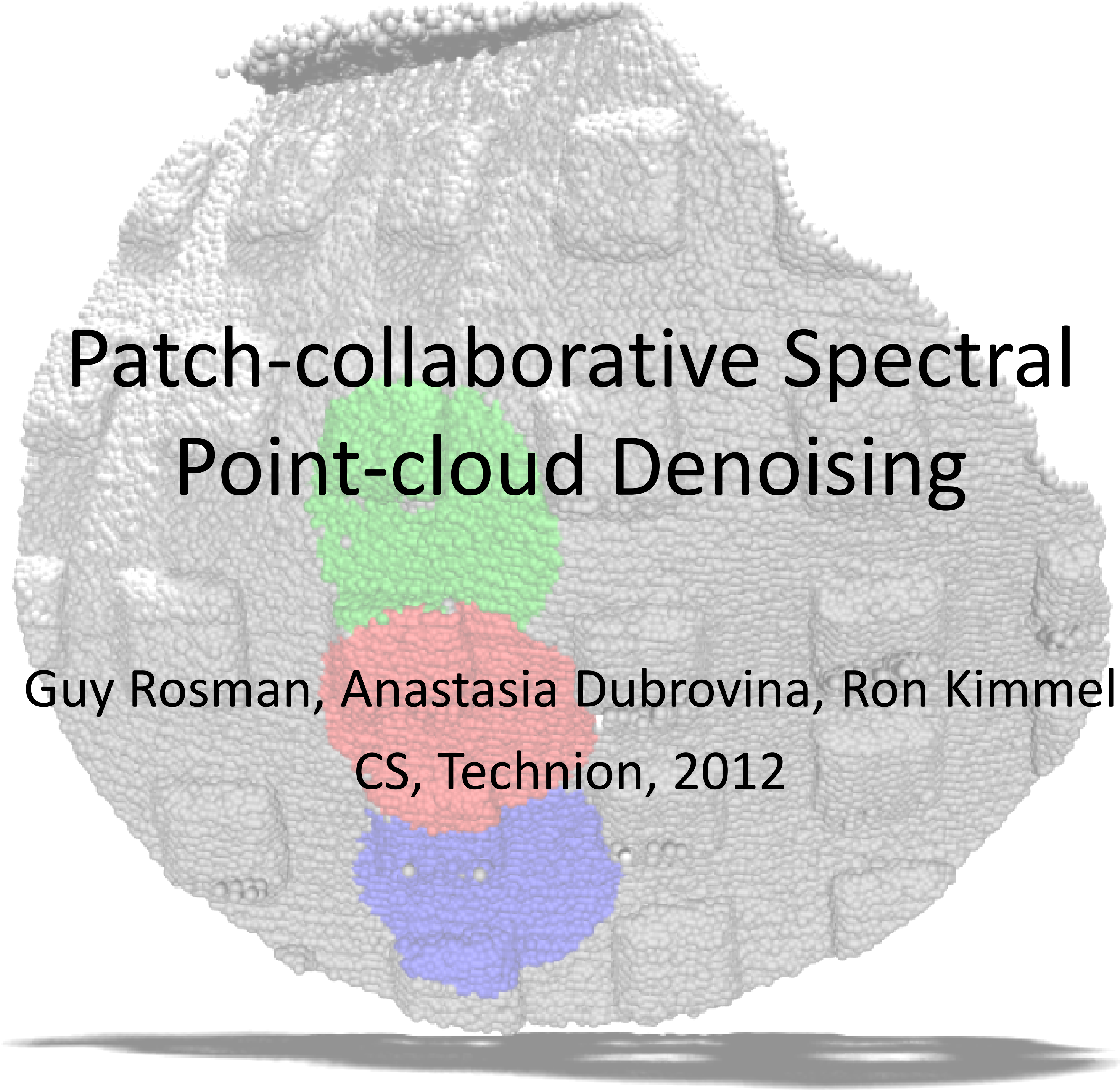
Patch-collaborative Spectral Point-cloud Denoising

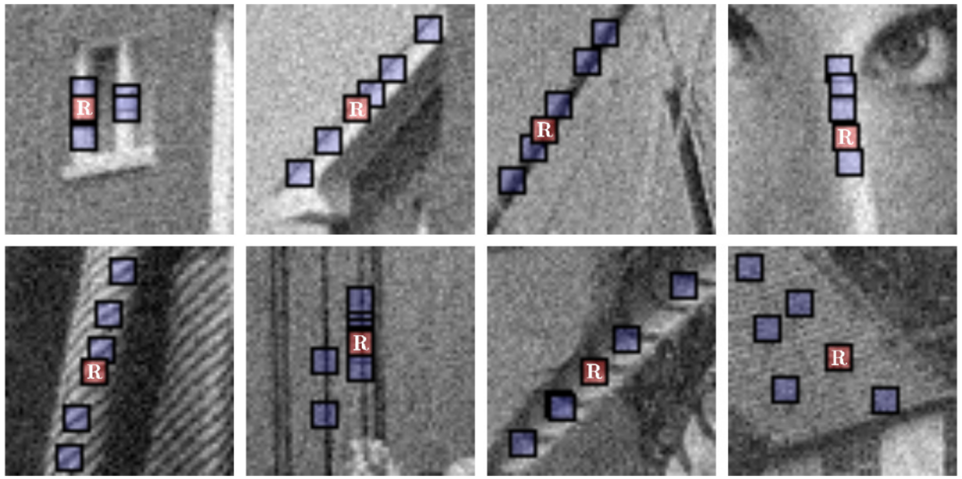


Guy Rosman, Anastasia Dubrovina, Ron Kimmel

CS, Technion, 2012

Patch-Collaborative Spectral Point-Cloud Denoising

- Spectral analysis is an important tools in surface and point cloud processing.
- In image processing, collaborative transforms reach state-of-the-art denoising results.

