Information Theoretic Accounts of Reaction Times in a Probabilistic Artificial Grammar Learning Task

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Abstract

In the present work, I utilize probabilistic artificial grammar learning via the probabilistic serial reaction time task (PSRTT) to study the effect of information-theoretic complexity metrics on reaction times. The probabilistic artificial grammars are designed to investigate the independent effects of one metric on reaction times by creating situations where one metric is manipulated while all other metrics remain constant. In Experiment 1, I designed a web-based probabilistic serial reaction time task to study the effect of surprisal on reaction times, in the form of a monster zapping game in which participants press keys to shrink growing animated monsters. Experiment 1 provides a web-based replication of in-lab studies which find a surprisal effect in the probabilistic serial reaction time task. In Experiment 2, I designed probabilistic artificial grammars to test the effect of global entropy measures on reaction times, where global entropy quantifies the uncertainty about the entire upcoming sequence of targets. Experiment 2 failed to find an effect of the entropy measures used. Future studies are required to investigate the essential conditions for entropy effects. In sentence processing, information-theoretic complexity metrics such as surprisal, quantifying how expected a word is, and entropy, quantifying the uncertainty about the sentence interpretation, have been supported to account for the reading times of words. The results in the present work suggest that analogous entropy measures do not have an effect on reaction times in the PSRTT.

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Introduction

Information-theoretic complexity metrics such as surprisal and entropy reduction are receiving considerable attention in research on human sentence processing, where they provide accounts of reading times and cognitive neuroscience measures. In the present work, we extend the use of information-theoretic complexity metrics to the domain of motor sequence processing. Understanding the effect of information-theoretic complexity metrics in other sequence processing domains would provide insight into the cognitive mechanisms of sentence processing, and would open up new avenues for these mechanisms to be studied. Additionally, investigating the effect of psycholinguistics inspired information-theoretic complexity metrics on reaction times in motor sequence learning and execution may provide novel insights into the cognitive mechanisms in motor sequence processing.

The present work uses probabilistic artificial grammar learning in a probabilistic serial reaction time task to investigate the effects of information-theoretic complexity metrics on reaction times in motor sequence processing. We construct probabilistic artificial grammars which create situations in which one information-theoretic complexity metric is manipulated while all others are held constant, providing a way to study the effect of each information-theoretic complexity metric.

We ran two web-based probabilistic serial reaction time experiments, formulated in a novel monster zapping game paradigm, in which participants press keys to shrink growing animated monsters, where the monsters grow in sequences dictated by probabilistic artificial grammars. In Experiment 1, we provide a web-based replication of previous studies which find an effect of surprisal on reaction times. In Experiment 2, we investigated whether or not entropy measures, analogous to those used in the sentence processing literature, account for reaction times. Our results suggest that analogous entropy measures to those that characterize reading times do not account well for reaction times in the probabilistic serial reaction time task.

The thesis is structured as follows. First, the thesis will provide a background of the related sentence processing literature which motivates the information-theoretic complexity metrics used and discuss related probabilistic artificial grammar learning studies. Next, I will explain the monster zapping game paradigm and the probabilistic artificial grammars used. Then, I will discuss the methods, results and conclusions from Experiments 1 and 2, and offer suggestions for future directions.

Background

The serial reaction time task (SRTT) is a widely used task to investigate the processes underlying learning, memory and language processing. In the probabilistic serial reaction time task (PSRTT), first developed by Schvaneveldt and Gomez (1998), targets appear probabilistically based on a probabilistic finite state grammar. Previous probabilistic serial reaction time task studies have demonstrated that the probabilistic structure of target appearances plays a central role in accounting for the reaction times to targets (Remillard, 2008, 2010; Remillard & Clark, 2001; Schvaneveldt & Gomez, 1998; Stadler, 1992).

Similar to the PSRTT, probabilistic structure plays an important role in accounting for processing difficulty in sentence processing. Information theory provides insights into how to formally quantify the information load of stimuli based on the underlying probabilistic structure (Hick, 1952; Shannon, 1948). Information-theoretic complexity metrics quantifying probabilistic structure such as surprisal, entropy and entropy reduction have been proposed to characterize the processing difficulty of a word in a sentence (Hale, 2016; Levy, 2008; Linzen & Jaeger, 2014). While surprisal quantifies how expected a word is given the context and entropy reduction quantifies the reduction of uncertainty about the sentence interpretation incurred by a word, both have been proposed as candidate complexity metrics to characterize which words are more difficult to process (Hale, 2006). The surprisal hypothesis proposes that words with higher surprisal are more difficult to process, whereas the entropy reduction hypothesis proposes that words with higher entropy reduction are more difficult to process. Entropy, quantifying the uncertainty about the sentence to come after processing a word, has also been proposed to explain processing difficulty such that words following a higher entropy state about the sentence to come are more difficult to process (Linzen & Jaeger, 2014; Roark, Bachrach, Cardenas, & Pallier, 2009). These hypotheses are not mutually exclusive, such that all three may play a unique role in characterizing processing difficulty of words in different situations (MacDonald & Hsiao, 2018).

Eye-tracking and self-paced reading studies on natural language suggest an effect of both surprisal and entropy reduction on reading times. Strong support for human readers' sensitivity to surprisal comes from Smith and Levy (2013), who found that reading times in eye-tracking and self-paced reading have a strong linear relationship with surprisal across several orders of magnitude. However, surprisal fails to explain reading times in relative clauses, which in part motivated the entropy reduction hypothesis (Hale, 2016).

Lowder, Choi, Ferreira, and Henderson (2018) conducted a study designed to test the effect of surprisal and entropy reduction values on reading times. Lowder et al. (2018) estimated the surprisal and entropy reduction of words by a cloze task, in which participants predict each word in a language corpus before reading it, and frequencies of the predictions are used to estimate probability distributions over the next word given the context. Subsequently, Lowder et al. (2018) engaged a separate group of participants in an eye-tracking reading task on the same text from the cloze task to obtain reading times on each word. Regressing reading times on surprisal and entropy reduction, Lowder et al. (2018) found that both suprisal and entropy reduction were significant predictors of reading times.

A second study, conducted by Frank (2013), similarly was designed to investigate the effects of surprisal and entropy reduction on processing difficulty in natural language reading. Frank (2013) computed the information-theoretic complexity metrics using a recurrent neural network (RNN) language model, trained to predict upcoming words and estimate probability distributions over the next possible words given a context. Then, Frank (2013) assessed reading times using a self-paced reading task. Similar to Lowder et al. (2018), Frank (2013) found that both entropy reduction and surprisal were significant predictors of reading times, providing support for both the surprisal and the entropy reduction hypothesis. Both Lowder et al. (2018) and Frank (2013) found entropy reduction effects while computing entropy reduction estimates in different ways and collected reading times via different tasks. This converging evidence suggests that entropy does have an effect on reading times in natural language.

It is challenging to approximate the probabilistic structure of natural language in order to estimate the information-theoretic complexity metrics. In response to this challenge, Linzen, Siegelman, and Bogaerts (2017) utilized a self-paced artificial grammar learning paradigm, involving probabilistic sequences of various shapes, to control surprisal, entropy reduction and entropy values. Understanding the effect of information-theoretic complexity metrics in a more bare-bones sequence processing task offers a controlled way to study how humans process probabilistic sequences, which can provide insights for sentence processing theory development. By using an artificial grammar learning paradigm, the probabilistic structure of sequences can be more tightly controlled than in natural language. Then, the effects of information-theoretic complexity metrics on processing difficulty can be investigated.

In Linzen et al.'s (2017) task, participants were instructed to try to remember shape sequences dictated by an artificial grammar. Next, participants pressed spacebar to reveal the next shape in the sequence, one by one, for 288 sequences total. Then, participants were given 24 test trials in which all three shapes of a sequence appeared at once and participants were asked to make grammaticality judgments. Participants who made worse than 18 out of 24 correct grammaticality judgments were excluded from the analysis. Linzen et al.'s (2017) self-paced artificial grammar learning task built on the work of Karuza, Farmer, Fine, Smith, and Jaeger (2014) who first developed this self-paced reading task for artificial grammar learning. See Figure 1 for visuals of the shapes used in this task.

Linzen et al. (2017) regressed log-transformed reaction times on the surprisal and entropy values associated with shapes in the sequences. Both surprisal and entropy were found to have significant effects on reaction times, such that surprisal and entropy were positively correlated with reaction times. Since entropy is inversely correlated with entropy reduction in their artificial grammar, their results can also be interpreted as a negative correlation of entropy reduction with reaction times, which is inconsistent with the entropy reduction hypothesis' prediction that higher entropy reduction incurs greater processing difficulty. As seen in Figure 2, the non-linear relationship between entropy and reading time suggests a weaker effect of entropy as compared to surprisal.

