Ross Miller

11/31/17

(A)

Starting with the initial system of non-linear equations F(X) = 0, where F = { and

= = 0

= = 0

= = 0

When using Newton’s method to approximate a solution we get the following iterations with an initial guess of X = {0,0,0}, and a tolerance of E = 1e-5 we get the following iterations:

k| x1| x2| x3| norm(x(n)-x(n-1))

-----------------------------------------------------

01 5.000 37.000 -39.000 37.000

02 4.351 18.491 -19.842 19.158

03 5.364 9.255 -11.619 8.223

04 5.696 4.665 -7.361 4.258

05 5.883 2.427 -5.310 2.051

06 5.966 1.413 -4.379 0.931

07 5.995 1.059 -4.054 0.325

08 6.000 1.002 -4.001 0.052

09 6.000 1.000 -4.000 0.001

10 6.000 1.000 -4.000 0.000

When using Quasi Newton’s method the iterations are identical, this means that in this case the approximation of the derivative value is very close to the actual derivative.

k| x1| x2| x3| norm(x(n)-x(n-1))

-----------------------------------------------------

01 5.000 37.000 -39.000 37.000

02 4.351 18.491 -19.842 19.158

03 5.364 9.255 -11.619 8.223

04 5.696 4.665 -7.361 4.258

05 5.883 2.427 -5.310 2.051

06 5.966 1.413 -4.379 0.931

07 5.995 1.059 -4.054 0.325

08 6.000 1.002 -4.001 0.052

09 6.000 1.000 -4.000 0.001

10 6.000 1.000 -4.000 0.000

(B)

Starting with the initial system of non-linear equations F(X) = 0, where F = { and

= = 0

= = 0

= = 0

When using Newton’s method to approximate a solution with an initial guess of X = {0,0,0}, and a tolerance of E = 1e-5 we get the following iterations:

k| x1| x2| x3| norm(x(n)-x(n-1))

-----------------------------------------------------

01 0.000 0.150 0.999 0.999

02 0.000 0.100 0.998 0.000

When using Quasi Newton’s method we get identical iterations.

k| x1| x2| x3| norm(x(n)-x(n-1))

-----------------------------------------------------

01 0.000 0.150 0.999 0.999

02 0.000 0.100 0.998 0.000

Converting the system into a system of fixed point equations we get:

Using fixed point iteration with an initial guess of X = {0,0,0} and a tolerance of E = 1e-5, we get the following iterations:

k| x1| x2| x3| norm

-------------------------------------------

01 0.000 0.000 1.000 1.000

02 0.000 0.100 0.998 0.100

03 0.000 0.100 0.998 0.000

(C)

Starting with the initial system of non-linear equations F(X) = 0, where F = { and

= = 0

= = 0

Which gives a system of fixed point equations:

Using fixed point iteration to approximate a solution with an initial guess of X = (15, 2} and a tolerance of E = 1e-5, we get the following iterations:

k| x1| x2| norm

----------------------------------

01 34.000 0.071 19.000

05 16.684 -0.902 4.771

10 13.990 -1.072 0.073

15 14.582 -1.034 0.267

20 14.440 -1.043 0.004

25 14.474 -1.041 0.015

30 14.466 -1.041 0.000

35 14.468 -1.041 0.001

40 14.467 -1.041 0.000

42 14.467 -1.041 0.000

Using Newton’s method with an initial guess of X = {15, 2} and a tolerance of E = 1e-5 we get the following iterations:

k| x1| x2| norm(x(n)-x(n-1))

-----------------------------------------------------

01 41.833 3.583 26.833

02 35.691 3.098 0.485

03 34.216 3.042 0.056

04 34.199 3.041 0.001

05 34.199 3.041 0.000

Using Quasi Newton’s method, we get an identical series of iterations with an initial guess of X = {15, 2}, an h = 1e-5, and a tolerance of E = 1e-5.

k| x1| x2| norm(x(n)-x(n-1))

-----------------------------------------------------

01 41.833 3.583 26.833

02 35.691 3.098 0.485

03 34.216 3.042 0.056

04 34.199 3.041 0.001

05 34.199 3.041 0.000

Using an algebraic approach to solving the system gives the equations:

,

then substituting for x1 in the second equation gives a simplified quadratic equation of:

Which we can of course solve using the quadratic formula which yields the following solutions for x2

Which we can substitute back into the original equation for x1 which gives the following values:

Solving a system of equations algebraically will always yield better results, in this case we find that there are two solutions to this system, whereas by using one of our iterative procedure we can only find one of these solutions, not only that but by solving this system algebraically we can get exact results.