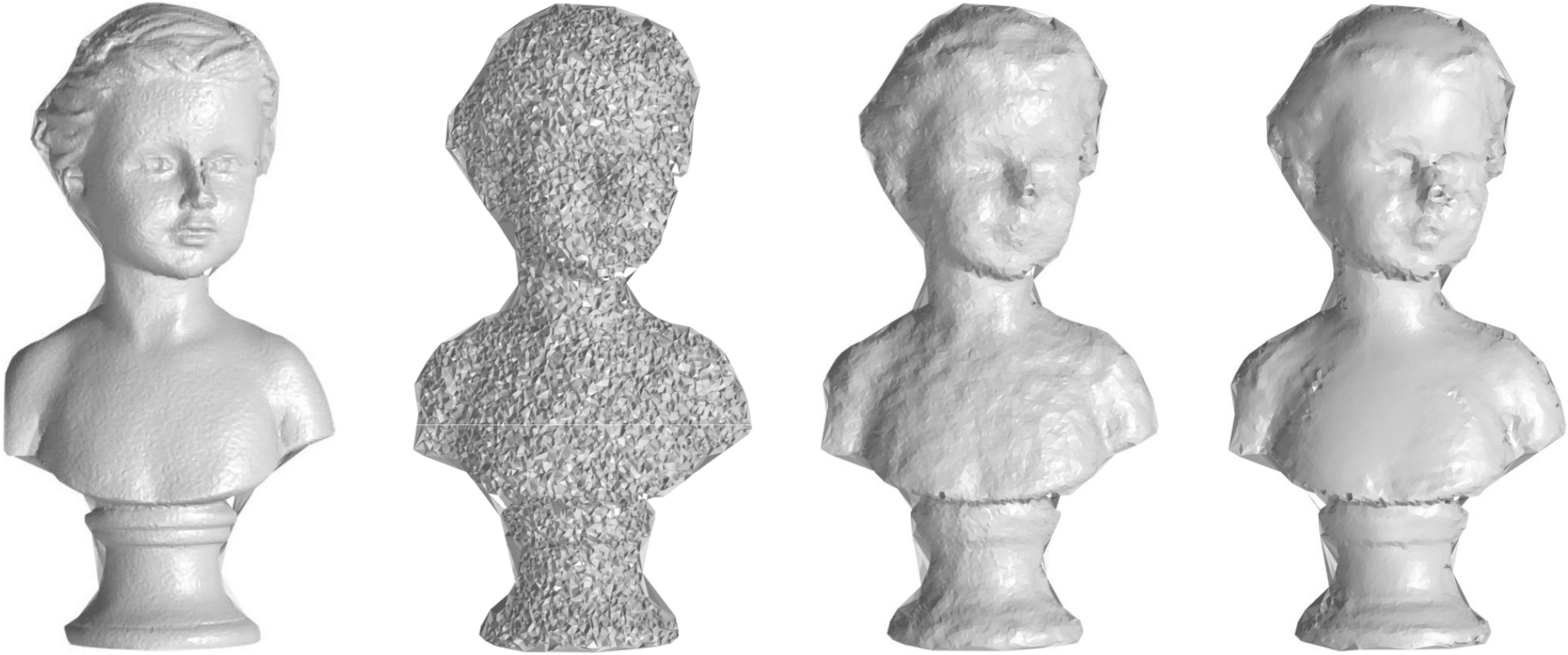


Dabov et al., '07

Patch-Collaborative Spectral Point-Cloud Denoising

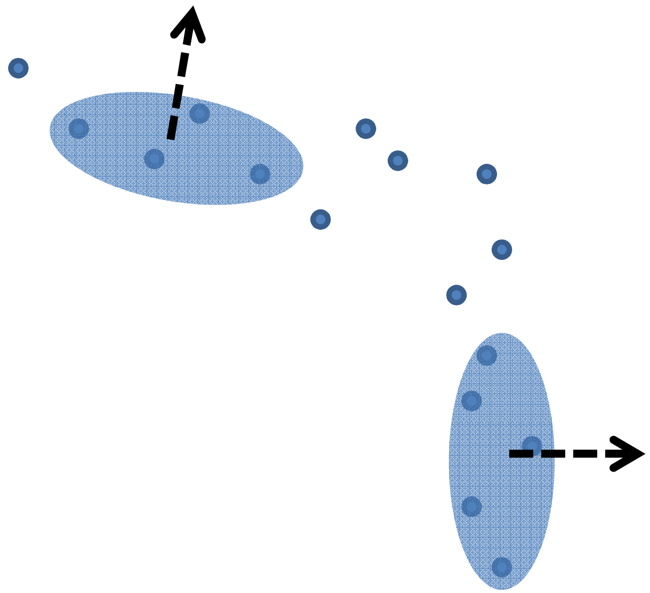
- Some of these use the spectral domain for the collaborative transform (notably BM3D)
- Need to fill in a gap for 3D point clouds.

Our main setting



- Assumption I: the surface is given by a point cloud.
- Assumption II: strong noise levels

Our main setting



- Assumption III: we assume a reasonable ratio between surface feature size, sampling density, and noise intensity.

  (Levin ’98, Fleishman et al. ’05, many other point-cloud denoising algorithms)

Surface denoising methods – several flavors

- Spectral / diffusion-based
  - Taubin’95, Desbruin’99, Clarenz’00, Schneider’01, Tasdizen’02, Lange’05, Zhao’06, etc.
- MLS
  - Levin’03, Fleishman’05, Lipman’07, etc.
- Signal processing-based
  - Taubin’95 (Spectral methods), Peng’01 (GSM), Yagou’02 (Mean and median normal filtering), Fleishman’03 (Bilateral filtering), Yoshizawa’05 (Non-local means), Lee’05 (Normal bilateral filtering), Mahmoudi’09 (Sparse representations), etc.

Collaborative-patch surface denoising

- Idea: use groups of similar surface patches to perform robust denoising.
- Motivation: a robust technique for image denoising
  - Block Matching and 3D Filtering (BM3D,).
- Intuitively:
  - Define patch similarity and construct groups of similar patches – then construct *collaborative patches.*
  - Define *denoising operator* and apply it to patch groups.
  - Return denoised patches to their original positions.

