

Over-parameterized Optical Flow using a Stereoscopic Constraint

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Contribution

- We present a new over-parameterized model for optical flow estimation for static scenes in the case of uncalibrated cameras.
- The parameterization stems from planar homographies.
- Results show high accuracy in scenarios conforming to the model assumptions.
- The regularization coefficient is closely related to the measure of plane boundaries in the $2\frac{1}{2}$ -D sketch.

Overparameterized Optical Flow

In [2] an over-parameterized approach for optical flow estimation is presented. The flow field between images is locally described as an over-complete set of vector fields,

$$[u(x, y; \mathbf{a}) \ v(x, y; \mathbf{a})]^T = \sum_i a_i(x, y) \phi_i(x, y);$$

The regularization term is defined in terms of the model coefficients a_i ,

$$E_{S,TV} = \int \psi_S \left(\sum_i \|\nabla a_i\|^2 \right)$$

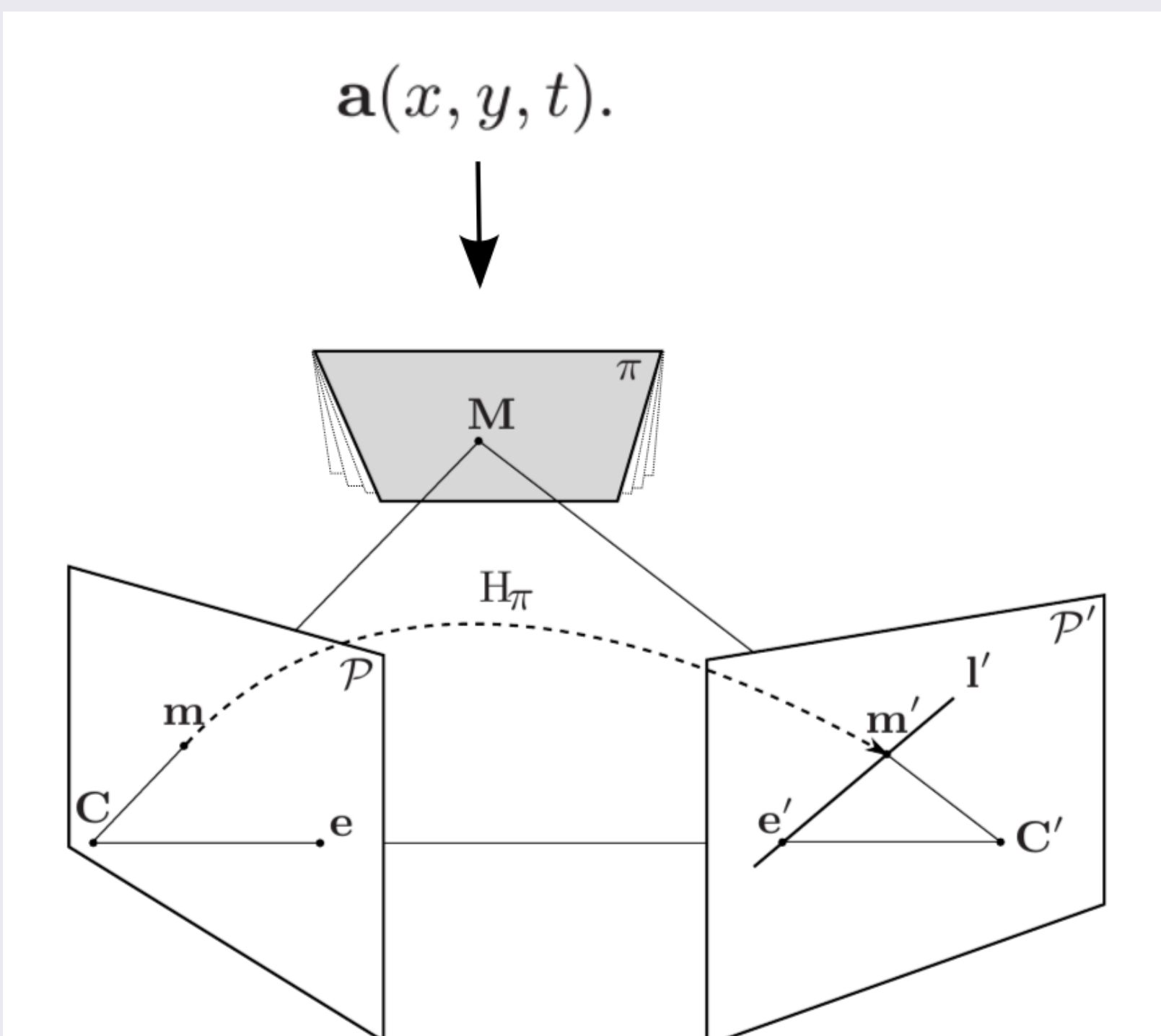
The Model

We extend the overparameterized model in the case of static scenes to a nonlinear parameter model.

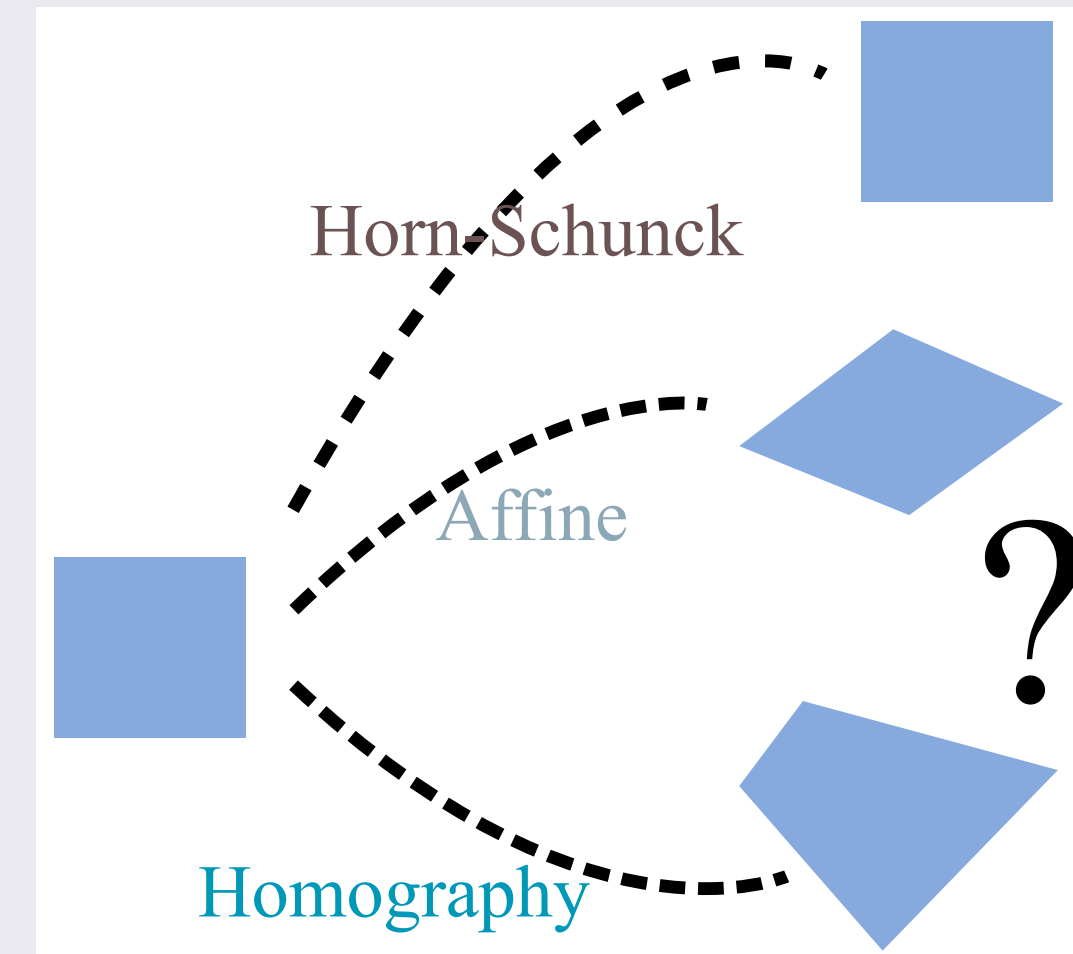
- In this model, the transformation between the images is locally a homography defined by the scene plane.
- Given 2 viewpoints of the scene and a local planar patch, each point is transferred to a point on the corresponding epipolar line.
- The set of homographies consistent with the fundamental matrix has 3 degrees of freedom, defining our model.
- We parameterize these homographies by

$$H(x, y, t) = H_0 + e' \mathbf{a}(x, y, t), \quad H_0 = [e'] \times \mathcal{F}.$$

- No need for metric calibration.



