Supplementary Information for:

Effects of Misinformation on Election Beliefs:
Disentangling Motivated Reasoning from Selective Exposure

A. Regression Analyses & Gelbach Decompositions

To provide a comparison to results that one may estimate based on a correlational regression framework, we also present the results of a Gelbach Decomposition (using OLS regression) of the belief that the 2020 U.S. Presidential Election was fraudulent by misinformation exposure in Table S1. Descriptively, we find that people exposed to misinformation were more likely to believe that the results of the 2020 Presidential Election were stolen than those who were not. Those exposed were 17.3% (95% CI = 12.7, 21.9) more likely to believe the election was stolen We decompose the 17.3% effect using a Gelbach Decomposition into two components: the part of the effect that can be explained by observed individual characteristics, such as demographics and presidential candidate support, and the unexplained portion, which may represent the motivated reasoning effect of exposure on election beliefs. The results of the Gelbach Decomposition (Table S1) reveal that 65.7% of this effect between those exposed and not exposed to false election beliefs can be explained by observed characteristics, such as demographics and political support. Therefore, 34.2% of the difference remains unexplained and may, in part, represent the effect of misinformation exposure on false election beliefs. Another way to interpret these results is that model (1) represents a regression with no control variables and model (10) is a regression with the full control variables. Thus, the misinformation exposure coefficient is akin to the results one could report in a simple regression-based analysis.

Table S1. OLS regression of the effect of misinformation exposure in the false belief in a stolen election, consecutively adding more covariates

	Dependent variable:									
	Belief that Trump won the 2020 U.S. Presidential Election									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Misinfo Exposures	0.173^{***}	0.077^{***}	0.079^{***}	0.083^{***}	0.083^{***}	0.085^{***}	0.093^{***}	0.089^{***}	0.082^{***}	0.059^{**}
	(0.023)	(0.020)	(0.021)	(0.021)	(0.021)	(0.021)	(0.021)	(0.021)	(0.021)	(0.023)
Constant	0.135^{***}	−0.014	−0.013	−0.019	0.006	0.006	0.061	-0.094^{*}	−0.022	−0.020
	(0.014)	(0.013)	(0.014)	(0.020)	(0.022)	(0.025)	(0.034)	(0.042)	(0.047)	(0.047)
Conservative	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Total Web Visits Wave 1	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Education	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Gender	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Race	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes
Political Knowledge	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Political Interest	No	No	No	No	No	No	No	Yes	Yes	Yes
Age	No	No	No	No	No	No	No	No	Yes	Yes
Misinfo Exposure Wave 1	No	No	No	No	No	No	No	No	No	Yes
Observations	1,194	1,194	1,194	1,194	1,194	1,194	1,194	1,194	1,194	1,194
R^2	0.044	0.336	0.337	0.340	0.343	0.346	0.349	0.368	0.378	0.382
Adjusted R^2	0.043	0.335	0.335	0.337	0.340	0.340	0.343	0.361	0.370	0.374
Residual Std. Error	0.393	0.327	0.327	0.327	0.326	0.326	0.325	0.321	0.318	0.318
F Statistic	54.485^{***}	302.005^{***}	201.214^{***}	101.992^{***}	88.627^{***}	62.551^{***}	57.656^{***}	57.235^{***}	47.808^{***}	45.492^{***}

Note: Gelbach Decomposition of the effect of misinformation website exposure on the belief that Donald Trump won the 2020 U.S. Presidential Election. Independent variables are sequentially added to the baseline model to the final model. This decomposition reveals that about 70 percent of the difference in the belief that Trump won the election between those who were exposed to misinformation websites and those who were not can be explained by observed differences. About 30 percent of difference remains unexplained and may partially be due to the effect of misinformation website exposure. Significance level: * p<0.05, ** p<0.01, *** p<0.001.

We repeat this analysis with dosage as the independent variable (Table S2).

Table S2: OLS regression of the dosage effect of misinformation exposure in the false belief in a stolen election, consecutively adding more covariates