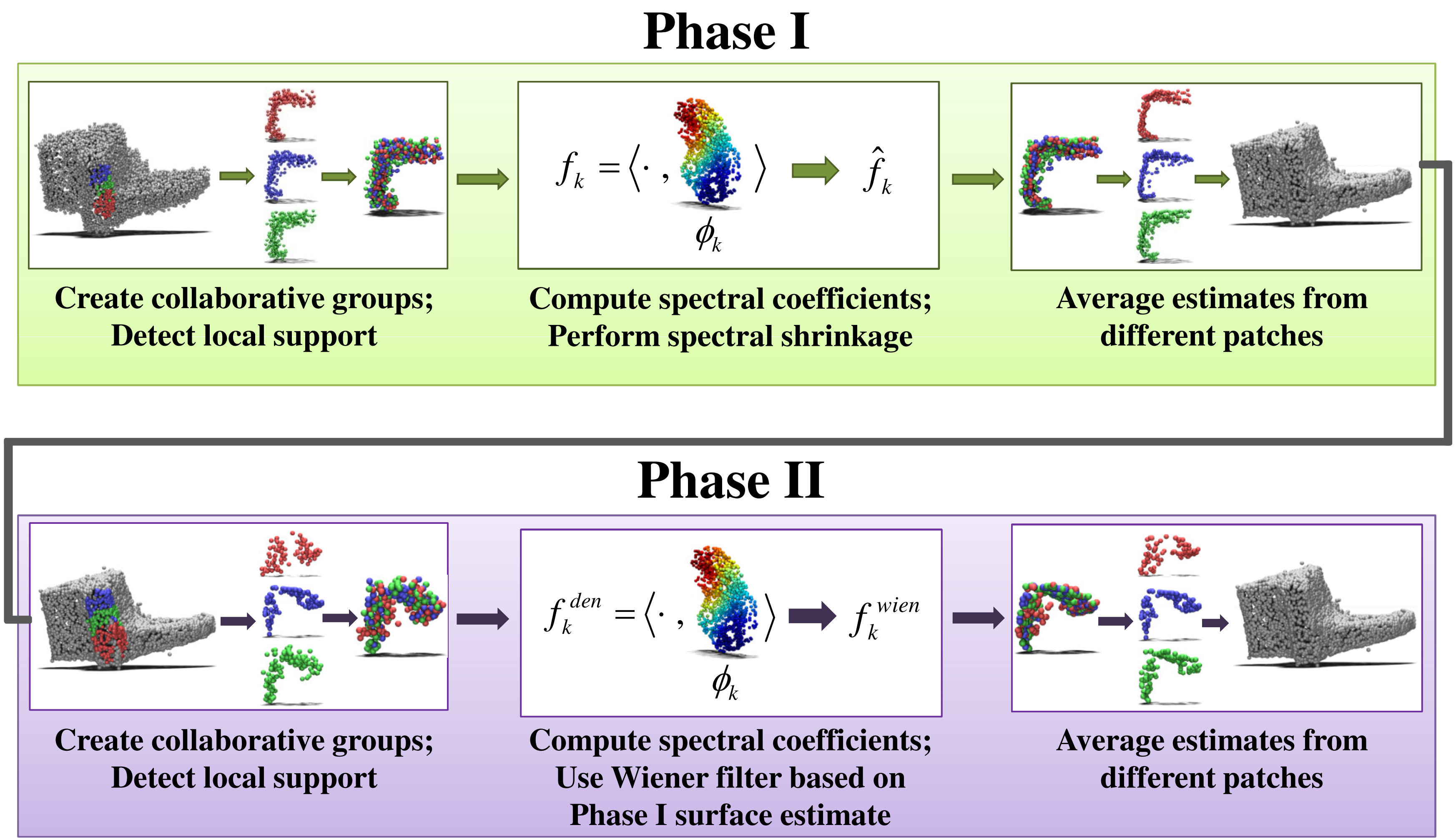
Algorithm outline

- The algorithm consists of 2 phases.

- Each phase consists of
  1. Collaborative patch construction and support estimation.
  2. Collaborative patch spectral denoising.
  3. Denoised estimates averaging.

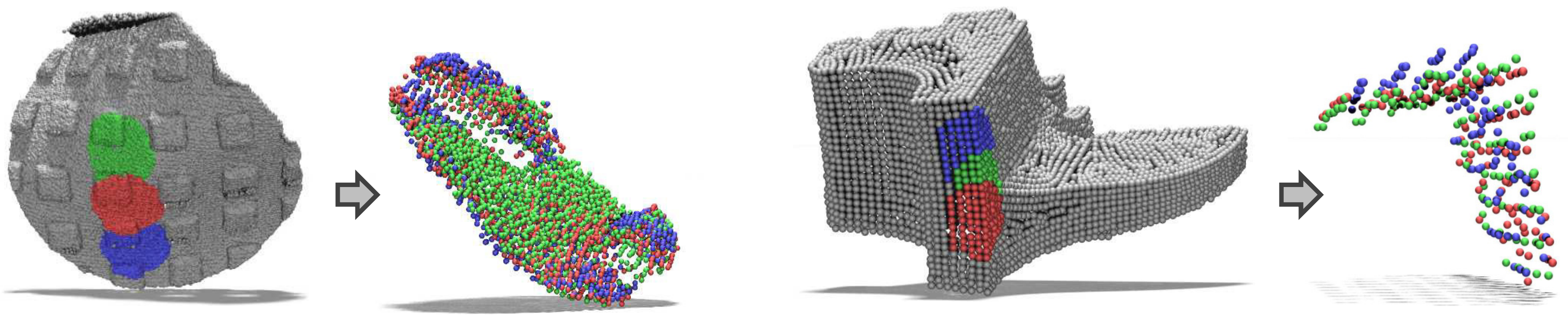
- The 2 phases differ by the spectral denoising method
  - Phase I – spectral shrinkage.
  - Phase II – Wiener filtering.

Collaborative surface denoising overview



Collaborative patch construction

- Similarity measure: $d_{ICP}\left(\mathcal{P}_i, \mathcal{P}_j\right) = \min_{R,t} d\left(R\mathcal{P}_j + t, \mathcal{P}_i\right)$
  - Find $R \in SO(3),\ t \in \mathbb{R}$ using Iterative Closest Point (ICP)
  - $d\left(R\mathcal{P}_j + t, \mathcal{P}_i\right)$ can be nearest neighbor L2 norm, point-to-plane distances, etc., over all the points in the patch.