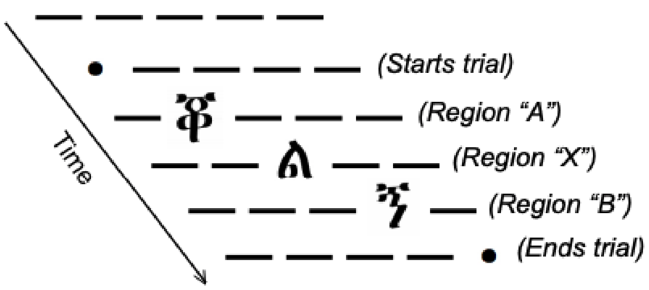

Figure 1. Stimuli for artificial grammar learning developed by Karuza et al. (2014) and used by Linzen et al. (2017). Each trial began with a row of dashes seen at the top. When space bar is pressed, the next shape appears while the previous shape disappears, akin to the self-paced reading task developed for natural language by Just, Carpenter, and Woolley (1982). Linzen et al. (2017) used sequences of length three instead of the length 5 sequences used by Karuza et al. (2014) in the visual above.

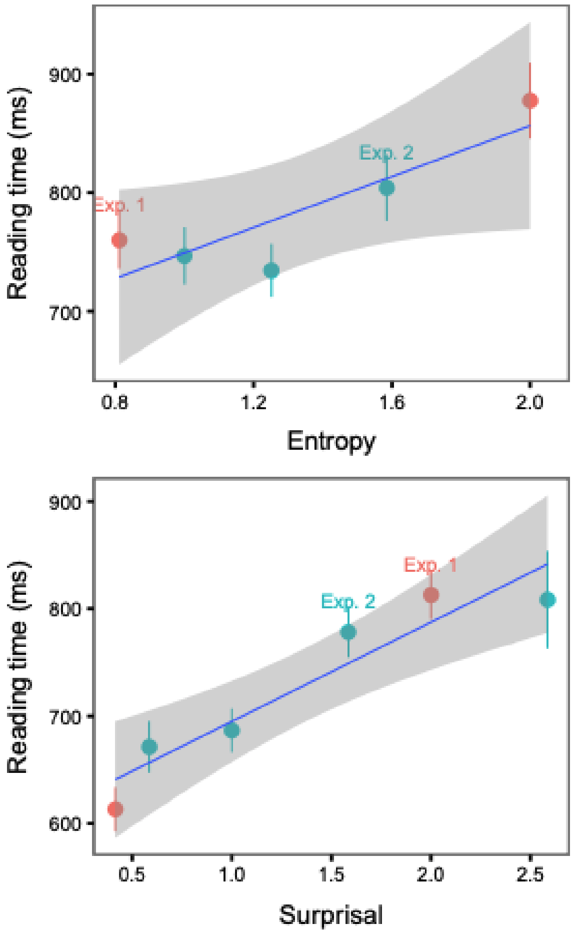

Figure 2. Linzen et al.'s (2017) results. Linzen et al. (2017) deployed different grammars in Experiments 1 and 2 which varied the surprisal and entropy values of shapes. These plots show the mean reading time, aggregated across all subjects, for various different surprisal and entropy values in Experiment 1 (red) and Experiment 2 (blue). The error bars convey the within-subject 95% confidence intervals using two standard deviations from the mean (Linzen et al., 2017). Here, we can see a clear linear relationship between the surprisal of the shape and reading times. However, the relationship between entropy and reading time appears non-linear, and suggests that the entropy effect is not as strong.

The present work builds on the work of Linzen et al. (2017) by deploying a similar artificial grammar but in a probabilistic serial reaction time task. In contrast to Linzen et al.'s (2017) task design, Experiments 1 and 2 have participants actively responding to multiple targets instead of passively pressing space bar to continue, participants are encouraged by bonus money to respond quickly and accurately, and participants are neither instructed to try to remember the sequences nor are they told that the target appearances have any structure. Therefore, the present work builds on Linzen et al.'s (2017) work, investigating the effect of information-theoretic complexity metrics on processing difficulty in probabilistic sequence processing under different task constraints and instructions.

In addition to building on the work of Linzen et al. (2017), the present study builds on a separate line of work by Pavão, Savietto, Sato, Xavier, and Helene (2016). Pavão et al. (2016) conducted probabilistic serial reaction time experiments to investigate the effect information theory inspired metrics on reaction times. In Pavão et al.'s (2016) probabilistic serial reaction time task, the grammar type dictating target appearances varied between-participants (see Figure 3b for the grammars). Inspired by information theory, Pavão et al. (2016) defined a quantity, the joint entropy, to quantify the uncertainty about the set of possible sequences of a grammar and each grammar type had its own joint entropy value. Joint entropy is a quantification of the uncertainty over all the sequences that the grammar can produce and is defined as follows: $H_{G_j}^{joint} = -\sum_{s \in S^j} P(s) log_2 P(s)$, where S^j is the set of all possible sequences in the grammar G_j and $P(s)$ is the joint probability of all the targets contained in the sequence s.

Pavão et al.'s (2016) analyses suggest joint entropy explains the variance of participants' median reaction times across various grammar types such that reaction times increase as the joint entropy of the grammar increases. Pavão et al. (2016) understood joint entropy as an information-theoretic complexity metric which quantified the degree to which a grammar is amenable to chunking in motor sequence execution. Joint entropy of the grammar increases as the number of valid transitions increases and as the number of deterministic transitions increases. Therefore, joint entropy may be capturing practice effects from participants getting more practice on particular transitions for small grammars, as well as chunking that occurs when the grammar is more deterministic.

The joint entropy takes the entropy of a very different random variable than the entropy measures which the present work will use. The present work will investigate the effect of different entropy measures in the probabilistic serial reaction time task, inspired by information-theoretic complexity metrics in the sentence processing literature. The present work will then contribute to Pavão et al.'s (2016) work by investigating different information theory inspired measures, measures which are known to have a significance already in sequence processing within natural language. In contrast to Pavão et al.'s (2016) paradigm which investigates participants' aggregated median reaction times between groups, our PSRTT paradigm contrasts reaction times within-participant so that a single participant receives two targets with different global entropy measures while other confounding variables, such as conditional probability, are held constant in the grammar. Therefore, our paradigm offers a controlled and interpretable way to investigate how entropy measures quantifying uncertainty about the global sequence structure characterize processing difficulty in the PSRTT.

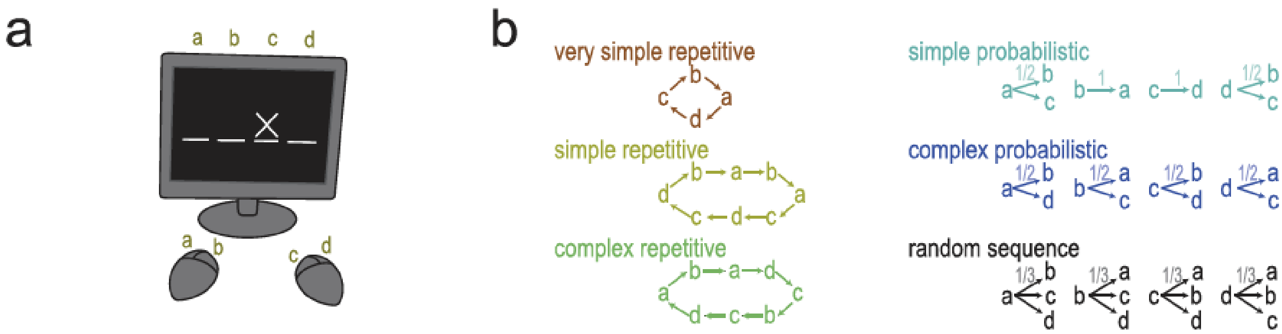

Figure 3. (a): Pavão et al. (2016) stimuli and task set up. (b): various grammars deployed between-participant with varying joint entropy values.

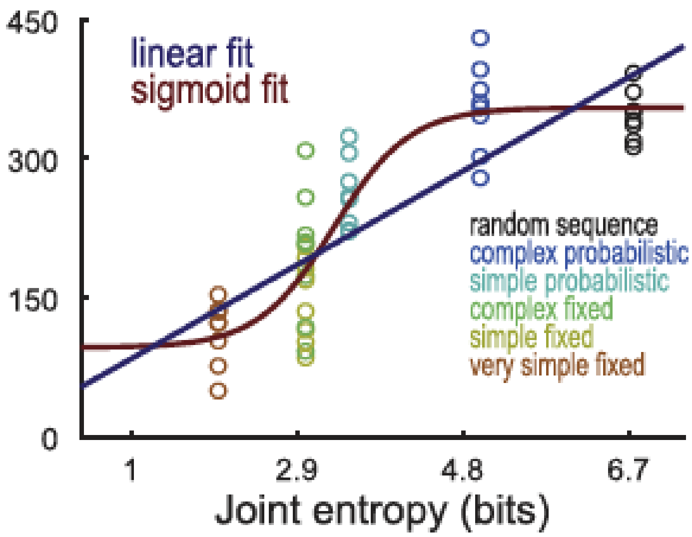

Figure 4. Pavão et al. (2016) results. Median reaction time in milliseconds for each participant in the various grammars, plotted as a function of the joint entropy (in bits) of the grammar. Here, we can see that grammars with higher joint entropy tend to have higher median reaction times on average. Joint entropy is a different metric than the entropy measures used in our paradigm.