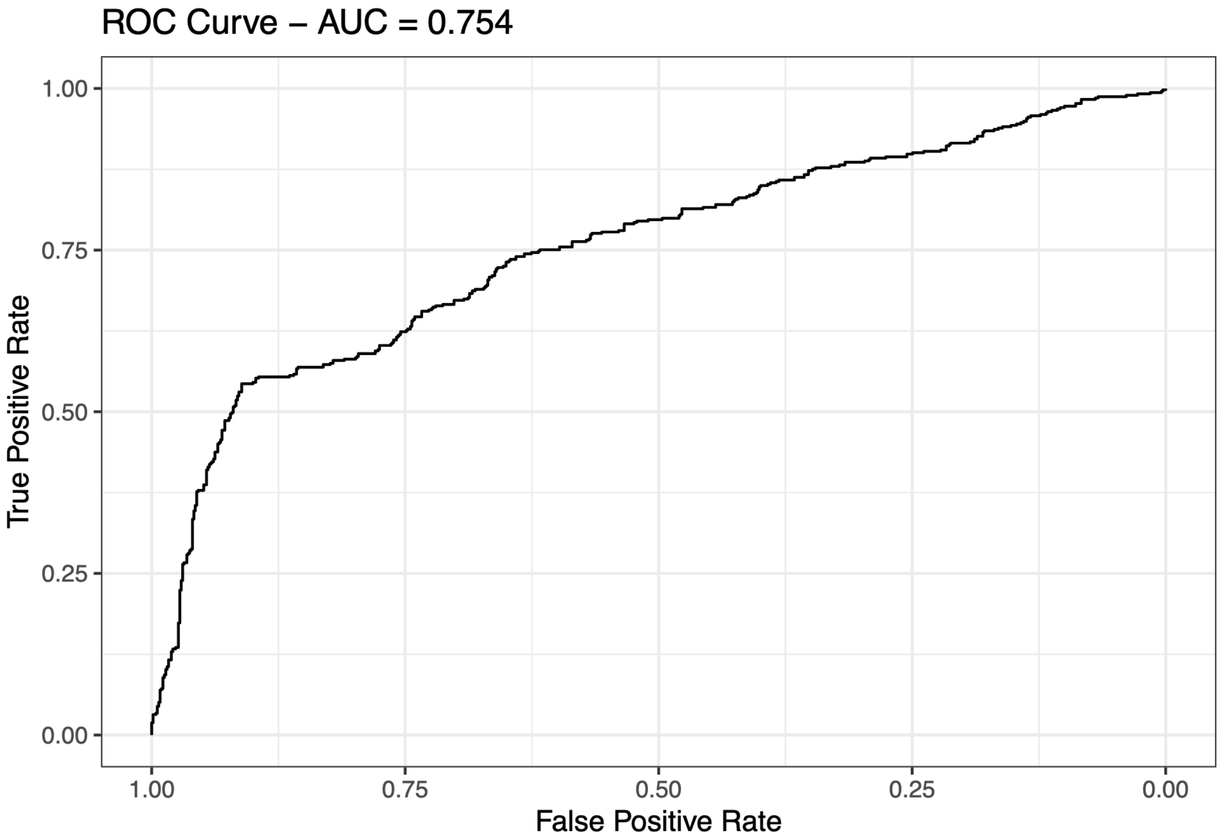
	Dependent variable:									
	Belief that Trump won the 2020 U.S. Presidential Election									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Misinfo Exposures	0.001^{***}	0.0005^{***}	0.0005^{***}	0.001^{***}	0.001^{***}	0.001^{***}	0.001^{***}	0.0005^{***}	0.0004^{***}	0.0004^{**}
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Constant	0.188^{***}	0.005	0.001	−0.004	0.018	0.021	0.067^{*}	-0.087^{*}	−0.018	−0.016
	(0.012)	(0.012)	(0.014)	(0.019)	(0.022)	(0.025)	(0.034)	(0.042)	(0.047)	(0.047)
Conservative	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Total Web Visits Wave 1	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Education	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Gender	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Race	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes
Political Knowledge	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Political Interest	No	No	No	No	No	No	No	Yes	Yes	Yes
Age	No	No	No	No	No	No	No	No	Yes	Yes
Misinfo Exposure Wave 1	No	No	No	No	No	No	No	No	No	Yes
Observations	1,194	1,194	1,194	1,194	1,194	1,194	1,194	1,194	1,194	1,194
R^2	0.025	0.338	0.338	0.341	0.344	0.346	0.348	0.367	0.378	0.384
Adjusted R^2	0.024	0.336	0.336	0.338	0.340	0.341	0.342	0.360	0.370	0.375
Residual Std. Error	0.396	0.327	0.327	0.326	0.326	0.326	0.325	0.321	0.319	0.317
F Statistic	30.334^{***}	303.499^{***}	202.325^{***}	102.540^{***}	88.889^{***}	62.630^{***}	57.465^{***}	56.978^{***}	47.650^{***}	45.810^{***}

Note: Gelbach Decomposition of the effect of misinformation website exposure on the belief that Donald Trump won the 2020 U.S. Presidential Election. Independent variables are sequentially added to the baseline model to the final model. This decomposition reveals that about 50 percent of the difference in the belief that Trump won the election between those who were exposed to misinformation websites and those who were not can be explained by observed differences. About 50 percent of difference remains unexplained and may partially be due to the effect of misinformation website exposure. Significance level: * p<0.05, ** p<0.01, *** p<0.001.