Collaborative patch construction

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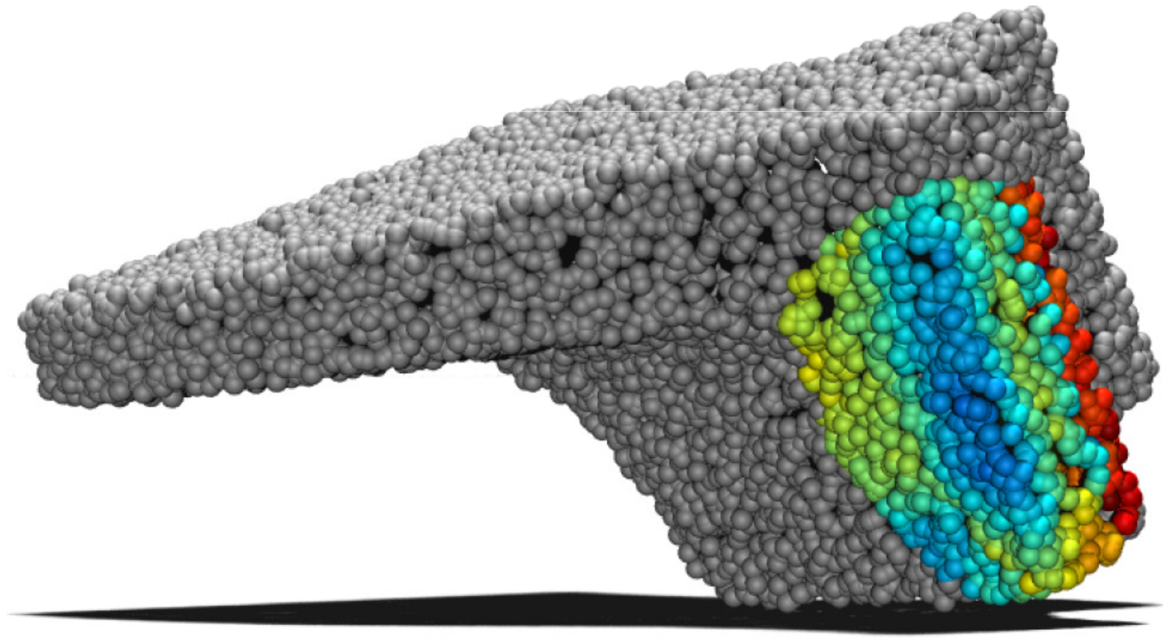
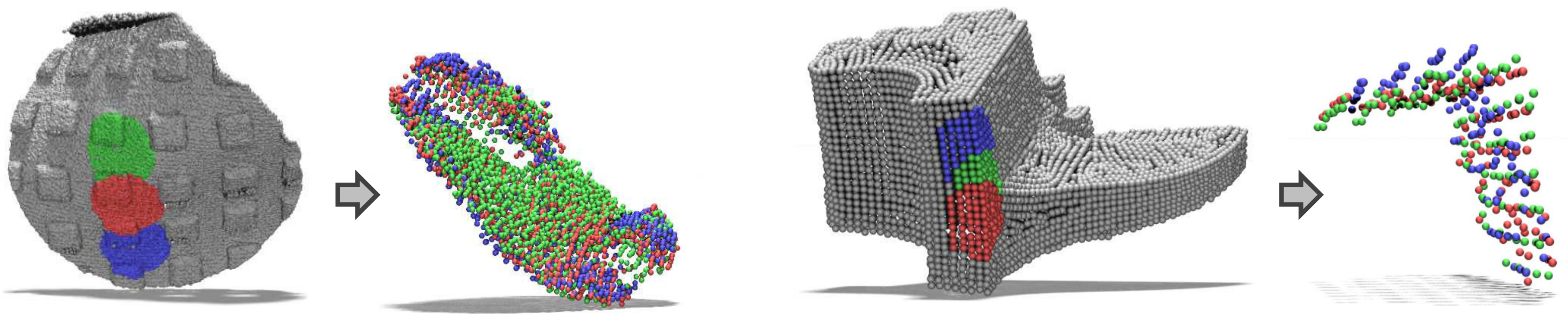


Collaborative patch construction

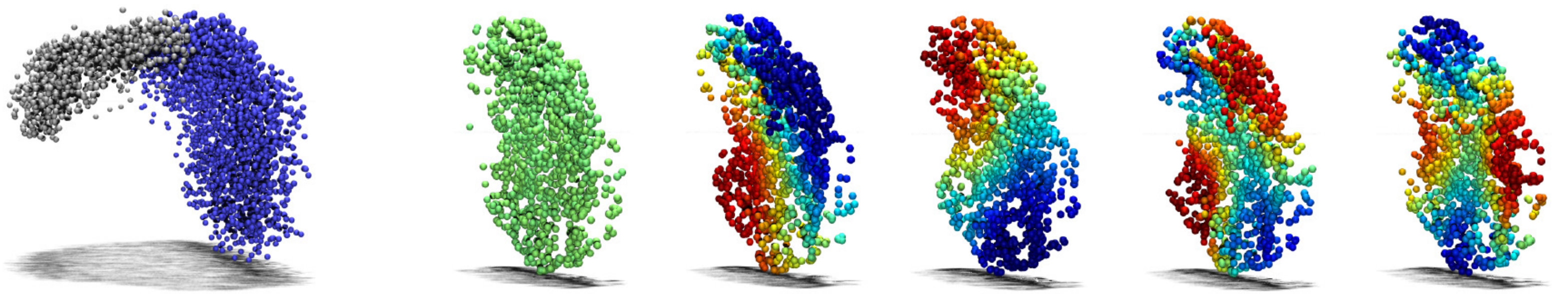
- Collaborative group: $G_i = \left\{ \mathcal{P}_j \quad \text{s.t.} \quad d_{ICP}\left(\mathcal{P}_i, \mathcal{P}_j\right) < \tau_1 \right\}$

- Collaborative patch: constructed by aggregating all the patches in G_i after alignment