Open Questions for Investigation

Below is a summary of questions I will address in the present work:

(1) Is web-based experimentation suitable to study the effect of probabilistic structure on reaction times in the probabilistic serial reaction time task?

(2) With different task constraints and instructions than Linzen et al. (2017), does entropy measures have an effect on reaction times?

(3) Do information-theoretic complexity metrics which explain reading times in sentence processing also explain reaction times in the probabilistic serial reaction time task?

(4) Pavão et al. (2016) found that characterizing grammars by joint entropy explained reaction times, but do different information-theory inspired measures at the level of an individual target explain reaction times?

The Monster Zapping Paradigm

Figure 5. Experiment 2 stimuli before a target monster shrinks but after it grows. In both experiments, the monsters grow and shrink in size in the same way. In Experiment 1, 4 monsters are used as targets, and in Experiment 2, 8 monsters are used.

In order to increase participant engagement we staged the web-based probabilistic serial reaction time task in Experiments 1 and 2 as a monster zapping game. In this game, participants are instructed to zap animated monsters when they grow in size by pressing the appropriate key to shrink the monster back to normal size (see Figure 5 for a target monster). The sequence of growing monsters is probabilistic and is dictated by a probabilistic finite state grammar where each target monster corresponds to a state in the grammar. If the participant presses the correct key corresponding to the enlarged target monster, within 500 milliseconds the target monster quickly shrinks to a size smaller than normal and then comes back to its normal baseline size before the next monster grows 500 milliseconds after the key press. If the participant presses the wrong key, corresponding to a monster other than the one that grew, the wrong monster that the participant pressed will shake horizontally, the target monster will just return to its baseline size, and the next monster grows 500 milliseconds after the key press in this case too. The deadline is 5 seconds, upon which the target monster shakes vertically up and down.

In both experiments, 60 trials are given as practice, and 600 trials are then given for bonus money, where a trial is the growth of one target monster. In the 600 trials for bonus money, participants are encouraged to respond quickly and accurately to earn bonus money. On each trial, a participant can earn up to 5 tenths of a cent, and the bonus pay on a trial as a function of response speed is visualized in the timeline in Figure 6. After every 20 trials, participants receive feedback about their performance, including how many monsters they zapped correctly and incorrectly, the bonus money they earned in the previous two blocks, and the total bonus money they accumulated so far. Although each trial can earn up to 5 tenths of a cent, with consistently strong performance, participants are able to almost double their base pay.

Figure 6. Bonus pay on a trial as a function of response speed.

After the 600 trials for bonus money, participants are instructed to now predict which monster grows next after seeing a sequence of monsters. Participants receive monster sequences from the grammar generated automatically as context, and then participants complete the sequence by making a prediction. Each correct prediction earns 1 cent and feedback is given at the end of the experiment. Correct predictions are those that are plausible given the grammar and the context of the automatically generated sequence. See Figure 7 for an illustration of the prediction task.

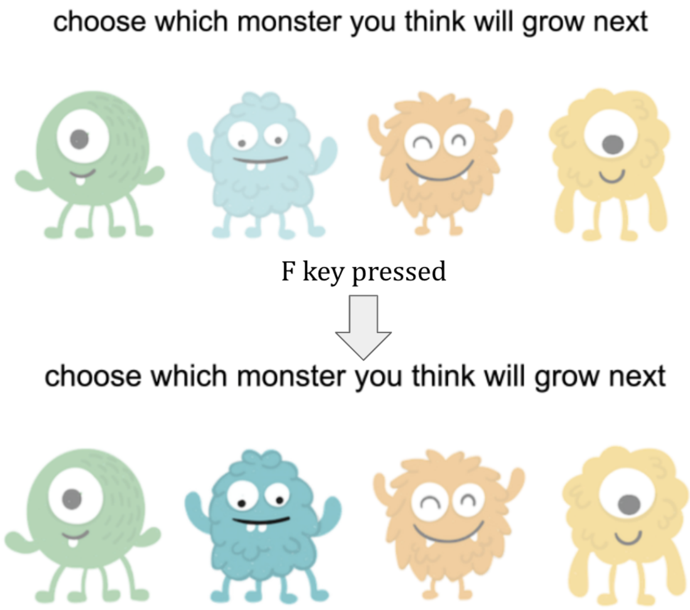

Figure 7. Prediction Task: after seeing a sequence of target monsters growing and shrinking, all monsters decrease in opacity and participants are prompted to choose the monster they think will grow next. Upon pressing a key to make a prediction, in this case F, the predicted monster returns to its normal opacity, and then the next sequence of automatically generated monsters initiates. The above figure is for Experiment 1, and in Experiment 2, the number of monsters increases to 8.

Assigning Surprisal and Entropy to Targets in the PSRTT

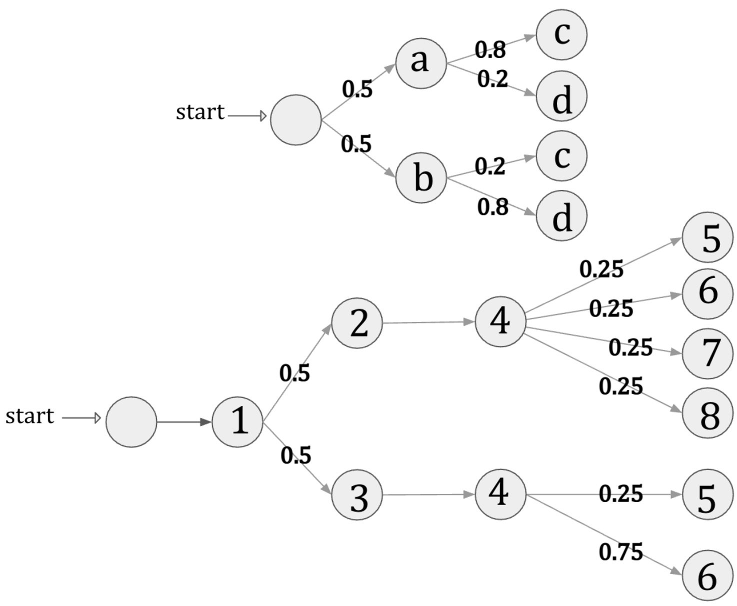

Figure 8. Probabilistic grammars from Experiment 1 (top) and Experiment 2 group A (bottom).

Surprisal and various entropy measures can be assigned to target monsters utilizing the probabilistic grammar which dictates the sequences of targets. Each symbol in the grammar corresponds to a target monster in the probabilistic serial reaction time task. See Figure 8 for example probabilistic grammars used in my experiments.

Surprisal quantifies how expected a target appearance is, given the context and is defined as follows:

$$-log_2 P(t_i|t_{i-1}\dots t_1) \tag{1}$$

where t_i is the target location in the current trial i. For example, in Figure 8 (top), the surprisal values of "a" and "b" are both $-log_2 0.5 = 1$.

Global entropy quantifies the uncertainty about the entire upcoming target sequence and the global entropy assigned to a target is the global entropy after processing the target. The global entropy of a target t_i is defined as follows:

$$H^{global}_{t_i} = -\sum_{x\in X^i} P(x) log_2 P(x) \tag{2}$$

where X^i denotes every possible upcoming target sequence after processing target t_i. For the Figure 8 (bottom) grammar, the global entropy of "4" in the context of "2" is 2 bits. After processing "4" in the context of "2", X^i is the following 4 possible upcoming targets each with probability 0.25: "5", "6", "7", "8". The global entropy of "4" in the context of "3" is 0.81 bits. Therefore, contrasting reaction times to "4" in the two contexts of "2" and "3" offers a way to test the effect of global entropy on reaction times.

Global entropy reduction quantifies the reduction in uncertainty about the entire upcoming target sequence incurred by a target. The global entropy reduction of a target is defined as follows:

$$ER^{global}_{t_i} = H^{global}_{t_{i-1}} - H^{global}_{t_i} \tag{3}$$

where $H^{global}_{t_{i-1}}$ is the global entropy before processing target t_i and $H^{global}_{t_i}$ is the global entropy after processing t_i. For the Figure 8 (bottom) grammar, the global entropy reduction assigned to "3" is 1.6 bits, while the global entropy reduction assigned to "2" is 0.4 bits, therefore contrasting reaction times to “2” and “3” offers a way to test the effect of global entropy reduction on reaction times.

Local entropy quantifies the uncertainty about only the single next upcoming target and the local entropy assigned to a target is the local entropy before processing the target. Before processing a target, the local entropy is

$$H_{t_i}^{local} = -\sum_{t \in T^i} P(t) log_2 P(t) \tag{4}$$

where T^i denotes every possible single next upcoming target. For example, the local entropy of “a” in the Figure 8 (top) grammar is $0.5 * log2\frac{1}{.5} + 0.5 * log2\frac{1}{.5} = 1$. There are two possible subsequent targets before processing “a”: “a” and “b” both with probability 0.5.

