is important to realize a possible ”shortcut”:

Multiplication Shortcut *If f(x, y) = z, a function of two variables, is in the form where each variable multiplied together separately (each variable cannot be a coefficient of one another), then the two integrals can be split respectively:*

$$\int_a^b \int_c^d (x)(y)dydx -- > \left\{\int_a^b (x)dx\right\} * \left\{\int_c^d (y)dy\right\}$$

however,

$$\int_a^b \int_c^d (xy)(yx)dydx \neq \left\{\int_a^b (x^2)dx\right\} * \left\{\int_c^d (y^2)dy\right\}.$$