$$y_t = \beta_0 + \beta_1 x_t + v_t,$$

as we showed in part (ii): $\text{Cov}(x_{t-1},v_t) = \text{E}(x_{t-1}v_t) = 0$. Second, x_{t-1} will often be correlated with x_t, and we can check this easily enough by running a regression of x_t of x_{t-1}. This suggests estimating the equation by instrumental variables, where x_{t-1} is the IV for x_t. The IV estimator will be consistent for β_1 (and β_0), and asymptotically normally distributed.

SOLUTIONS TO COMPUTER EXERCISES

C15.1 (i) The regression of log(*wage*) on *sibs* gives

$$\widehat{\log}(wage) = \underset{(0.022)}{6.861} - \underset{(.0059)}{.0279}\, sibs$$

$$n = 935,\quad R^2 = .023.$$

This is a reduced form simple regression equation. It shows that, controlling for no other factors, one more sibling in the family is associated with monthly salary that is about 2.8% lower. The *t* statistic on *sibs* is about –4.73. Of course *sibs* can be correlated with many things that should have a bearing on wage including, as we already saw, years of education.

(ii) It could be that older children are given priority for higher education, and families may hit budget constraints and may not be able to afford as much education for children born later. The simple regression of *educ* on *brthord* gives

$$\widehat{educ} = \underset{(0.13)}{14.15} - \underset{(.046)}{.283}\, brthord$$

$$n = 852,\quad R^2 = .042.$$

(Note that *brthord* is missing for 83 observations.) The equation predicts that every one-unit increase in *brthord* reduces predicted education by about .28 years. In particular, the difference in predicted education for a first-born and fourth-born child is about .85 years.

(iii) When *brthord* is used as an IV for *educ* in the simple wage equation we get

$$\widehat{\log}(wage) = \underset{(0.43)}{5.03} + \underset{(.032)}{.131}\, educ$$

$$n = 852.$$

(The *R*-squared is negative.) This is much higher than the OLS estimate (.060) and even above the estimate when *sibs* is used as an IV for *educ* (.122). Because of missing data on *brthord*, we are using fewer observations than in the previous analyses.

(iv) In the reduced form equation

$$educ = \pi_0 + \pi_1 sibs + \pi_2 brthord + v,$$

we need $\pi_2 \neq 0$ in order for the β_j to be identified. We take the null to be H_0: $\pi_2 = 0$, and look to reject H_0 at a small significance level. The regression of *educ* on *sibs* and *brthord* (using 852 observations) yields $\hat{\pi}_2 = -.153$ and se($\hat{\pi}_2$) = .057. The *t* statistic is about –2.68, which rejects H_0 fairly strongly. Therefore, the identification assumptions appears to hold.

(v) The equation estimated by IV is

$$\widehat{\log}(wage) = \underset{(1.06)}{4.94} + \underset{(.075)}{.137}\, educ + \underset{(.0174)}{.0021}\, sibs$$

$$n = 852.$$

The standard error on $\hat{\beta}_{educ}$ is much larger than we obtained in part (iii). The 95% CI for β_{educ} is roughly –.010 to .284, which is very wide and includes the value zero. The standard error of $\hat{\beta}_{sibs}$ is very large relative to the coefficient estimate, rendering *sibs* very insignificant.

(vi) Letting $\widehat{educ}_i$ be the first-stage fitted values, the correlation between $\widehat{educ}_i$ and $sibs_i$ is about –.930, which is a very strong negative correlation. This means that, for the purposes of using IV, multicollinearity is a serious problem here, and is not allowing us to estimate β_{educ} with much precision.

C15.2 (i) The equation estimated by OLS is

$$\widehat{children} = \underset{(0.241)}{-4.138} - \underset{(.0059)}{.0906}\, educ + \underset{(.017)}{.332}\, age - \underset{(.00027)}{.00263}\, age^2$$

$$n = 4.361, \quad R^2 = .569.$$

Another year of education, holding *age* fixed, results in about .091 fewer children. In other words, for a group of 100 women, if each gets another of education, they collectively are predicted to have about nine fewer children.

(ii) The reduced form for *educ* is

$$educ = \pi_0 + \pi_1 age + \pi_2 age^2 + \pi_3 frsthalf + v,$$

and we need $\pi_3 \neq 0$. When we run the regression we obtain $\hat{\pi}_3 = -.852$ and se($\hat{\pi}_3$) = .113. Therefore, women born in the first half of the year are predicted to have almost one year less education, holding *age* fixed. The *t* statistic on *frsthalf* is over 7.5 in absolute value, and so the identification condition holds.