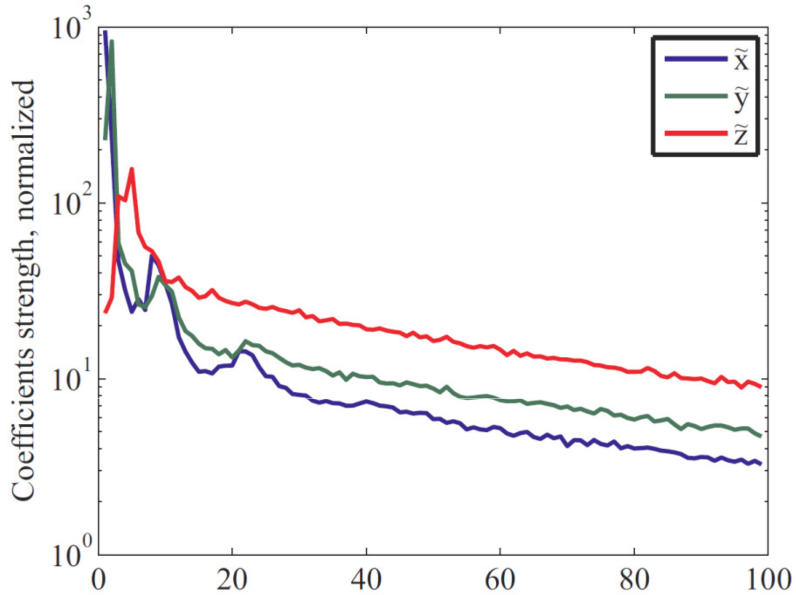


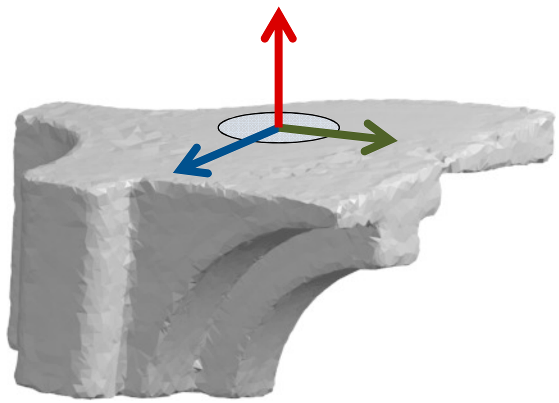
Spectral point cloud processing

- Calculate the Laplace-Beltrami operator (LBO) for the collaborative patch:
  - Use either graph Laplacian using *k*-NN,
  - or Belkin'09 discretization.
- Calculate its eigenvalues and eigenfunctions (λ_i, ϕ_i).
- Calculate the LBO over a support estimated as suggested in Fleishman'05.



Spectral coefficients





- Average normalized coefficient strength for the collaborative patch Laplacian eigenfunctions, over 400 patches.

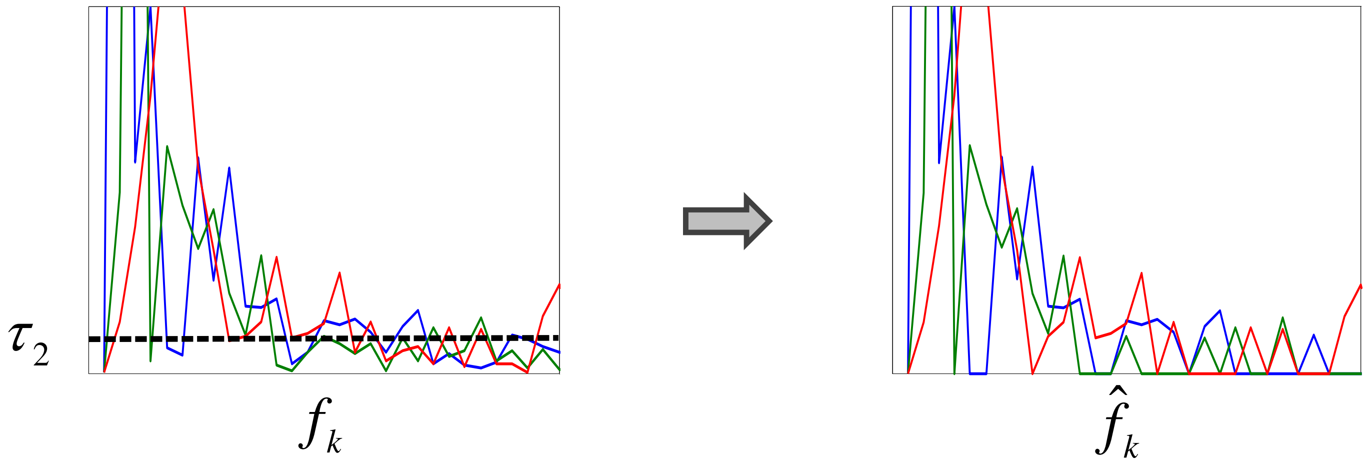
  Red, green and blue represent the absolute magnitude of the normal (red) and two tangent coordinates in the local frame as estimated by our algorithm.

Spectral patch denoising – Phase I

- *Spectral coefficients* of collaborative patch f

$$f_k = \langle f, \phi_i \rangle$$

- *Shrinkage operator*: $S_{\mathcal{P}_i,\tau} = \sum_k \hat{f}_k \phi_k$, where $\hat{f}_k = \begin{cases} f_k, & |f_k| \geq \tau_2 \\ 0, & \text{o.w.} \end{cases}$.



Spectral patch denoising – Phase II

- Empirical spectral Wiener filter:

$$f_k^{wien} = \left(\frac{\left(f_k^{den}\right)^2}{\left(f_k^{den}\right)^2 + \underbrace{\left(f_k^{orig} - f_k^{den}\right)^2}_{= \sigma_{noise}^2}} \right) f_k^{orig}$$

where

– f_k^{orig} are the original noisy surface spectral coefficients

– f_k^{den} is the denoised estimate from Phase I

Averaging denoised estimates

- Each point $\mathbf{x}_j$ belongs to several patches.
- Each patch belongs to several collaborative groups.
- Each collaborative group gives us a denoised estimate for the point $\mathbf{x}_j$.
- We need to combine these estimates.

