Ross Miller

1. [0, 2π]

We can see that the function above is clearly continuous on the interval [0, 2π]. Since g(x) ϵ [0, 2π] for all x ϵ [0, 2π] we know their must exist atleast one fixed point. Moreover we can say that the |g’(x)| <= k and k < 1 for cos(x) x ϵ (a,b). However it’s much more difficult to come up with an exact value for k since cos(2π) = 1 but we know that on x ϵ (a,b) x < 2π, g’(x) < 1, therefore k does exist and k is less than 1 but very close to 1. When supposing k = .9 the number of iterations needed is greater than 59.

n | g(Pn-1) = Pn | Pn - Pn-1 | E

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010 3.2242 0.3481 .01

020 3.2746 0.2428 .01

030 3.3084 0.1729 .01

040 3.3320 0.1243 .01

050 3.3488 0.0899 .01

060 3.3609 0.0651 .01

070 3.3697 0.0473 .01

080 3.3760 0.0343 .01

090 3.3807 0.0249 .01

100 3.3840 0.0181 .01

110 3.3864 0.0132 .01

119 3.3978 0.0099 .01

On a reexamination of the previous problem I’ve realized that I made several mistakes. In the function it is easy to tell the existence of a fixed point since the function h is clearly continuous, and g(x) ϵ [0,2π] for all x ϵ [0,2π]. However, it’s on the second theorem where I make my first mistake. g(x) fails the second test since g’(x) = cos(x) and |cos()| = 1 so it’s impossible to find a k < 1 so uniqueness is not certain. At this point I started making outright wrong statements and crazy assumptions, and it was just by chance that my guess of P0 = happened to converge on the correct answer (although very slowly). I believe the function I was supposed to analyze was .