Minimum-Energy Landing Trajectory Optimization for a Reusable Launch Vehicle

An Example GNC Project by Ross O'Hara

Objective

The ability to land a reusable launch vehicle with precision and efficiency is essential for advancing cost-effective spaceflight and enabling long-term space infrastructure. This project presents a reduced-fidelity simulation of a generic vertically landing space vehicle. A trajectory optimization routine was developed to minimize energy expenditure while satisfying dynamic and terminal constraints. The project demonstrates core skills in dynamic modeling, optimal control, and simulation of spaceflight systems.

Vehicle Model

A reduced-order, planar model was developed to approximate the dynamics of a vertically landing, reusable space vehicle. The model assumes a rigid body of uniformly distributed mass exposed to a uniform gravitational field without atmospheric drag. The vehicle is controlled via a single gimbaled rocket engine.

Based on the free-body diagram shown in Figure 1, the differential equations of motion for the vehicle are as follows, derived using Newtonian laws of motion. Where m is the mass of the vehicle, I is the moment of inertia, F_T is the throttled thrust from the rocket engine, δ is the angle of deflection of the gimble, and L is the length of the vehicle.

$$\dot{x} = v_x$$

$$\dot{z} = v_z$$

$$\dot{v_x} = \frac{\sum F_x}{m}$$

$$\dot{v_z} = \frac{\sum F_z}{m}$$

$$\dot{\theta} = \omega$$

$$\dot{\omega} = \frac{\sum M}{I}$$

$$\sum F_x = F_T[\sin(\delta)\cos(\theta) + \cos(\delta)\sin(\theta)]$$

$$\sum F_z = F_T[\sin(\delta)\cos(\theta) + \cos(\delta)\sin(\theta)] - mg$$

$$\sum M = F_T \frac{L}{2} \sin(\delta)$$

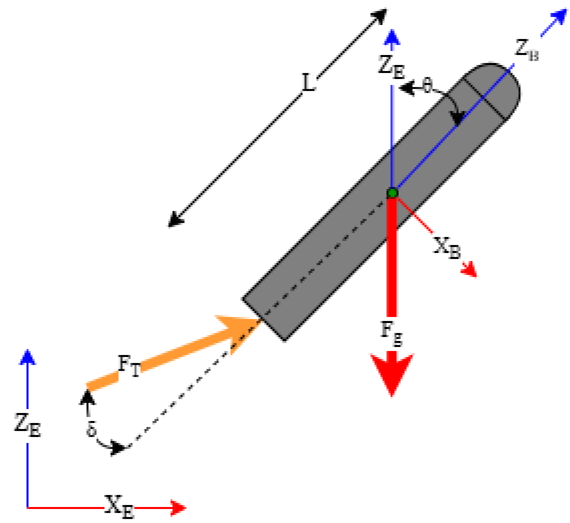

Figure 1. Free-body diagram of a simplified reusable launch vehicle during powered descent. Thrust $\boldsymbol{F_T}$ is gimbaled by angle $\boldsymbol{\delta}$ and applied below the center of mass, producing torque. Frames $\boldsymbol{X_E}$, $\boldsymbol{Z_E}$ (inertial) and $\boldsymbol{X_B}$, $\boldsymbol{Z_B}$ (body) are shown along with pitch angle $\boldsymbol{\theta}$.

The model parameters were estimated from online sources and implemented as follows. The total fixed mass of the vehicle is 100 *t*, the local acceleration due to gravity is 9.81 $\frac{m}{s^2}$, the vehicle is 50 *m* tall, and the moment of inertial is estimated as a solid cylinder. The maximum thrust is 15 *MN,* which can be throttled from 0-100%. The gimbal can articulate from positive to negative 22.5 degrees.

Trajectory Optimization

A trajectory optimization problem was formulated to compute a minimum-energy descent for the vehicle under thrust vector control. The objective was to minimize the integral of squared thrust magnitude and gimbal position over the descent, with the final time treated as a free variable. The trajectory was constrained by the vehicle's nonlinear dynamics, actuator limits, and terminal conditions enforcing zero velocity and upright orientation at touchdown. The problem was transcribed using direct collocation and solved as a nonlinear program using MATLAB's fmincon. State and control trajectories were discretized over the collocation points, with initial guesses generated from a straight-line descent and constant-thrust profile.