B. Model Calibration and Robustness

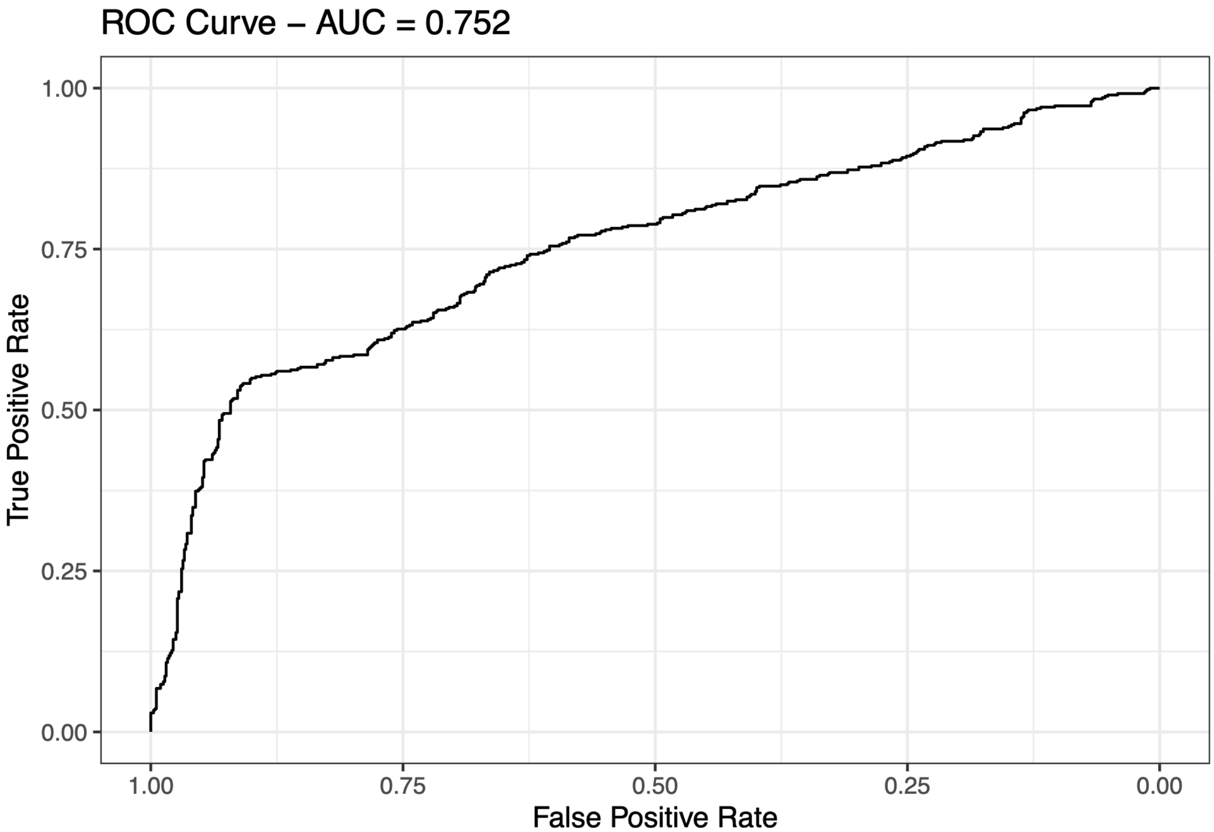
We examine the calibration and robustness of the models in two main ways: calibration testing and estimating effects using an alternative double machine learning methodology. First, the calibration tests show that two models (predicting exposure and predicting effect) used for assessing the effect of misinformation exposure on the belief that the election was fraudulent are well-calibrated. The calibration test is done as “best linear fit using forest predictions (on held-out data) as well as the mean forest prediction as regressors, along with one-sided heteroskedasticity-robust (HC3) SEs” (grf v2.1.0 50), implemented directly in the grf package. The first model is well-calibrated and accurately predicts exposure ($p < .001$; AUC = 0.754; see Figure S1).

Figure S1. ROC Curve of the First Machine Learning Model



Note: Receiver Operating Characteristic (ROC) curve for the predictive model of misinformation website exposure. The area under the curve (AUC) of 0.754 indicates a good level of accuracy in the model's ability to distinguish between individuals who were and were not exposed to misinformation websites. The ROC curve plots the true positive rate (sensitivity) against the false positive rate (1 - specificity) at various threshold settings. The closer the curve follows the left-hand border and then the top border of the ROC space, the more accurate the test. The point where the curve touches the top border represents the threshold with the best balance between sensitivity and specificity. The calibration of the model was assessed using a best linear fit on held-out data with the inclusion of mean forest predictions as regressors, and the statistical significance was established using one-sided heteroskedasticity-robust (HC3) standard errors.