The probabilistic finite-state grammars create contexts in which two targets have different values of one metric while other metrics are held constant. In the Experiment 1 grammar, “c” and “d” in the contexts of “a” and “b” have varying surprisal values although all entropy measures are constant. In the Experiment 2 grammar, “2” has different global entropy measures than “3” although both have equal local entropy and surprisal and similarly for “4” in the two different contexts. See Table 1 below for the information-theoretic complexity metric values of the symbols in the Experiment 2 grammars.

Experiment 2 Group A Grammar:

Symbol	Surprisal	Global Entropy	Global Entropy Reduction	Local Entropy
1	0	2.4	N/A	0
2	1	2	0.4	1
3	1	0.8	1.6	1
4 after 2	0	2	0	0
4 after 3	0	0.8	0	0

Experiment 2 Group B Grammar:

Symbol	Surprisal	Global Entropy	Global Entropy Reduction	Local Entropy
1	0	2.5	N/A	0
2	1	2	0.5	1
3	1	1	1.5	1
4 after 2	0	2	0	0
4 after 3	0	1	0	0

Table 1

Information-theoretic complexity metrics (in bits) for Experiment 2 grammars in groups A (top) and B (bottom).

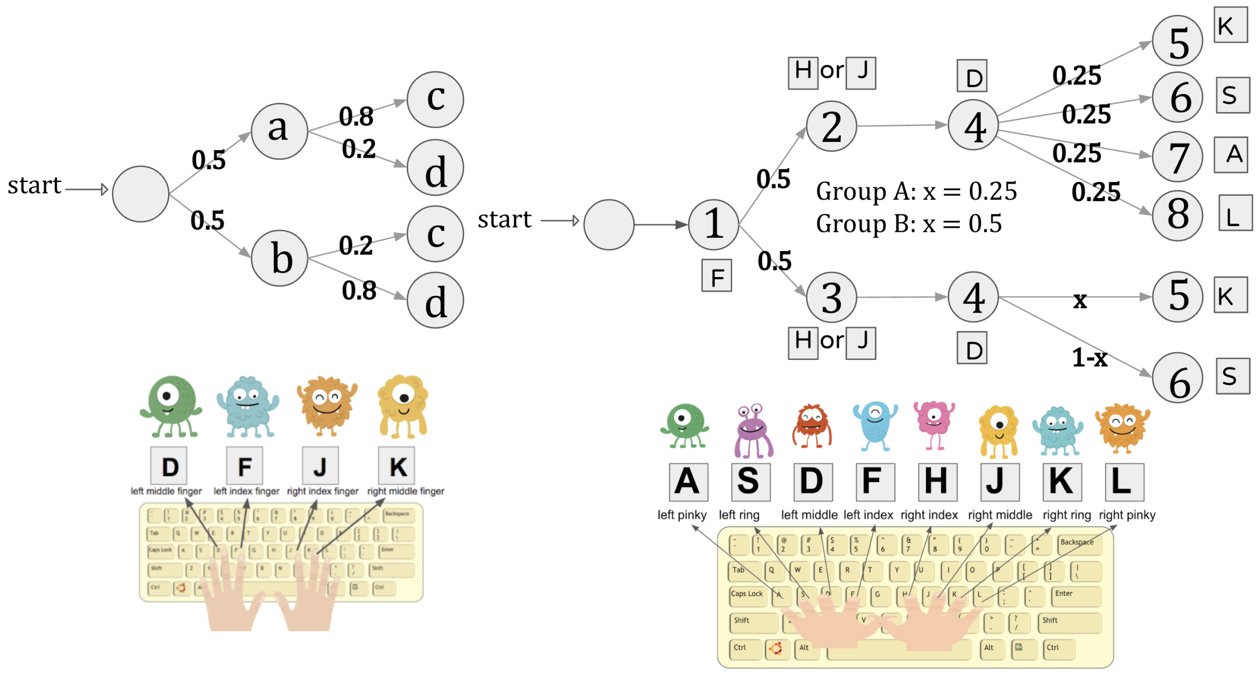

Figure 9. Probabilistic Finite-State Grammars, Stimuli, and Finger to Key to State Mapping. Left: Experiment 1. Right: Experiment 2.

Experiment 1

Experiment 1 is a web-based probabilistic serial reaction time task investigating the relationship of surprisal, the log of the inverse conditional probability of a target appearing, with reaction times in a novel web-based setting formulated as a monster zapping game.

Methods

Data Collection, Materials & Procedure. 150 participants total were recruited from Prolific.co to complete the 30-minute study; however, data from 4 participants were lost due to normal data loss via Pavlovia servers. Participants were paid a $3.25 base pay and can make up to $6.50 total with bonus money. Exclusion criteria ensured that participants' first language is English and participants are fluent in English. Participants from our pilot studies are excluded by Prolific.co. The experiment could only be taken in a chrome browser, due to technical issues I discovered with other browsers. Utilizing Bridges, Pitiot, MacAskill, and Peirce's (2020) study which compared the web-based reaction time precision of various web browsers, I confirmed that PsychoPy on Chrome browsers offered sufficient reaction time precision. Participants were required to use a monitor with a 60 Hz refresh rate so that the stimulus display rate was constant across participants' different monitors.

Experiment 1 has two parts. The first is a probabilistic serial reaction time task formulated as a monster zapping game where participants press keys D, F, J, and K to "zap" 4 corresponding monsters that get larger and shrink when participants press the corresponding key. See Figure 9 (left) for finger to key mappings.

The prediction task asked participants to complete 32 predictions, 8 predictions in each of the 4 possible adjacent target contexts.

Participants are randomly split approximately in half into groups A and B. Group A receives sequences sampled from the probabilistic finite-state grammar in Figure 9 (left). In group B, participants receive random sequences where on each trial, a target is sampled from a uniform distribution over the 4 possible targets. For each participant in group A, length 2 sequences were sampled 330 times for 660 trials total from a probabilistic finite state grammar with sequences "ac", "ad", "bc", and "bd" with corresponding probabilities of 0.4, 0.1, 0.1 and 0.4. Each of the four symbols in these sequences are assigned to a unique finger and key out of the 4 finger-key mappings seen in Figure 9 (left).

The experiment code is written using the PsychoPy builder and utilized PsychoPy builder's automatic JavaScript export for web-based experimentation. Condition files for each participant were created via custom Python scripts. Participants were randomly assigned to a treatment group upon beginning the study via JavaScript's Math.random() function. To experimentally control for the effect of finger transitions, we randomized the mapping from symbols ("a", "b", "c","d") of the grammar to keys (D, F, J, K) between-participant by sampling the mapping from a discrete uniform distribution with 24 possible outcomes. Each of the 24 symbol-to-key mappings were assigned to at least one participant; however, some of the mappings were more frequent than others due to random sampling from a uniform distribution. Additionally, we randomized the order of the monsters on the screen between-participants.

Statistical Analysis Model. I utilized the brms R library to specify Bayesian models in R while stan is executed under the hood (Bürkner, 2017a, 2017b, 2017c). The effect of surprisal on reaction times was analyzed via Bayesian mixed effects lognormal regression models (Baayen, Davidson, & Bates, 2008). Thereby, taking $y_{i,j}$ to be the reaction time of the i^{th} individual and j^{th} trial, the reaction time is assumed to have been generated by the following equation under the lognormal model:

$$y_{i,j} = e^{\mu_{i,j} + e_{i,j}}$$

where $\mu_{i,j}$ is the mean function on the log-milliseconds scale and $e_{i,j} \sim \mathcal{N}(0, \sigma^2)$ is the error term sampled for the i^{th} individual and j^{th} trial. Furthermore,

$$log(y_{i,j}) = \mu_{i,j} + e_{i,j}.$$

The mean function on the log-milliseconds scale for the i^{th} individual and j^{th} trial is described as follows:

$$\mu_{i,j} = \beta_{0,i} * 1 + \beta_{1,i} * x_{i,j,1} + \cdots + \beta_{p,i} * x_{i,j,p}$$

where p is the number of predictors.

Each $\beta_{k,i}$, for $k = 1, \ldots, p$ will be equal to the sum of a parameter α_k which is shared amongst all individuals and a random effect value $\epsilon_{k,i}$ for each individual:

$$\beta_{k,i} = \alpha_k + \epsilon_{k,i}$$

. The random effect values for each regression coefficient of each participant will be sampled from a multivariate Gaussian distribution, and is described as follows:

$$\epsilon_{\mathbf{i}} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$$

where a set of parameters specifies the covariance matrix, $\mathbf{\Sigma}$. Note that some predictors did not receive random effects due to model complexity constraints.

Reaction times were regressed on data from both groups A and B using the following fixed effects: intercept, surprisal value computed from ideal observer, finger transition (16-level categorical variable), trial index to capture motor learning speed up, random sequence or grammar sequence mode (2-level categorical variable), and interaction between sequence mode and surprisal. Random effects included surprisal and the intercept.

