A New Physically Motivated Warping Model for Form Drop-Out

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The Challenge

- Documents scanned by sheet-fed scanners often exhibit distortions due to the feeding and scanning mechanisms
- Many applications require registration to a given template

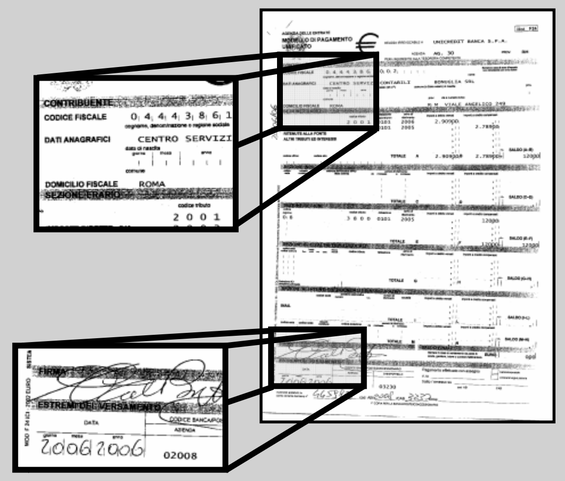
The Solution

- A novel warping model that enables a robust and accurate registration process

Problem Illustration

When the form advances at an uneven rate, it may cause a rotation – like distortion

The right side of the form progresses faster than the left side



Problem Definition

Given a set of n point pairs

$$(\mathbf{p}_i, \mathbf{q}_i), \mathbf{p}_i \in \mathbb{R}^2,\ \mathbf{q}_i \in \mathbb{R}^2, i = 1 \ldots n$$

Output a continuous function $f : \mathbb{R}^2 \to \mathbb{R}^2$

Such that $\forall i = 1 \ldots n \quad \{ f(\mathbf{p}_i) \}$

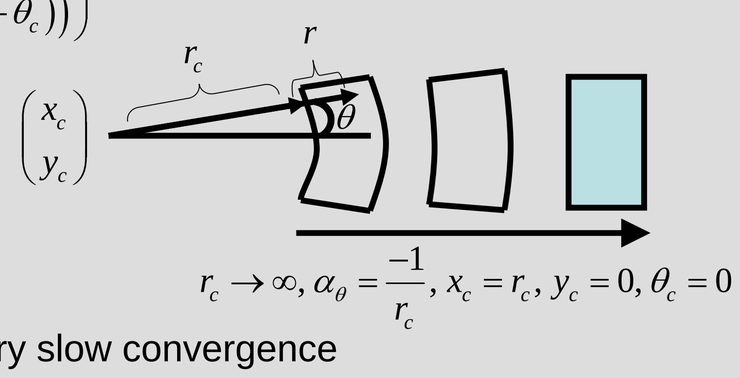
Best approximates $\{\mathbf{q}_i\}$ in some sense

And f is well behaved in some sense

Polar Model

Approximates the deformations caused by the feeding and scanning process

$$\begin{pmatrix} \tilde{x}(r,\theta) \\ \tilde{y}(r,\theta) \end{pmatrix} = \begin{pmatrix} x_c \\ y_c \end{pmatrix} + \alpha_r (r - r_c) \begin{pmatrix} \cos(\alpha(\theta - \theta_c)) \\ \sin(\alpha(\theta - \theta_c)) \end{pmatrix}$$



$$r_c \to \infty, \alpha_\theta = \frac{-1}{r_c}, x_c = r_c, y_c = 0, \theta_c = 0$$