Figure S2. ROC Curve of the Second Machine Learning Model



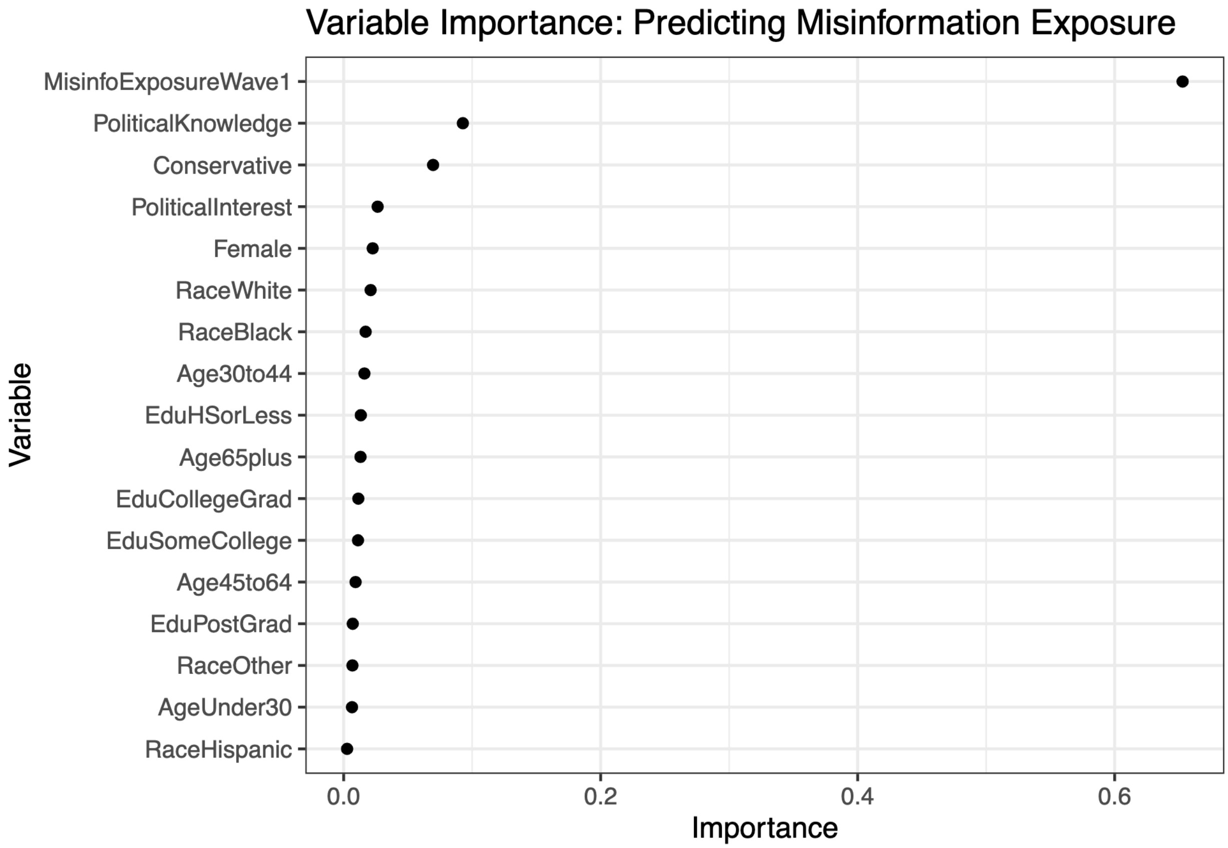
Note: Receiver Operating Characteristic (ROC) curve for the predictive model of the belief that the 2020 U.S. Presidential Election was fraudulent. The area under the curve (AUC) of 0.752 indicates a good level of accuracy in the model's ability to distinguish between individuals who were and were not exposed to misinformation websites. The ROC curve plots the true positive rate (sensitivity) against the false positive rate (1 - specificity) at various threshold settings. The closer the curve follows the left-hand border and then the top border of the ROC space, the more accurate the test. The point where the curve touches the top border represents the threshold with the best balance between sensitivity and specificity. The calibration of the model was assessed using a best linear fit on held-out data with the inclusion of mean forest predictions as regressors, and the statistical significance was established using one-sided heteroskedasticity-robust (HC3) standard errors.

Furthermore, we calculate results under related methodologies, finding that the results remain robust to these additional analyses. First, we calculate results using Chernozykov's (2015) double machine learning method. Using this alternative method, we find the results remain significant, with an average effect of 5.7% (95% CI = 1.4, 10.1) for misinformation exposure and .035% (95% CI = .023, .048) for each additional exposure. Using a propensity matching model (Stuart et al., 2011), the results are also significant, with an average effect of 6.6% (95% CI = 2.4, 10.9, p-value = .002) for misinformation exposure and .037 (95% CI = .022, .051, p-value < .001).

C. Model Explanation

To provide an additional understanding of the machine learning models, we provide underlying statistical results used in the ultimate (C)ATE calculations. First, we provide variable importance information from the first causal forest model predicting exposure (Figure S3). Variable importance is a measure of the relative contribution of each predictor variable to the model's predictive power. It is derived from the model's algorithm, indicating how much the model's accuracy decreases when data for that variable is permuted. In the context of a causal forest, this measure helps identify which variables are most predictive of treatment assignment—in this case, exposure to misinformation. High variable importance suggests that the variable plays a significant role in the model's decisions, whereas low importance indicates a smaller role. The visualization in Figure S3 presents these importance scores, ranking the variables from the most to the least important in predicting misinformation exposure. Variables such as prior misinformation exposure, political knowledge, and conservative political alignment appear to be strong predictors, highlighting the role of pre-existing beliefs and information consumption patterns in determining exposure to misinformation.