The two continuous predictors in the analysis, surprisal and trial number, were centered to have mean of 0. The sequence mode categorical variable is 1 for group B participants, who receive random sequences, and -1 for group A, who receive sequences from the grammar.

The Ideal Observer Model. In order to compute surprisal estimates which reflect what the participant could have learned about the grammar as the experiment progresses, I constructed two ideal observer models, one for each of the sequence conditions. In the grammar sequence condition, I dynamically construct 4 conditional probability distributions of the current target in trial i given the previous target in trial $i-1$, utilizing a transition count matrix in which an element at position j, k increments when target k appeared after target j. The transition count matrix is 4 by 4 in Experiment 1 since we have 4 targets, where the rows denote the previous target and the columns denote the current target. Before the first trial, the transition matrix is set to include all 1's to assume no prior knowledge about the transition probabilities from one target to another. The ideal observer model only tracks the previous adjacent target, and therefore assumes that the probability of the target in trial i only depends on the previous target in trial $i-1$.

When computing the surprisal value for target k in the context of target j, first I compute the sum of the row corresponding to target j. Then, I divide the frequency count of target k in this row by the row sum to estimate the probability of target k in the context of target j at the current trial of the experiment. Surprisal is then estimated by taking the negative log_2 of this probability estimate.

Since the sequences are sampled probabilistically from the grammar, it is likely that two participants receive a different sample from the grammar. Surprisal estimates from the ideal observer model take this into account, since the ideal observer model will dynamically compute the surprisal estimates for the current trial based on what the participant has received so far in the experiment. As the experiment progresses, the ideal observer surprisal estimates converge to the surprisal based on the probability set in the grammar.

Additionally, a separate ideal observer model is used to capture fluctuations in surprisal in the random sequence group. Instead of assuming that the probability of the current target depends on the previous target, this ideal observer model assumes that the current target does not depend on the previous targets. Similar to the first ideal observer model, this ideal observer model initially assumes that all targets are equally probable. This second ideal observer model dynamically updates a vector of frequencies, recording how many times a particular target has appeared in the experiment. The probability of target k on trial i is estimated by taking the frequency recorded for target k and dividing it by the sum of all of the frequencies in the vector. Then, the surprisal value for a target in the random sequence group is computed by taking the negative $log2$ of that probability.

Data Exclusions. I excluded trials where the target monsters corresponded to "a" and "b," since the contrasts of interest were "c" and "d" in the two different contexts, whose surprisal values vary. Only the 600 trials for bonus money are included in the analysis. I excluded trials where the participant is inaccurate since those reaction times are not meaningful when evaluating the effect of surprisal. Trials where the previous trial is inaccurate are also excluded, since the finger transition is less interpretable for those trials. Trials with reaction times greater than 1000 milliseconds are excluded, since those trials are clear outliers in Experiment 1.

Results

With weakly informative priors for the surprisal fixed effects, centered at zero, the posterior distribution on the surprisal fixed effect is precisely centered at 0.07 with a 95% credible interval of [0.06,0.08]. The surprisal and sequence mode interaction is centered around -0.02 with a 95% credible interval of [-0.03,-0.01]. These results suggest that the geometric mean of reaction times on the milliseconds scale increases by 9% for an increase of surprisal by 1 bit for group A (grammar sequences), and by 5% for group B (random sequences). See Figure 10 for the priors, and Figure 11 for the posteriors over the surprisal fixed effects regression coefficients.

As seen in Figure 12, the reaction times to high surprisal targets are clearly higher than to low surprisal targets, and the difference appears to increase as the trials increase.

Accuracies were high, with an average above 90% for both groups and no abnormally low performance which would require data exclusion.

In the prediction task, participants in group A appeared to learn the grammar and participants in group B appeared to learn the lack of structure. As seen in Figure 13, the predictions match the true conditional probability distributions of targets given the context.

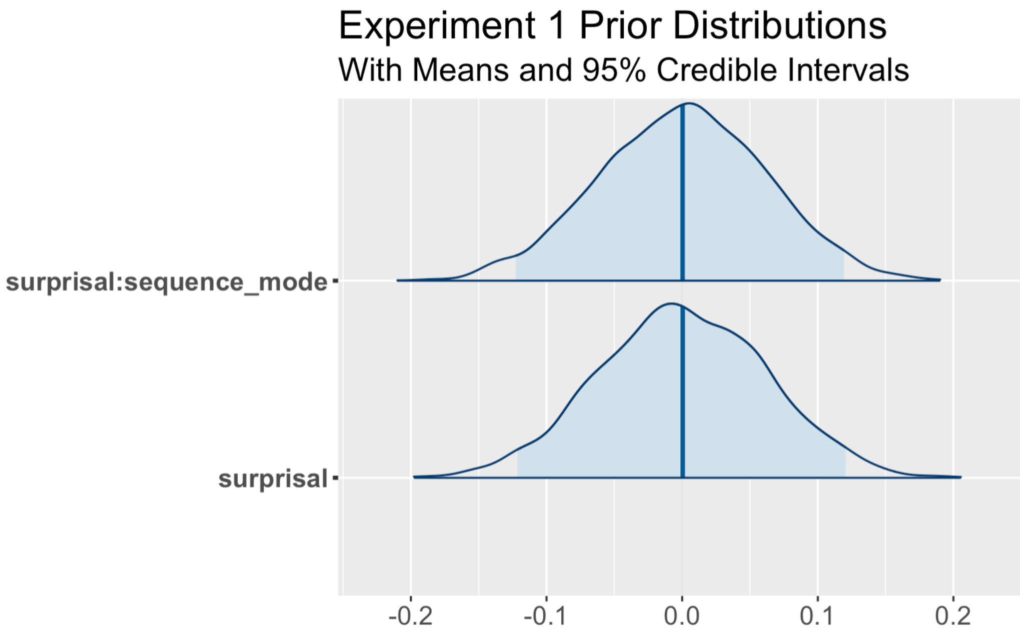

Figure 10. Experiment 1 prior distributions placed on fixed effect of surprisal and the interaction between surprisal and sequence mode. The priors are weakly informative and centered at zero.

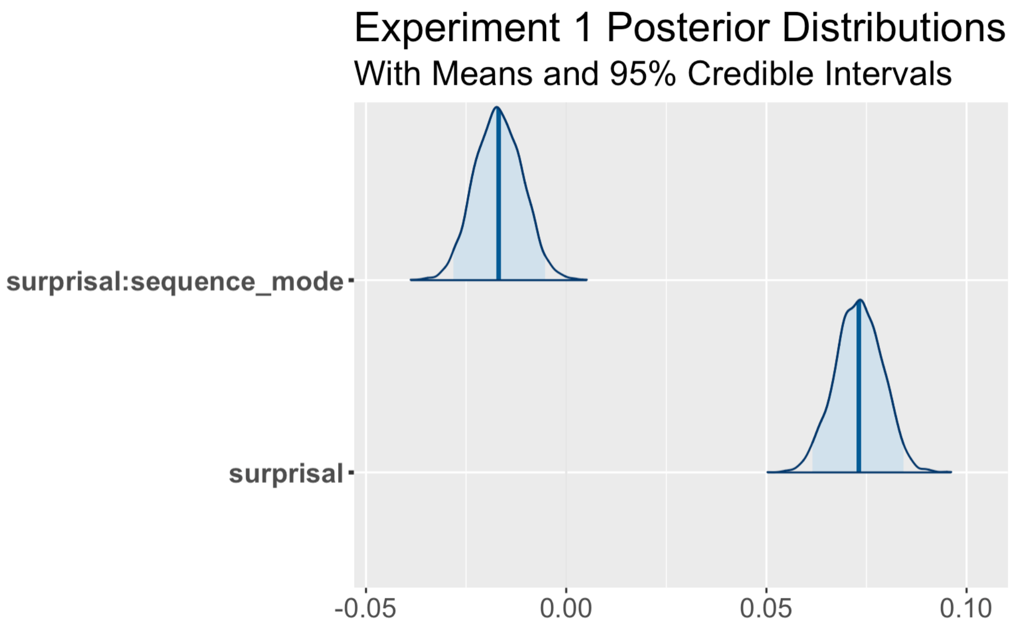

Figure 11. Experiment 1 posterior distributions on the surprisal fixed effect and the interaction between surprisal and sequence mode.