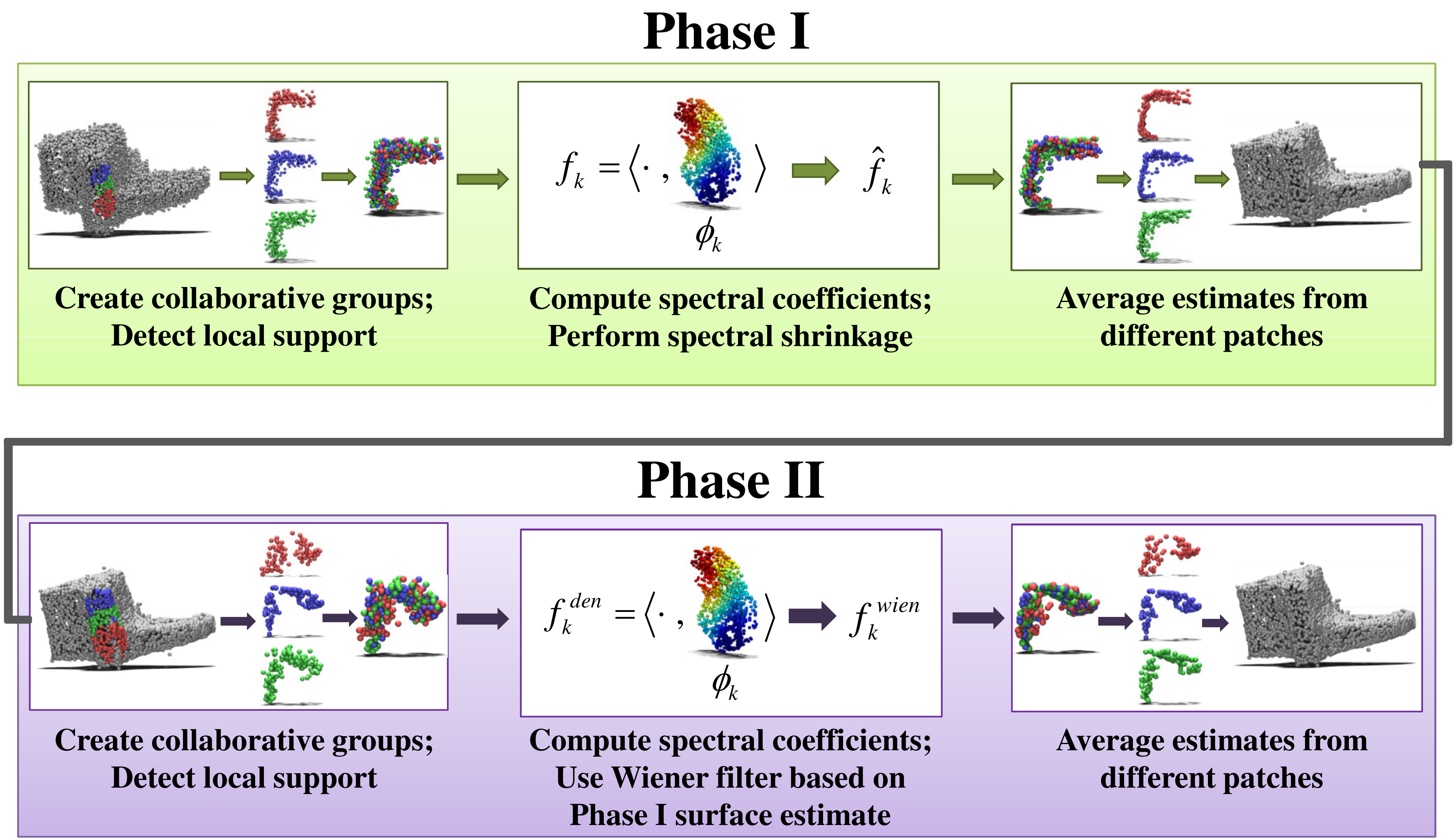
Averaging denoised estimates

- Input:
  - Set of denoised collaborative patches $\hat{\mathcal{P}}_k$ around $\mathbf{x}_i$.
- Given $\mathbf{x}_j$ belonging to several patches $\forall k : \mathbf{x}_j \in \mathcal{P}_k$
  - Average its estimates from the denoised patches $\left\{\hat{\mathcal{P}}_k\right\}$ with weights

    $$w_{ji} = \exp\left\{-\left\|\mathbf{x}_j - \mathbf{x}_i\right\|^2 \Big/ \sigma_D^2\right\} \cdot w_{Q,ji}$$

    - σ_D^2 - order of the patch size $\mathbf{x}_j$
    - $w_{Q,ji}$ - depends on the position of $\mathbf{x}_j$ in $\hat{\mathcal{P}}_k$ (inner part/boundary) and the point density at $\mathbf{x}_j$

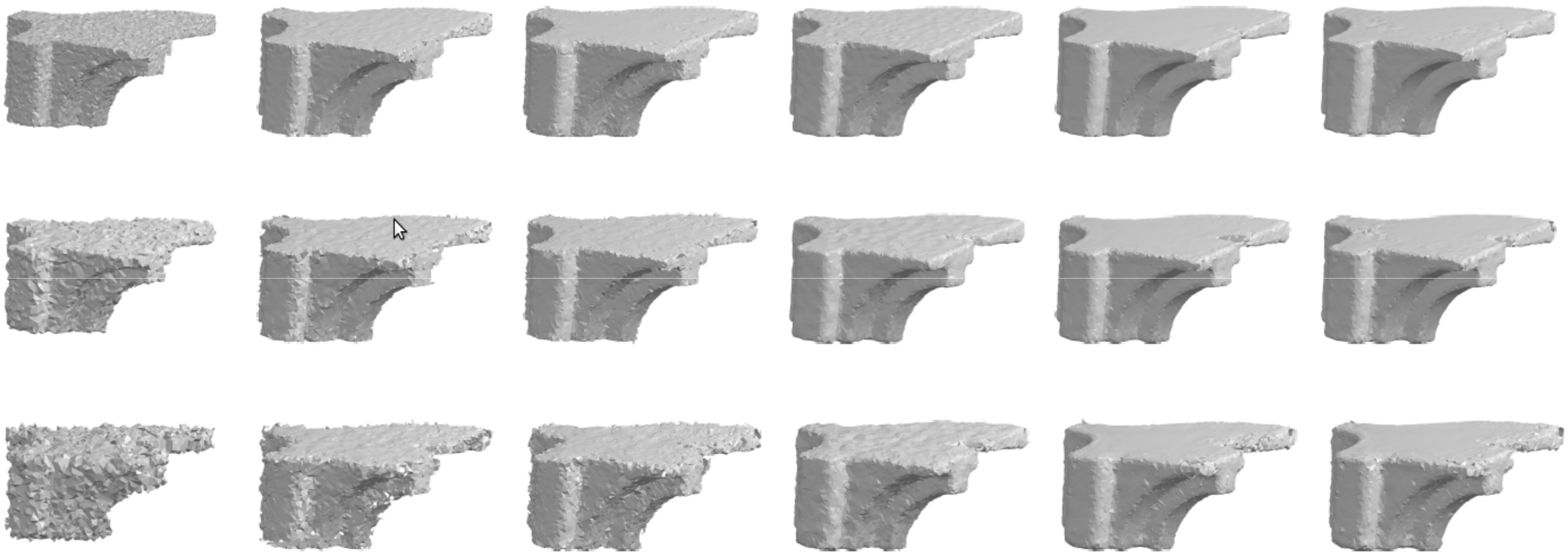
Collaborative surface denoising overview