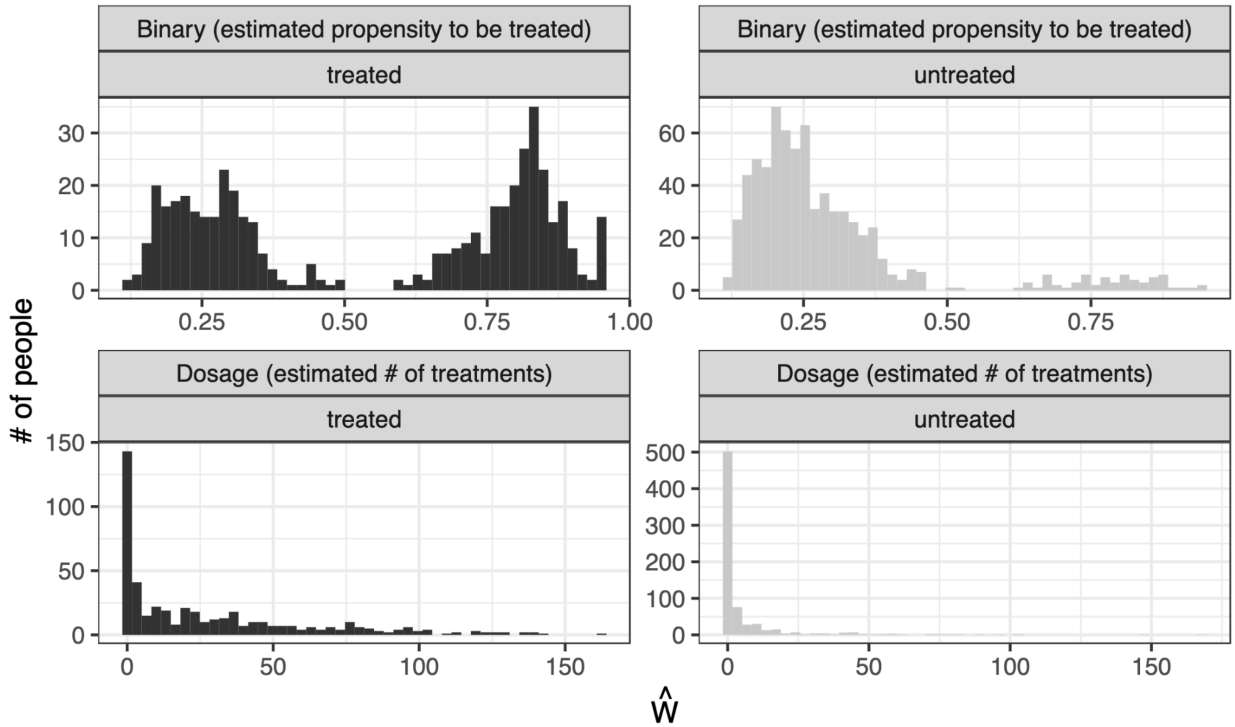
Figure S3. Variable Importance in Predicting Misinformation Exposure



Note: Variable importance scores as determined by the causal forest model for predicting exposure to misinformation websites. These scores quantify the contribution of each predictor variable to the model's predictive accuracy, with higher values indicating greater importance. The determination of variable importance is based on the mean decrease in accuracy when the variable's values are permuted across the out-of-bag samples, a common method in random forest algorithms. This metric reflects the relative significance of each variable within the model, for example, with 'MisinfoExposureWave1' showing the highest importance, suggesting it is the most critical predictor in the model.

Furthermore, to better show the underlying predictions of this first model, we present Figure S4. This figure shows the distribution $\hat{W}$, the predicted propensity to be treated or the number of predicted exposures. Again, this figure shows that the models are generally accurate in predicting exposure to misinformation.