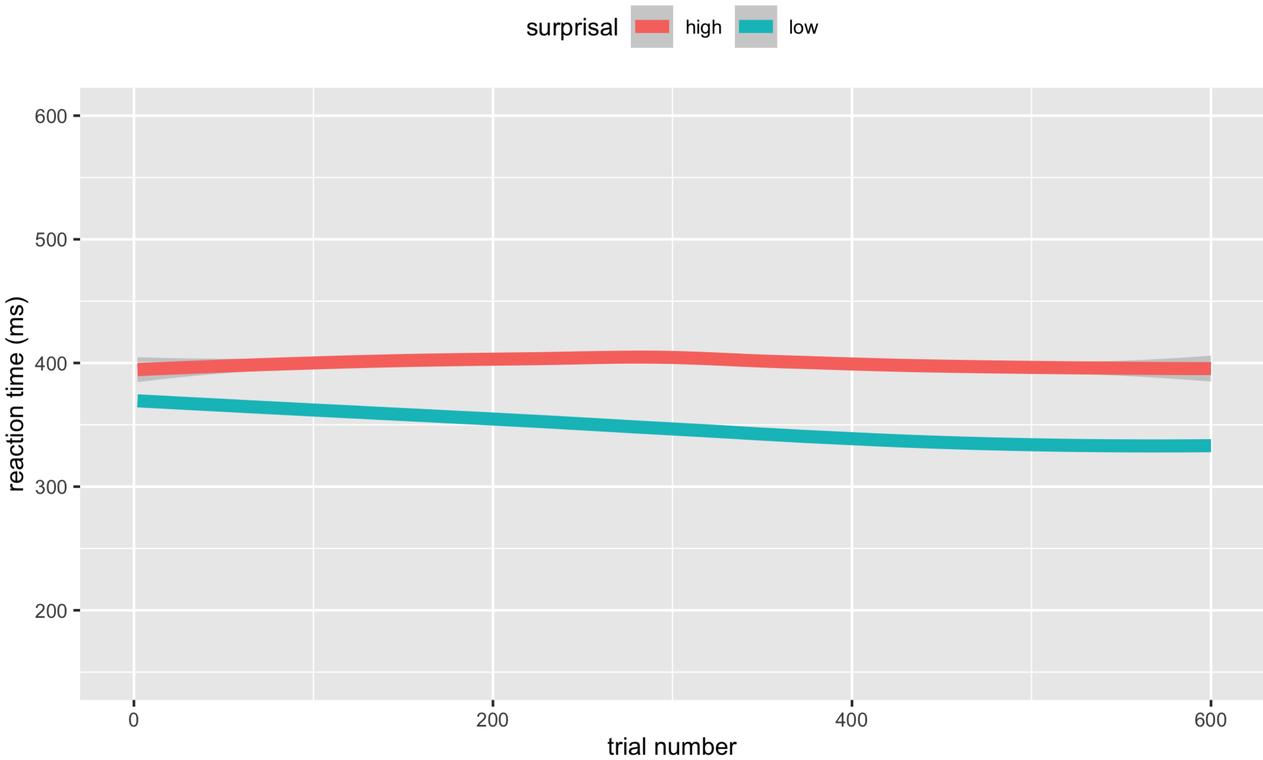

Figure 12. Reaction times across trials for surprisal contrasts for group A participants (grammar sequences). Surprisal contrasts are for reaction times to “c” and “d” in Experiment 1 where “c” and “d” have a high surprisal of 2.3 bits or a low surprisal of 0.3 bits depending on the context.

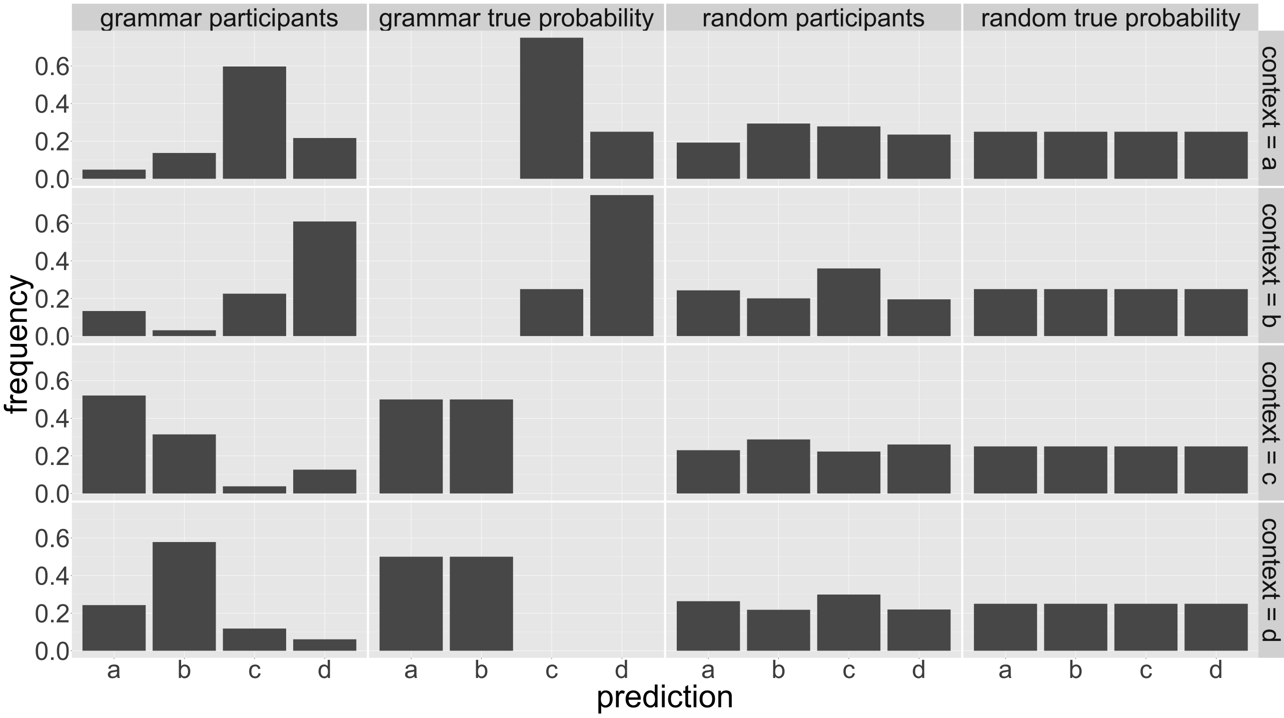

Figure 13. Experiment 1 prediction task results. Here each row denotes a different preceding target context in the automatically generated sequence in the prediction task, and the histograms plot the frequencies of the 4 possible predictions in each context. Participants in the grammar group appear to have acquired knowledge about the grammar. The "grammar participants" column contains the aggregated and normalized prediction frequency for the group A participants in the four different contexts. Comparing the "grammar participants" and "grammar true probability" histograms, we see that participants appear to have acquired knowledge about the grammar. The participants who received random sequences learned the lack of structure in the sequences, as seen by comparing the "random participants" column to the "random true probability column."

Discussion. Experiment 1 provides a web-based replication of transition probability effects in the probabilistic serial reaction time task, first found by Schvaneveldt and Gomez (1998). Surprisal had a clear effect on reaction times, and the effect became more pronounce as the trials increased, presumably as a result of grammar learning. This web-based replication sets the stage for Experiment 2, demonstrating that the web-based monster zapping paradigm is at least sensitive to surprisal effects which have been only been demonstrated in-lab to date. Sævland and Norman (2016) provided a web-based replication of various implicit learning tasks excluding the probabilistic serial reaction time task. Therefore Experiment 1 also builds upon Sævland and Norman (2016), demonstrating that another seminal implicit learning task can be deployed on the web and detect effects which have been previously found in-lab.

Experiment 2

Experiment 2 investigates the effects of entropy measures related to the global sequence structure on reaction times, akin to the entropy measures used in the sentence processing literature. Experiment 2 utilizes a variation of the probabilistic serial reaction time task deployed in Experiment 1, but with 8 target monsters in order to accommodate more complex grammars. The grammars provide a way to test the effect of global entropy measures while holding local conditional probability measures constant in the grammar.

Methods

Data Collection, Materials & Procedure. The experiment code and condition files were created via the same software as Experiment 1. 225 participants, 75 in each of the 3 treatment groups, were recruited from prolific.co to complete the 35-minute study. Data from 15 participants were lost due to normal data loss on Pavlovia servers and data from three participants were excluded due to unreasonably poor performance. Participants were paid a $3.80 base pay and could make up to $7.30 total with bonus money. Exclusion criteria were the same as Experiment 1.

Experiment 2 has two parts. The first is a probabilistic serial reaction time task formulated as a monster zapping game where participants press keys A, S, D, F, H, J, K, and L to “zap” 8 corresponding monsters that get larger and shrink when participants press the corresponding key. The structure of Experiment 2 has the same two parts as Experiment 1. In order to accommodate the additional sequence contexts of the more complex grammars, the second part of Experiment 2 contains 2 practice predictions and then 56 predictions rewarded by 1 cent for correct predictions.

Participants are randomly split into three groups: A, B, and C. Groups A and B receive sequences sampled from a probabilistic finite-state grammar. In group C, participants receive sequences where the target monster location in each trial is sampled from a uniform distribution over all 8 monsters.