The shortcoming of this model is a very slow convergence

Approximated Polar Model

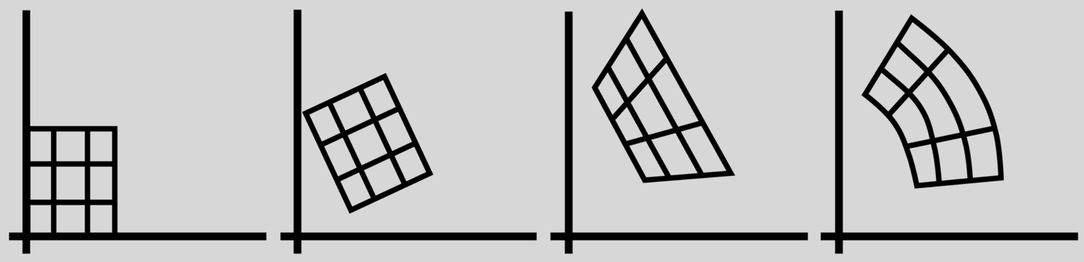
- Looking for a model that converges faster, we use the following approximation of the polar model:

$$T_{\mathbf{P}} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{R}_\theta \mathbf{S} \mathbf{P}_{\alpha,\beta} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \Delta x_c \\ \Delta y_c \end{pmatrix}$$

- Where $\mathbf{R}_\theta$ is a rotation matrix, $\mathbf{S}$ is a scale matrix and $\mathbf{P}_{\alpha,\beta}$ operates as follows:

$$\mathbf{P}_{\alpha,\beta} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x(1 + \alpha(y - K)^2) \\ y(1 + \beta x) \end{pmatrix}$$

- From left to right: the truncated polar model can be viewed as a shift, scale, and rotation operation along a trapezoid and parabolic distortion



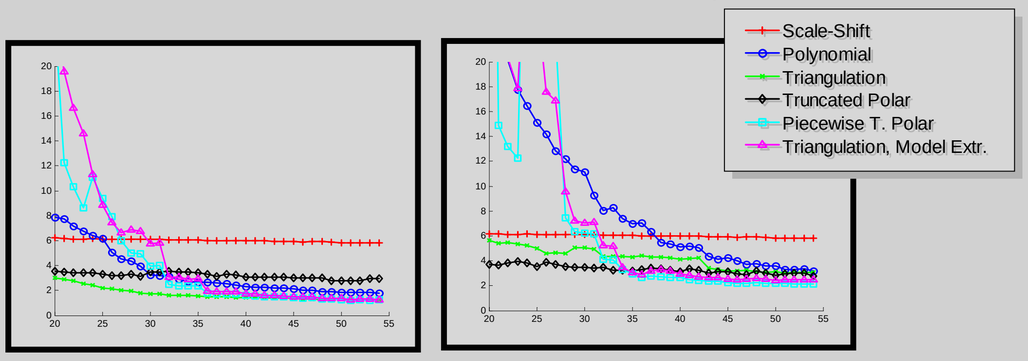
- To see the relation between this model and the polar model, we look at the case of a small distortion caused by a large radius and small angular proportional factor
- Using the Taylor series expansion for the trigonometric functions, we obtain the approximated polar model, up to the redefinition of constants

Piecewise Approximated Polar Model

- In practice, the speed at which the paper advances with respect to the feeder can change during the scan, due to stains or paper elasticity, folds, and wrinkles that affect friction
- The model is naturally extended into a compound model, consisting of a handful of regions, where each region is modeled separately

Results

- Registration results:
  - From left to right: the resulting image de-warping using the scale and shift, triangulation, polynomial, truncated polar, and piecewise polar models.
  - White and black pixels denote pixels in which the template and the de-warped form agree
  - Blue indicates what is black in the form and white in the template
  - Red indicates what is white in the form and black in the template



Without noise — With noise ($\sigma^2 = 4$)

- Estimation results:
  - Around 80 point correspondences were available, of which we use between 20 to 55 points for training. The remaining points are estimated using the model. The RMS values of their errors are displayed in the graphs.
  - In the right graph, we show the model accuracy when Gaussian noise is added to the coordinates of the point matches. The truncated polar model reaches a low RMS even with a small number of correspondences.

Applications

- De-warping deformed forms for drop-out purposes
- Registration regulation – evaluating registration likelihood
- Model-based registration (e.g., RANSAC)

Current status

- This model is currently in use as part of an Italian tax form automation project

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