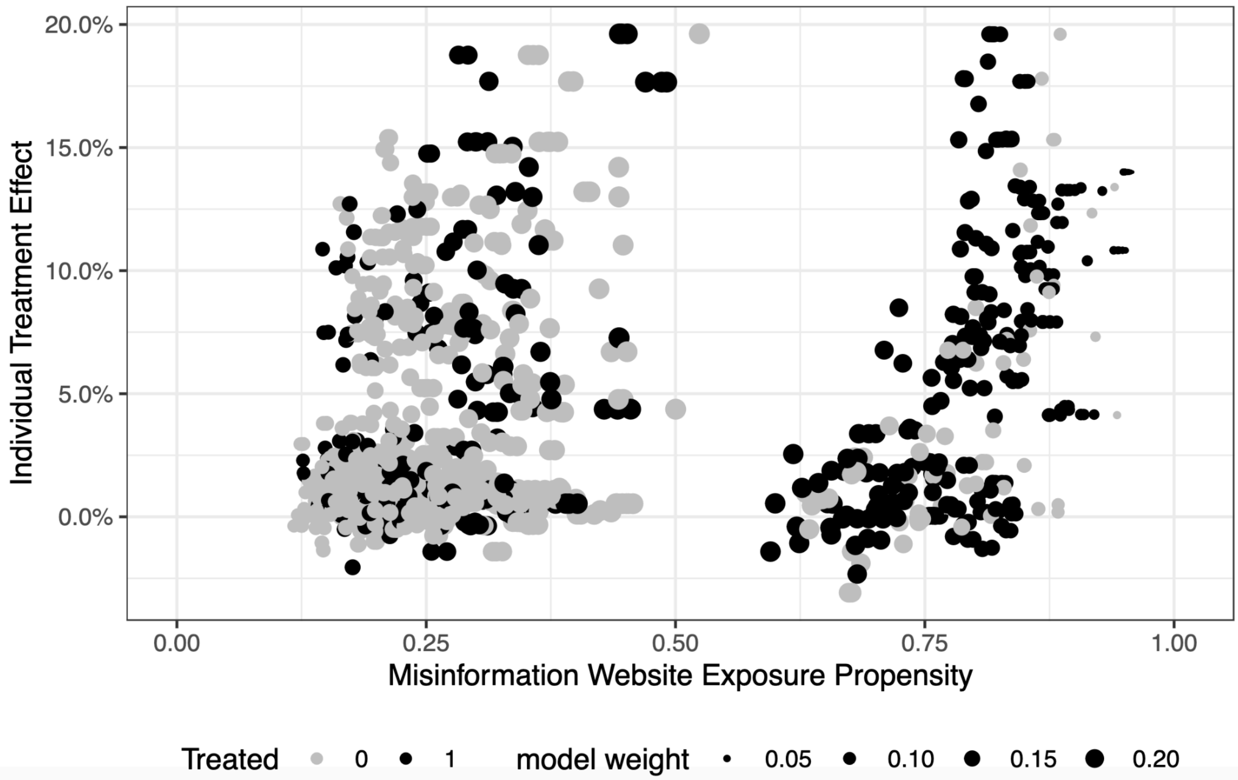
Figure S4. Distribution of $\hat{W}$ for binary and dosage exposure.



Note: Distribution of propensities for binary exposure and estimated number of misinformation websites an individual was exposed to. On the x-axis is this estimated number. In the top panel, the x-axis is the estimated propensity to be treated, the likelihood of being exposed to a misinformation article. In the bottom panel, the x-axis is expected number of misinformation websites each individual is exposed to. The y-axis for both panels is the count of people. The black bars represent people who were actually treated, exposed to at least one misinformation websites. The grey bars are people who were not exposed.

Next, we show how these propensities are incorporated into the second machine learning model output by showing the propensity to be exposed, calculated individual treatment effect, and the model weight in Figure S5.

Figure S5. Propensity to be Exposed, Individual Treatment Effect, and Model Weight