In groups A and B, length 4 sequences will be sampled 165 times for 660 trials total from a probabilistic finite-state grammar utilizing the random sampling functionality of the Python numpy library. The probabilistic finite-state grammar in Experiment 2 contains symbols “1” through “8” and the following sequences which correspond to sequences of target locations: “1-2-4-5”, “1-2-4-6”, “1-2-4-7”, “1-2-4-8”, “1-3-4-5”, and “1-3-4-6.” The sequences have corresponding probability of 0.125, 0.125, 0.125, 0.125, 0.5x and 0.5 - 0.5x. Here, x is a variable which varies between groups A and B. In group A, x = 0.25, thereby “1-3-4-5” has probability 0.125 and “1-3-4-6” has probability 0.375. In group B, x = 0.5. See Figure 9 (right) for the grammars for groups A and B.

To experimentally control for the effect of finger transitions on reaction times in the groups A and B, we randomized the mapping from symbols of the probabilistic finite-state grammar to keys on the keyboard. In the two groups, all symbol to key mappings are fixed, except “c” and “d” are randomly assigned to the H key or J key. See Figure 9 (right) for the finger to key to symbol mapping for all groups in Experiment 2. The condition files were generated such that participants in groups A and B are approximately split in half to one of these two mapping types. Additionally, we randomized the order of the monsters on the screen between-participants as in Experiment 1.

Statistical Analysis Methods. We used a Bayesian mixed effects lognormal regression model to analyze Experiment 2, similar to Experiment 1. We ran two analyses. The first investigated the effect of global entropy reduction on reaction times by contrasting reaction times to “2” and “3” in groups A and B. The second investigated the effect of global entropy on reaction times by contrasting reaction times to “4” in groups A and B.

Statistical Analysis 1. In the first analysis, a model was fit to test the effect of global entropy reduction on reaction times, by contrasting the reaction times at “2” and “3” in the grammar shown in Figure 9 (right). Surprisal and local entropy is the same for “2” and “3”; however, global entropy reduction is higher on “3.” It is impossible to perfectly tease apart global entropy and global entropy reduction: here the global entropy after processing “2” is higher than for “3.” In order for there to be global entropy reduction differences between two targets, the global entropy either before processing or after processing the targets must be different.

This first analysis was run on a data set exclusively on trials where “2” and “3” are the targets, for groups A and B only. Trials only in the second half of the experiment are considered for the analysis, since learning the grammar before the first half of the trials is unlikely.

In analysis 1, the following fixed effects were used: global entropy reduction (in bits), surprisal computed from ideal observer model (in bits), finger transition (2-level categorical variable: left index (F) to right index (H) or left index (F) to right middle (J)), trial index to capture motor learning speed up (treated as a continuous variable). Random effects included the global entropy reduction and intercept, by participant.

We used an ideal observer model, described in the Experiment 1 methods, to compute the surprisal values of “2” and “3” as the task unfolds by dynamically approximating the knowledge participants gain about the probabilistic structure of the sequences. While “2” and “3” have the same probability following “1” in the grammar, our sequences are sampled from a probability distribution so we expect minor fluctuations of the surprisal values, which will be accounted for by the ideal observer surprisal predictor.

Statistical Analysis 2. In the second analysis, we fit a model to test the effect of global entropy on reaction times, by contrasting the reaction times at "4" in the two different contexts in the grammar shown in Figure 9 (right). Surprisal, local entropy, and global entropy reduction are the same for "4" in the two different contexts; however, global entropy is higher for "4" after "2." This second analysis was run on a data set exclusively on trials where "4" are the targets, for groups A and B.

For analysis 2, we used the following fixed effects: intercept, global entropy (in bits), surprisal of previous trial computed from the ideal observer model to capture frequency effects of the particular finger transition, finger transition (2-level categorical variable: right index (H) to left middle (D) or right middle (J) to left middle (D)), and the trial index to capture motor learning speed up (treated as a continuous variable). Random effects included the intercept and global entropy.

Results

In analysis 1: with a weakly informative prior centered at zero on the global entropy reduction fixed effect, the posterior of the global entropy reduction fixed effect has mean 0.01 with a 95% credible interval of [-0.03,0.04]. The fixed effect for surprisal has a posterior with mean 0.28 and 95% credible interval of [0.10,0.45]. As seen in Figure 16, global entropy reduction does not appear to have a consistent effect on reaction times across the four global entropy reduction levels in groups A and B.

In analysis 2: with a weakly informative prior centered at zero on the global entropy fixed effect, the posterior of the global entropy was centered with mean 0.01 and 95% credible interval of [-0.01,0.03]. As seen in Figure 17, the global entropy does not appear to have an effect on reaction times across the global entropy levels assigned to the target corresponding to "4," in groups A and B.

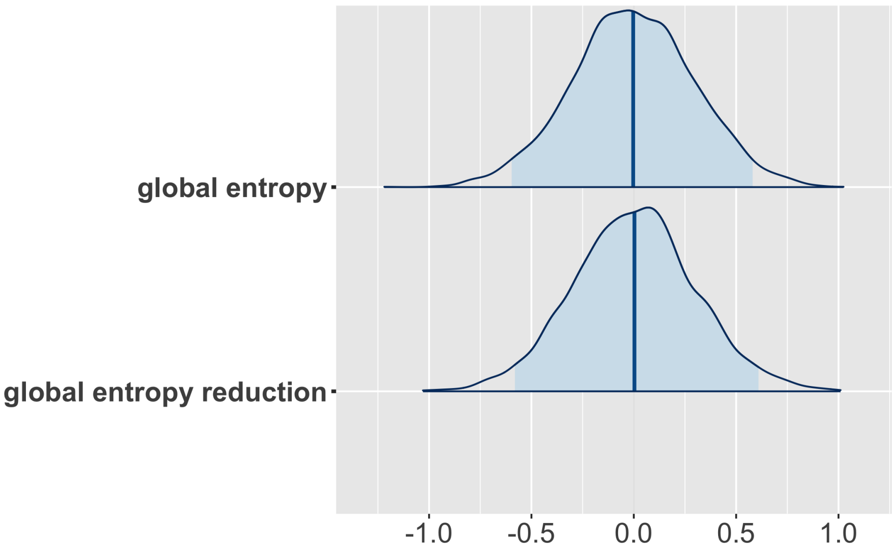

Figure 14. Experiment 2 priors for global entropy reduction fixed effect in analysis 1 (bottom) and global entropy fixed effect in analysis 2 (top). The mean is marked by the dark blue vertical line and the light blue shaded region marks the 95% credible interval.

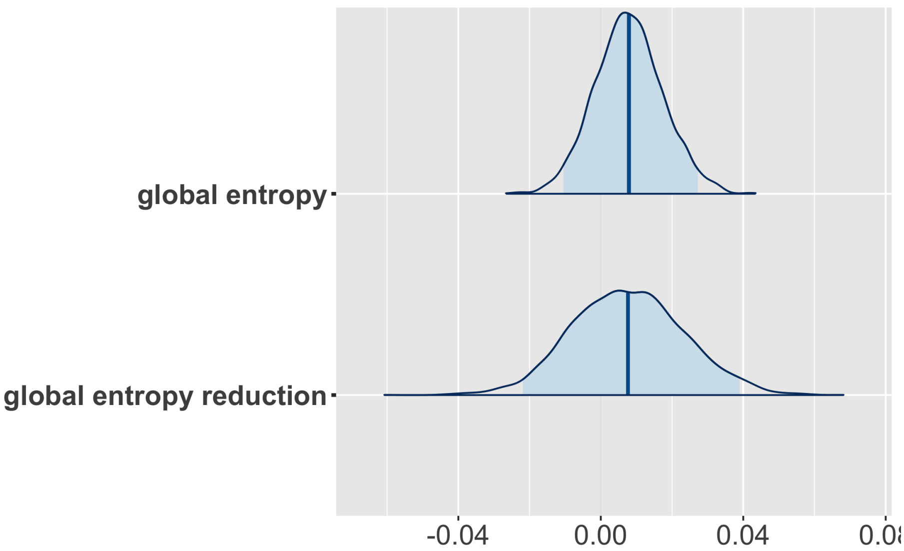

Figure 15. Experiment 2 posteriors for global entropy reduction fixed effect in analysis 1 (bottom) and global entropy fixed effect in analysis 2 (top). Posterior expectations are marked by the dark blue vertical line and the light blue shaded region marks the 95% credible interval of the posteriors.

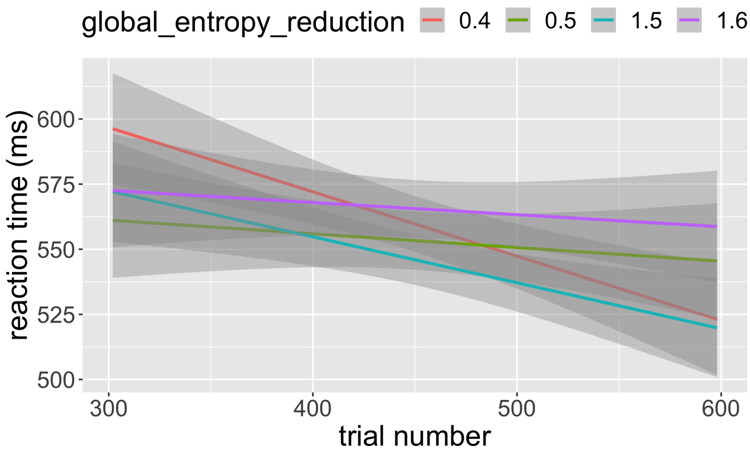

Figure 16. Experiment 2 results: reaction times in the second half of the part 1 trials for different global entropy reduction values of "2" and "3" in groups A and B.