Algorithm parameters

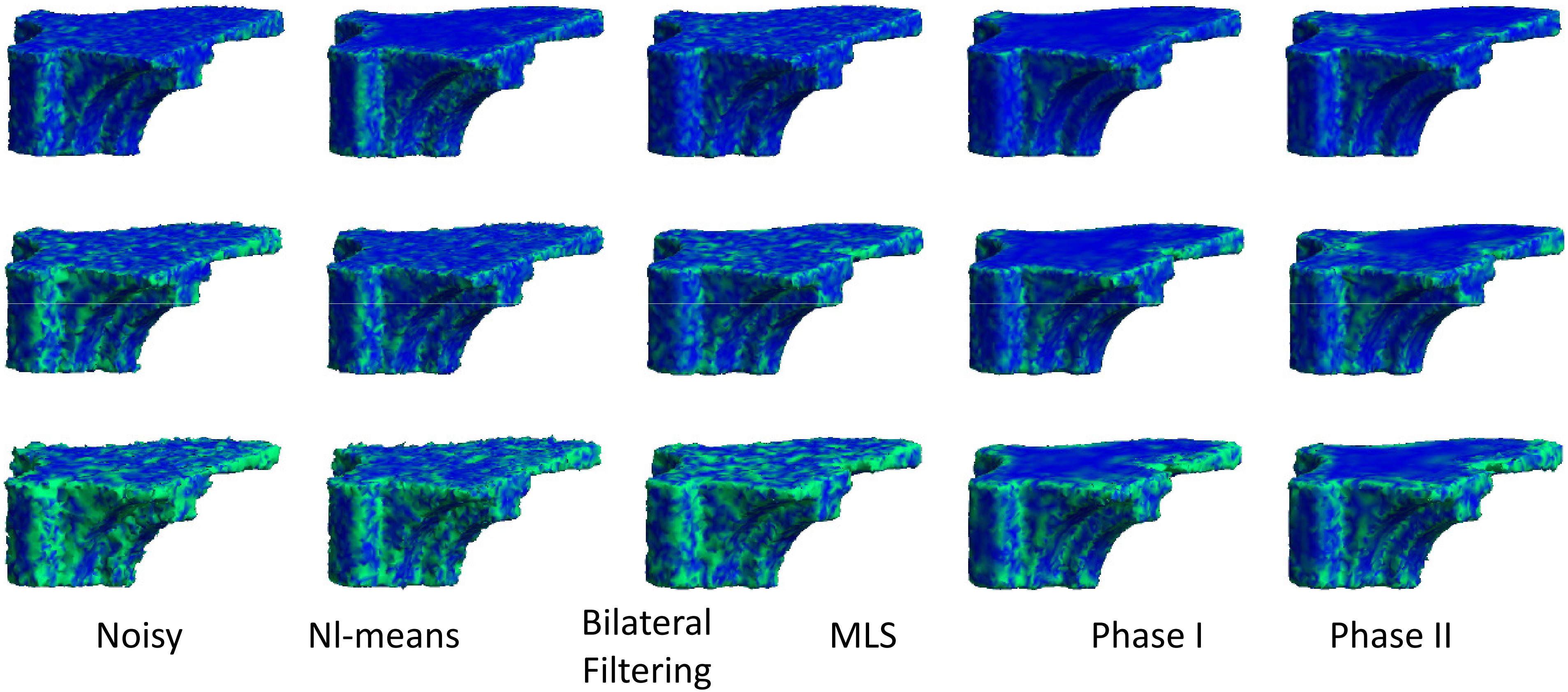
- The patch size: patches contain a few hundreds of points
- Number of patches: we used 400 candidate patches.
- Number of the LBO eigenfunctions: 100 eigenfunctions computed.
- Other parameters - Similarity threshold, Local support estimation stopping threshold – same values for all examples.

Fandisk examples



- MLS: Point Cloud Library (PCL), Rusu'11.
- Triangulation: Surface Reconstruction Toolbox 2.0, Giaccari'11.

Fandisk – method noise



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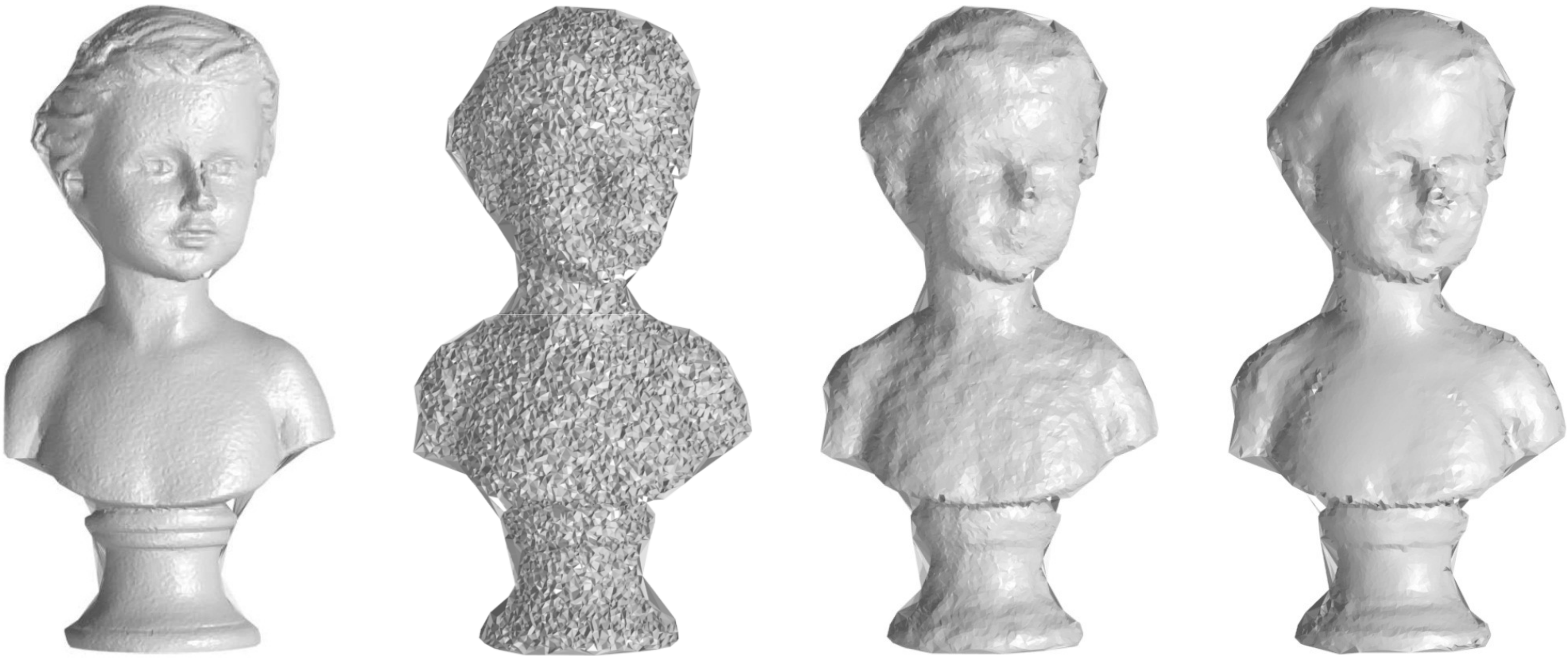
Bust example