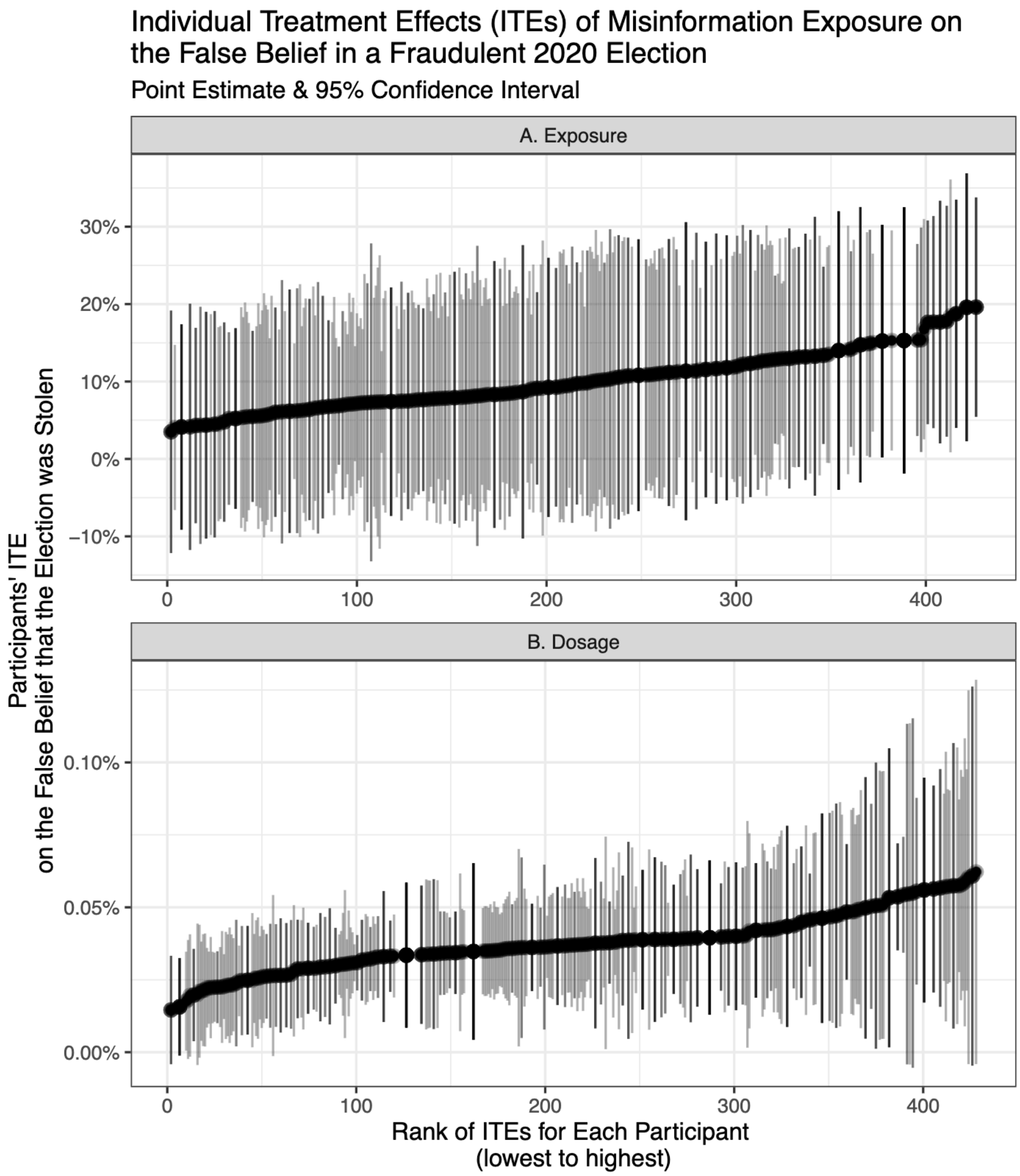


Note: Plot of the propensity of being exposed to a misinformation website and the Individual Treatment Effect. Points are sized by the model weight, which is based on the propensity to be exposed. Points are colored by whether the individual was “treated” (actually exposed to a misinformation website).

D. Individual Treatment Effects

Under the expected outcomes framework, we calculate each individual's individual treatment effects (ITE) and show conservatives' ITEs in a caterpillar plot in Figure S6.

Figure S6. Individual Treatment Effects (ITEs) of Misinformation Exposure on the False Belief in a Fraudulent 2020 Election for Conservatives



Note: Plot of estimated Individual Treatment Effects (ITEs) and 95% confidence interval of misinformation website exposure on the belief that Donald Trump won the 2020 U.S. Presidential Election for each conservative individual in our sample. Participants are ordered along the x-axis from the lowest conditional estimate to the highest, and the y-axis is the estimated conditional average treatment effect

E. Omitted Variable and Confounder Robustness Checks

We test for omitted variable bias test using the method and robustness cutoff suggested by (Oster, 2019), as implemented by the robomit R package (v1.0.6 52). Specifically, we conduct bootstraps with replacement for 1,000 simulations with 1,000 draws each. The R_{max} value we use is the suggest R^2 of the full controlled model multiplied by 1.3, as suggested by (Oster, 2019). The bias-adjusted coefficients are in table S3. Although this omitted variable bias robustness test is conducted on the regression analysis, the fact that the coefficients remain positive suggest that the results are robust to omitted variable bias. This test is fairly conservative, with only 45% of nonrandomized results in top Economics journal passing this robustness test (Oster, 2019).

Table S3. Omitted Variable Bias Test

Model	$\hat{\beta}^{base}_{ExposedMisinfo}$	$\hat{\beta}^{full}_{ExposedMisinfo}$	$\hat{\beta}^{bias-adjusted}_{ExposedMisinfo}$
Binary exposure	0.173302	0.057813	0.000965
Dosage	0.000809	0.000381	0.000239

Note: Omitted variable bias test as found in Oster (2019). The Model column distinguishes between the binary exposure and dosage models. The $\hat{\beta}^{full}_{ExposedMisinfo}$ column is the coefficient in the base model from the Gelbach decompositions, and $\hat{\beta}^{bias-adjusted}_{ExposedMisinfo}$ is the full model from the decompositions (see Table 1 and 2). The $\hat{\beta}^{bias-adjusted}_{ExposedMisinfo}$ is the bias-adjusted coefficient using the method suggested by Oster (2019).

We also examine the robustness of the assumption of non-confoundedness via a causal sensitivty analysis. To operate under the conservative assumption that we do not know the specific parameters of a potential confounding variable (53), we calculate a Robustness Value (r-value) (54) using the tipr R package (v1.0.1). The Robustness Value (RV) represents the percentage of the residual variance that an unobserved confounder would have to explain of both the treatment and outcome that would “explain away” the observed treatment effect. We report these RVs in Table S4.