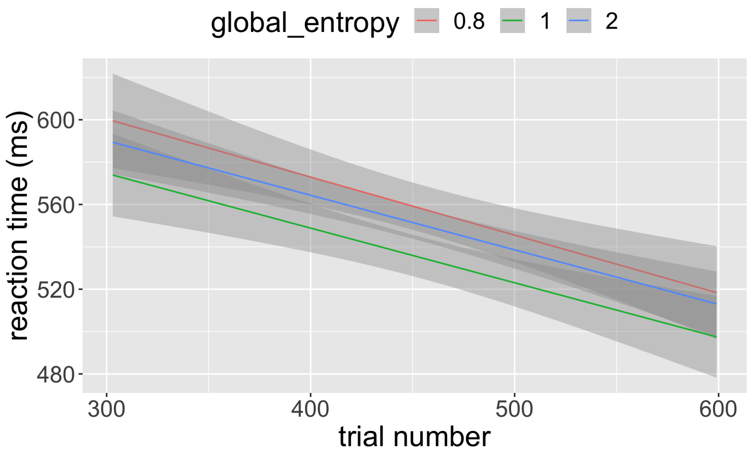

Figure 17. Experiment 2 results: reaction times in the second half of the part 1 trials for different global entropy values of "4" in groups A and B.

Discussion

From a weakly informative prior centered at zero, the Bayesian mixed effects lognormal regression model estimates that both global entropy measures have a non-zero postive center; however, the estimation is not very precise and well-includes zero in the credible interval. Therefore, we fail to find strong evidence for an effect of the global entropy measures. In analysis 1, the effect of the ideal observer estimated surprisal was detected although a clear entropy effect was not. Since the contrasts in analysis 1 involved targets with constant surprisal values in the grammar, this result demonstrates the humans are extremely sensitive to mild fluctuations in surprisal in the probabilistic serial reaction time task. However, either there is no effect of the entropy measures on reaction times, or the web-based paradigm we deployed acquired too much noise in the reaction time data for a small entropy effect to more clearly reveal itself in our posterior estimates.

General Discussion & Conclusion

Below is a summary of the present work's contributions:

(1) The creation of a novel web-based probabilistic serial reaction time task paradigm, the monster zapping game.

(2) A web-based replication of a surprisal effect in the probabilistic serial reaction time task, including a demonstration of humans' high sensitivity to mild changes in surprisal values.

(3) The creation of an ideal observer model to dynamically compute conditional probabilities and surprisal values in the probabilistic serial reaction time task.

(4) Experimental evidence suggesting that global entropy measures do not have an effect on reaction times in the probabilistic serial reaction time task.

In Experiment 1 we detected an effect of surprisal, demonstrating a web-based replication of an effect of transition probability on reaction times in the probabilistic serial reaction time task. In Experiment 2, we did not find strong evidence for an effect of global entropy and global entropy reduction on reaction times. However, in Experiment 2, we can not rule out that participants failed to learn the grammar in the limited number of trials since the grammar complexity increased in Experiment 2 although the number of trials remained constant. Subsequent studies and analyses could be run to investigate whether the grammar could be learned in the given number of trials.

Our results suggest that global entropy and global entropy reduction are not particularly informative metrics to characterize reaction times in the probabilistic serial reaction time task. This suggests that analogous entropy measures to those shown to have an effect on reading times do not have an effect on reaction times in motor learning and execution.

Subsequent studies can explore whether global entropy effects occur in motor sequence learning when sequences of targets in the probabilistic serial reaction time task stand for a particular meaning. For example, a task in which sequences are mapped to various photos could investigate whether global entropy becomes a relevant metric when sequences are mapped to a single structure, similar to how words in sentences correspond to a particular meaning.

Additionally, the monster zapping paradigm may be deployed without the bonus for quick responses, and with explicitly instructing participants to remember the sequences akin to Linzen et al.'s (2017) task. These follow up manipulations may reveal what manipulations are essential for entropy effects, providing insights about the necessary conditions for the entropy effects found by Linzen et al. (2017), and the absence of clear global entropy effects in the present work.

Acknowledgments

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References

Baayen, R. H., Davidson, D. J., & Bates, D. M. (2008). Mixed-effects modeling with crossed random effects for subjects and items. *Journal of memory and language*, *59*(4), 390–412.

Bridges, D., Pitiot, A., MacAskill, M. R., & Peirce, J. W. (2020). The timing mega-study: comparing a range of experiment generators, both lab-based and online. *PeerJ*, *8*, e9414.

Bürkner, P.-C. (2017a). Advanced bayesian multilevel modeling with the r package brms. *arXiv preprint arXiv:1705.11123*.

Bürkner, P.-C. (2017b). Bayesian distributional non-linear multilevel modeling with the r package brms. *arXiv preprint arXiv:1705.11123*.

Bürkner, P.-C. (2017c). brms: An R package for Bayesian multilevel models using Stan. *Journal of statistical software*, *80*(1), 1–28.

Frank, S. L. (2013). Uncertainty reduction as a measure of cognitive load in sentence comprehension. *Topics in cognitive science*, *5*(3), 475–494.

Hale, J. (2006). Uncertainty about the rest of the sentence. *Cognitive science*, *30*(4), 643–672.

Hale, J. (2016). Information-theoretical complexity metrics. *Language and Linguistics Compass*, *10*(9), 397–412.

Hick, W. E. (1952). On the rate of gain of information. *Quarterly Journal of experimental psychology*, *4*(1), 11–26.

Just, M. A., Carpenter, P. A., & Woolley, J. D. (1982). Paradigms and processes in reading comprehension. *Journal of experimental psychology: General*, *111*(2), 228.

Karuza, E. A., Farmer, T. A., Fine, A. B., Smith, F. X., & Jaeger, T. F. (2014). On-line measures of prediction in a self-paced statistical learning task. In *Proceedings of the annual meeting of the cognitive science society* (Vol. 36).

Levy, R. (2008). Expectation-based syntactic comprehension. *Cognition*, *106*(3), 1126–1177.

Linzen, T., & Jaeger, T. F. (2014). Investigating the role of entropy in sentence processing. In *Proceedings of the fifth workshop on cognitive modeling and computational linguistics* (pp. 10–18).

Linzen, T., Siegelman, N., & Bogaerts, L. (2017). Prediction and uncertainty in an artificial language. In *Cogsci.*

Lowder, M. W., Choi, W., Ferreira, F., & Henderson, J. M. (2018). Lexical predictability during natural reading: Effects of surprisal and entropy reduction. *Cognitive science*, *42*, 1166–1183.

MacDonald, M. C., & Hsiao, Y. (2018). Sentence comprehension. *The Oxford Handbook of Psycholinguistics*, 171–196.

Pavão, R., Savietto, J. P., Sato, J. R., Xavier, G. F., & Helene, A. F. (2016). On sequence learning models: Open-loop control not strictly guided by hick's law. *Scientific reports*, *6*(1), 1–9.

Remillard, G. (2008). Implicit learning of second-, third-, and fourth-order adjacent and nonadjacent sequential dependencies. *Quarterly Journal of Experimental Psychology*, *61*(3), 400–424.

Remillard, G. (2010). Implicit learning of fifth-and sixth-order sequential probabilities. *Memory & Cognition*, *38*(7), 905–915.

Remillard, G., & Clark, J. M. (2001). Implicit learning of first-, second-, and third-order transition probabilities. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *27*(2), 483.

Roark, B., Bachrach, A., Cardenas, C., & Pallier, C. (2009). Deriving lexical and syntactic expectation-based measures for psycholinguistic modeling via incremental top-down parsing. In *Proceedings of the 2009 conference on empirical methods in natural language processing* (pp. 324–333).

Sævland, W., & Norman, E. (2016). Studying different tasks of implicit learning across multiple test sessions conducted on the web. *Frontiers in psychology*, *7*, 808.

Schvaneveldt, R. W., & Gomez, R. L. (1998). Attention and probabilistic sequence learning. *Psychological Research*, *61*(3), 175–190.

Shannon, C. E. (1948). A mathematical theory of communication. *The Bell system technical journal*, *27*(3), 379–423.

Smith, N. J., & Levy, R. (2013). The effect of word predictability on reading time is logarithmic. *Cognition*, *128*(3), 302–319.

Stadler, M. A. (1992). Statistical structure and implicit serial learning. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *18*(2), 318.