Original surface | Noisy surface | MLS | Patch-Collaborative

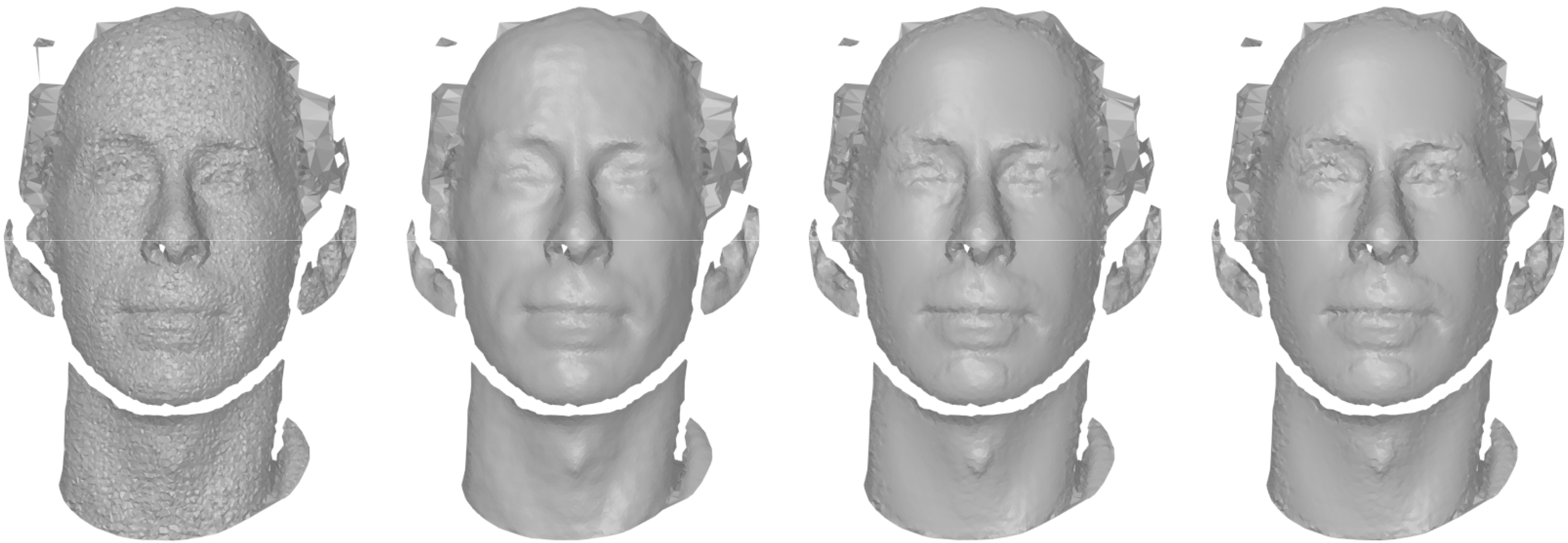
Quantitative results

Model	Fandisk				Bust	
Noise Level	$\sigma = 0.05$		$\sigma = 0.1$		$\sigma = 0.01$	
	MSE	LMedSq	MSE	LMedSq	MSE	LMedSq
Noisy surface	2.31×10^{-3}	9.14×10^{-3}	9.69×10^{-3}	4.54×10^{-3}	1×10^{-4}	4.63×10^{-5}
NL Means	6.29×10^{-4}	2.45×10^{-4}	2.37×10^{-3}	9.63×10^{-4}	1.55×10^{-5}	5.02×10^{-6}
MLS	4.83×10^{-4}	1.52×10^{-4}	2.02×10^{-3}	1.2×10^{-3}	1.46×10^{-5}	4.81×10^{-6}
Proposed approach (phase I)	4.73×10^{-4}	1.58×10^{-4}	1.80×10^{-3}	7.46×10^{-4}	1.43×10^{-5}	4.21×10^{-6}
Proposed approach (phase II)	3.83×10^{-4}	1.24×10^{-4}	1.38×10^{-3}	6.81×10^{-4}	1.54×10^{-5}	4.30×10^{-6}

- Mean squared error (MSE) and
- Least-median of squares (LMedSq)

of the point cloud after denoising for the Fandisk and the Bust models, for the noise levels shown in the figures.

Surface obtained using coded-light scanner



Original scan MLS denoising Phase I Phase II

Conclusions

- We presented a method for spectral denoising of point clouds.
- It employs spectral filtering of similar patches (the collaborative group)
  - Inspired by image processing algorithms (in particular, BM3D).
  - Runs in two steps a shrinkage operator and Wiener filter, both in a similar domain.

- Experimental results: preserves sharp features and smoothes flat regions.

Thank you!