- The proposed parameterization better suits 3D scene understanding.



The Functional

- We further extend the smoothness term to a generalized Ambrosio-Tortorelli scheme [1].
- The smoothness term encourages solutions that are piecewise smooth in terms of local planar coefficients.
- The functional over \mathbf{a} and the diffusivity v_{at} reads

$$E_D + \alpha \left(E_{S,AT} + \int \frac{(v_{AT} - 1)^2}{4\epsilon} + \epsilon |\nabla v_{AT}|^2 \right),$$

$$\text{with } E_{S,AT} = \int v_{AT}^2 \psi_S \left(\sum_i \|\nabla a_i\|^2 \right),$$

- and the E_D enforcing the brightness constancy term,

$$E_D = \int \Psi(I_z^2), \quad I_z = I(x+u, y+v, t+1) - I(x, y, t)$$

- The flow is a nonlinear function of the parameters $\mathbf{a}(x, y, t)$.

A Motivating Thought Experiment

- Consider a piecewise-planar static scene.
- Assume a solution \mathbf{a}^* with correct plane coefficients at each interior plane point.
- Assume a diffusivity function such that $v_{AT}^* = 1$ at interior points and $v_{AT}^* = 0$ at boundary points
- The cost of the smoothness term converges with $\epsilon \rightarrow 0$ to a measure of discontinuities in the $2\frac{1}{2}$ -D sketch.

References

- [1] L. Ambrosio and V. M. Tortorelli. Approximation of functional depending on jumps by elliptic functional via Γ -convergence. *Communications on Pure and Applied Mathematics*, 43(8):999–1036, 1990.
- [2] T. Nir, A. M. Bruckstein, and R. Kimmel. Over-parameterized variational optical flow. *International Journal of Computer Vision*, 76(2):205–216, 2008. ISSN 0920-5691.