Results

The optimized trajectory begins with the vehicle in a horizontal orientation at 1000 meters altitude, offset 200 meters laterally, and descending at 90 m/s with a forward velocity of 25 m/s. These conditions approximate the start of a terminal landing sequence, in which the vehicle performs a flip maneuver to reorient from horizontal flight into a vertical position for landing. As shown in Figure 2, the optimizer generates this flip early in the trajectory, visible in the pitch angle plot (top right), where the vehicle rotates by approximately 90 degrees. This reorientation is coordinated with thrust vectoring and throttling (bottom center and bottom right) to control both attitude and deceleration. After the flip, the vehicle transitions into a vertical descent, reducing velocity smoothly and aligning with the landing target. The flight path plot (bottom left) confirms the lateral translation followed by vertical alignment at touchdown. The vehicle lands upright in roughly 10 seconds, meeting all terminal constraints within a solver tolerance of 5e-2.

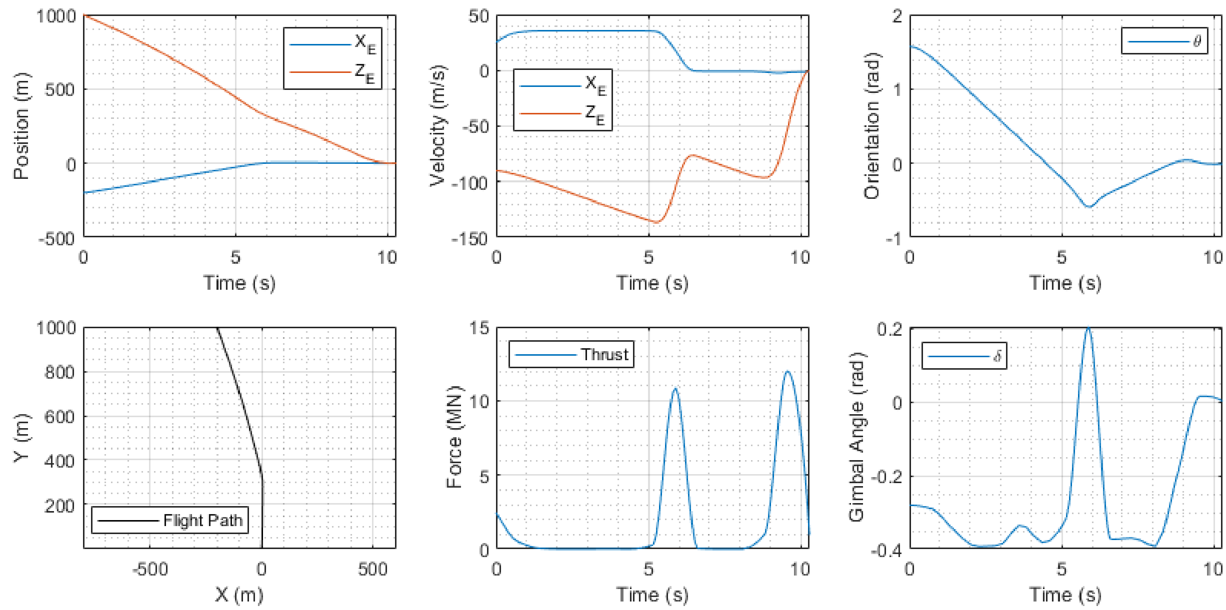

Figure 2. Optimized state and control trajectories for a terminal landing maneuver. Top row (left to right): inertial position, velocity, and pitch angle. Bottom row: 2D flight path, thrust magnitude, and gimbal angle. The trajectory features a flip maneuver from horizontal to vertical orientation, followed by a controlled descent and soft landing using thrust vectoring and modulation.

Discussion

This project demonstrates several key competencies relevant to guidance, navigation, and control engineering. By modeling the coupled translational and rotational dynamics of a reusable launch vehicle and solving an energy-optimal landing trajectory, the work highlights an ability to integrate physical modeling, constrained control, and numerical optimization into a unified solution. The results illustrate how thrust vectoring can be leveraged not only for deceleration but also for attitude reorientation, enabling a smooth transition from horizontal flight to vertical landing. The flip maneuver emerged naturally from the optimization, emphasizing the value of trajectory-level control strategies in reusable vehicle design. While the model simplifies real-world conditions such as neglecting atmospheric drag and assuming constant mass, the framework establishes a solid foundation for higher-fidelity simulations. Future work could extend this approach to full 6-DOF dynamics, variable mass modeling, and uncertainty-aware optimization, bringing the analysis closer to flight-ready GNC algorithms.

Code, data, and simulation videos are available upon request by contacting: rossohara.rlo@gmail.com