Table S4. Robustness Values for significant Causal Forest results.

Result	RV
Exposure	0.0602685
Exposure - Trump Supporters	0.1052042
Dosage	0.1183523
Dosage - Trump Supporters	0.1652413

Note: The Robustness Value (RV) represents the percentage of the residual variance that an unobserved confounder would have to explain of both the treatment and outcome that would “explain away” the observed treatment effect.

To put these values into perspective, we go variable-by-variable in X, removing the given value in the effect estimation in the Causal Forest process. We then examine whether this removed variable confounds the analysis given the RV for the analysis and the residual variable explained by the removed variable. These results are in Table S5. For example, in our main analysis of the effect of exposure to misinformation, we find that only political ideology and college education would confound the results. In other words, any unobserved confounder would need to confound our results more than other variables we observe, such as misinformation exposure in Wave 1, gender, race, political knowledge, political interest, and age. Given these variables would not confound the analysis, we believe our results are robust to potential confoundedness.

Table S5. Confounder Analysis.

	Exposure		Exposure - Trump Supporters		Dosage		Dosage - Trump Supporters	
variable	res. var.	confounder	res. var.	confounder	res. var.	confounder	res. var.	confounder
MisinfoExposureWave1	0.040	No	0.001	No	0.000	No	0.006	No
Conservative	0.749	Yes	0.000	No	0.032	No	0.000	No
EduCollegeGrad	0.038	No	0.128	Yes	0.018	No	0.034	No
EduHSorLess	0.001	No	0.120	Yes	0.025	No	0.069	No
EduPostGrad	0.043	No	0.065	No	0.335	Yes	0.233	Yes
EduSomeCollege	0.114	Yes	0.525	Yes	0.428	Yes	0.425	Yes
Female	0.001	No	0.004	No	0.021	No	0.097	No
RaceBlack	0.028	No	0.001	No	0.004	No	0.000	No
RaceHispanic	0.004	No	0.009	No	0.004	No	0.009	No
RaceOther	0.000	No	0.005	No	0.001	No	0.004	No
RaceWhite	0.028	No	0.010	No	0.009	No	0.013	No
PoliticalKnowledge	0.000	No	0.019	No	0.127	Yes	0.157	No
PoliticalInterest	0.005	No	0.002	No	0.032	No	0.034	No
AgeUnder30	0.009	No	0.000	No	0.000	No	0.000	No
Age30to44	0.003	No	0.015	No	0.000	No	0.000	No
Age45to65	0.005	No	0.021	No	0.079	No	0.092	No
Age65plus	0.034	No	0.059	No	0.100	No	0.117	No

Note: Confounder analysis assessing whether a given observed control variable would confound the results if left out of the analysis.

Furthermore, we also estimated the effect of misinformation exposure on a variable of political behaviors in Table S6. We do not find significant results for any of these behaviors, providing further evidence that our main analysis is robust to picking up a latent confounding variable.

Table S6. Other Dependent Variables

DV	ATE	se	CI	z-value	p-value
Support - Trump	0.019	0.015	[-0.011, 0.049]	1.240	0.215
Attend Rally - Trump	0.002	0.009	[-0.016, 0.019]	0.208	0.835
Attend Rally - Biden	-0.003	0.007	[-0.017, 0.011]	-0.447	0.655
Attend Rally - Other	0.016	0.010	[-0.003, 0.036]	1.621	0.105
Volunteer - Biden	-0.014	0.013	[-0.039, 0.01]	-1.128	0.259
Volunteer - Trump	-0.004	0.006	[-0.016, 0.008]	-0.626	0.531
Volunteer - Other	-0.007	0.011	[-0.029, 0.015]	-0.643	0.520
Volunteer - Poll Worker	-0.012	0.008	[-0.027, 0.004]	-1.436	0.151
Donation - Trump	-0.001	0.015	[-0.031, 0.029]	-0.088	0.930
Donation - Biden	-0.003	0.023	[-0.047, 0.042]	-0.123	0.902
Donation - Other	0.006	0.024	[-0.041, 0.053]	0.248	0.804
Social Media - Trump	0.028	0.020	[-0.012, 0.068]	1.387	0.165
Social Media - Biden	-0.014	0.026	[-0.065, 0.038]	-0.512	0.608
Social Media - Other	0.047	0.025	[-0.001, 0.095]	1.919	0.055
Yard Sign - Trump	0.010	0.015	[-0.019, 0.039]	0.654	0.513
Yard Sign - Biden	-0.023	0.017	[-0.057, 0.012]	-1.297	0.195
Yard Sign - Other	-0.018	0.017	[-0.052, 0.016]	-1.054	0.292

Note: Additional analysis of other outcome variables. “DV” = dependent variable. “ATE” = average treatment effect. “se” = standard error. “CI” = 95% confidence interval. “z-value” = z-value. “p-value” = p-value.