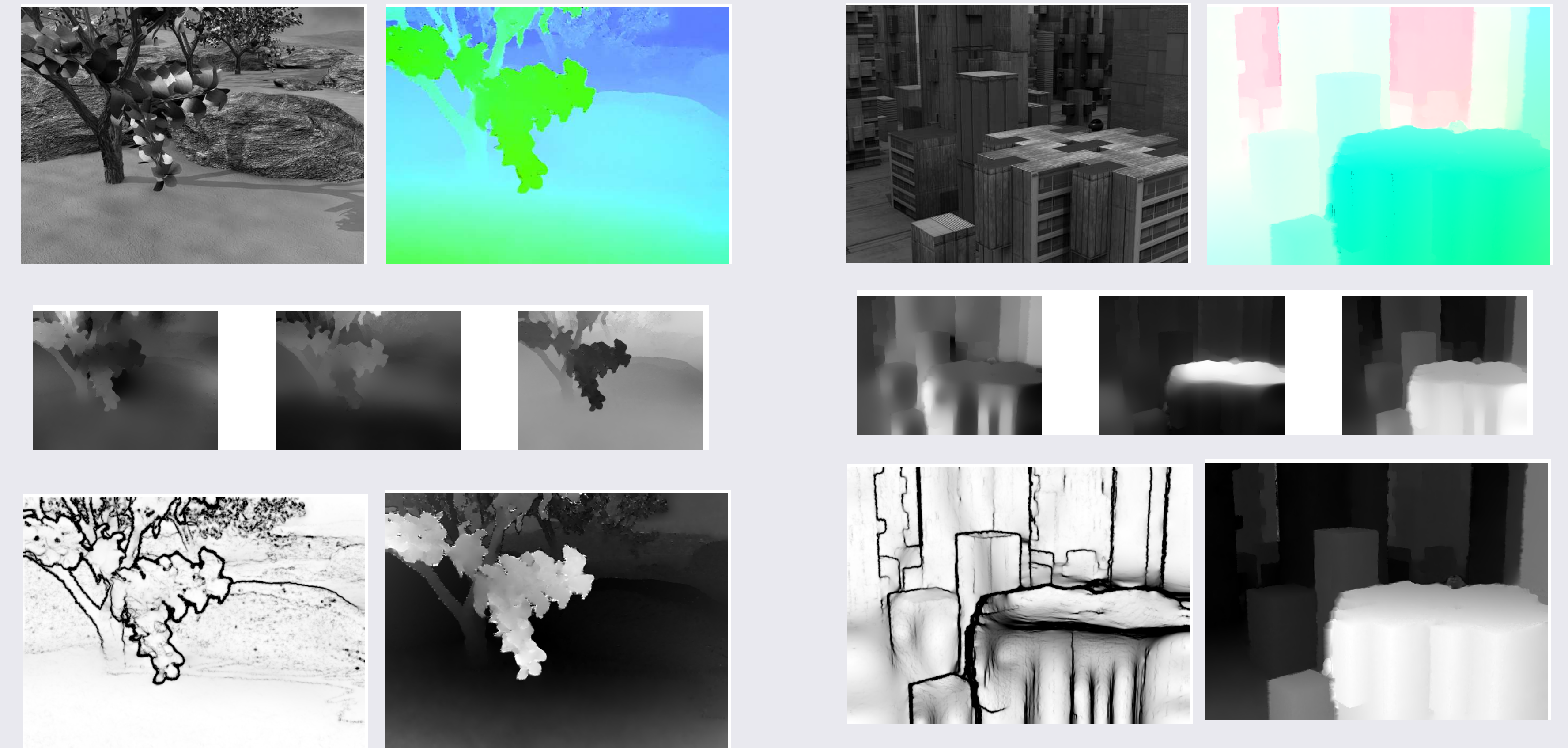
Results - Middlebury Test Set

Average angle error	Grove (Synthetic)			Urban (Synthetic)			Yosemite (Synthetic)			Teddy (Stereo)		
	GT	im0	im1	GT	im0	im1	GT	im0	im1	GT	im0	im1
	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext
MDP-Flow2 [40]	2.87 5	3.73 5	2.32 4	3.15 2	11.1 4	2.65 3	2.04 5	3.64 8	1.60 5	1.88 5	4.49 8	1.49 2
Layers++ [38]	2.35 1	3.02 1	1.96 1	3.81 7	11.4 5	3.22 10	2.74 17	4.01 21	2.35 18	1.45 2	3.05 1	1.79 4
LSM [41]	2.82 3	3.68 3	2.36 5	3.38 4	9.41 2	2.81 6	2.69 16	3.52 8	2.84 23	1.59 3	3.38 3	1.80 5
Classic+NL [31]	2.83 4	3.68 3	2.31 3	3.40 5	9.09 1	2.76 5	2.87 21	3.82 12	2.86 28	1.67 4	3.53 4	2.26 8
MDP-Flow [26]	3.03 6	3.87 6	2.60 11	3.43 6	12.6 8	2.81 6	2.19 8	3.88 15	1.60 5	4.13 21	9.96 22	3.86 25
Proposed Method	4.60 40	5.05 36	5.52 42	2.38 1	11.5 6	1.77 1	1.25 1	2.92 1	0.71 1	4.49 23	10.3 25	4.23 27

Average angular errors (AAE) results for static scenes in the Middlebury test set. Bold numbers represent best results for the test scene.

Results - Middlebury Training Set

We demonstrate results of the optical flow estimation on the Middlebury test and training datasets.



At each subfigure, left to right, top to bottom: The scene itself, color coded flow field; the motion parameters; the diffusivity function and the disparity norm. Results are shown for the Middlebury Grove2 and Urban2 training sequences.

Middlebury Training Set			Yosemite Sequence			
	AAE	STD	Method	AAE	Method	AAE
Grove2	2.41	7.16	Brox , '04	1.59	Roth/Black '08	1.43
Grove3	5.53	15.76	Mémin/Pérez, '02	1.58	Valgaerts , '08	1.17
Urban2	2.15	9.22	Bruhn '05	1.46	Nir , '08	1.15
Urban3	3.84	16.88	Amiaz '07	1.44	Our method	0.85
Venus	4.29	12.01				
Yosemite	0.85	1.24				

AAE results for static Middlebury Training set sequences.

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