Name: Solutions

Homework Week # 8

Momentum
Due Thurs 10/17/19

Reading

C&J Physics:	**Tues – Ch. 8: 1-4**	**Thurs – Ch. 8: 4-7**
OS Coll Phys:	**Tues – Ch. 10: 1-4**	**Thurs – Ch. 10: 4-7**

Problems

Problem 1.	**FOC 7.1**	
Problem 2.	**FOC 7.2**	
Problem 3.	**7.1**	
Problem 4.	**7.12**	
Problem 5.	**7.14**	
Problem 6.	**7.18**	
Problem 7.	**7.25**	
Problem 8.	**7.38**	(a) Solve (b) set up only
Problem 9.	**7.43**	set up only

Prob. 1 FoC 7.1

Two cars with equal mass and speeds but moving in different directions.

KE doesn't account for direction, but $\vec{p}$ does. Therefore the answer is

(b) They have the same kinetic energies, but different momenta.

Prob. 2 FOC 7.2

Which of the following runners have the same momenta?

$\vec{p}_A = \frac{1}{2} m_o v_o$ North $\vec{p}_B = m_o v_o$ East

$\vec{p}_C = 2 m_o v_o$ South $\vec{p}_D = 2 m_o v_o$ West

$\vec{p}_E = \frac{1}{2} m_o v_o$ North $\vec{p}_F = 4 m_o v_o$ West

Even though they have different masses and speeds, runners A and E have the same momenta.

(d) A and E

Prob. 3 7.1

Find $\vec{F}_w$, force on wall, if it's equal and opposite to the force of the impulse on the skater

$$\vec{F}_w = -\vec{F}_{ave} \quad \text{for} \quad \vec{F}_{ave}\Delta t = \Delta \vec{p}$$

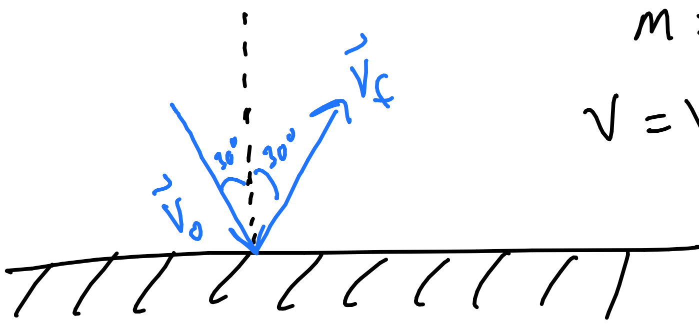
$$\vec{F}_{ave} = \frac{\Delta \vec{p}}{\Delta t} = \frac{(46\,kg)(-1.2\,\frac{m}{s})}{0.80\,s}$$

$$\vec{F}_w = -\left[-\frac{(46)(1.2)}{0.80}\,N\right]$$

$$\boxed{\vec{F}_w = +69\,N}$$

Prob. 4 7.12 4/18

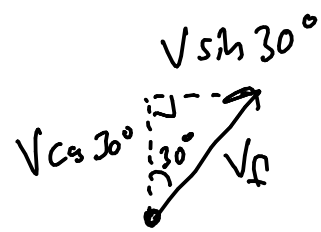
Find $\vec{J} = \Delta \vec{p}$ for the described rebound.



$m = 0.047$ kg

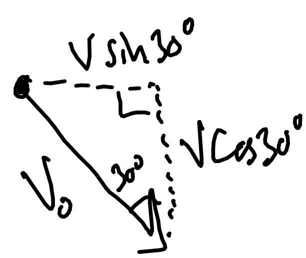
$V = V_0 = V_f = 45 \frac{m}{s}$

$$\Delta \vec{p} = m(\vec{V}_f - \vec{V}_0)$$



$$\vec{V}_f = V\left(+\sin 30°\,\hat{x} + \cos 30°\,\hat{y}\right)$$

$$-)\quad \vec{V}_0 = V\left(+\sin 30°\,\hat{x} - \cos 30°\,\hat{y}\right)$$



$$\vec{V}_f - \vec{V}_0 = V\left(\cos 30° + \cos 30°\right)\hat{y}$$

$$\Delta \vec{p} = 2mV\cos 30°\,\hat{y} = \boxed{3.66\ \frac{kg \cdot m}{s}\ \hat{y}}$$

Prob. 5 7.14

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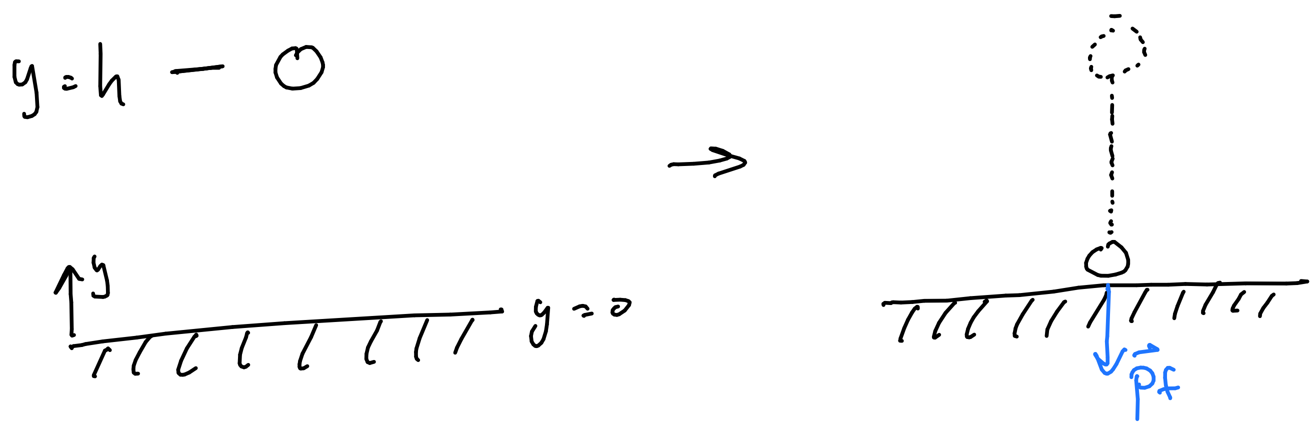
Given a "final" momentum and initial speed, find an initial drop height.



$m = 0.60\,kg$

$p_f = 3.1\ \frac{kg \cdot m}{s}$

$E = PE + KE$

$KE = \frac{p^2}{2m}$, $PE = mgh$

Cons. of E: $E_0 = E_f$ with $KE_0 = 0$, $PE_f = 0$

$$PE_0 = KE_f$$

$$mgh = \frac{p^2}{2m}$$

$$\boxed{h = \frac{p^2}{2m^2 g} = 1.3\,m}$$

Prob 6 7.18

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This is another example of internal forces of the "explosion" type. This time two objects are moving as one and after the internal impulse, the less massive object is at rest

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$\vdash\!\rightarrow +$ $\quad \rightarrow \vec{v}_0$ $\qquad\qquad$ $\vec{v}_1 = 0 \quad \rightarrow \vec{v}_2$

t_0: $\boxed{m_1 \,/\, m_2}$ $\quad \xrightarrow[\text{released}]{\text{spring}}$ $\quad t_f$: $\boxed{m_1}$ $\quad \boxed{m_2}$

$m_1 = 1.2\ \text{kg} \qquad m_2 = 2.4\ \text{kg} \qquad \vec{v}_0 = +5.0\ \frac{m}{s}$

$\vec{J} = 0$ b/c internal forces only so $\vec{p}_f = \vec{p}_0$

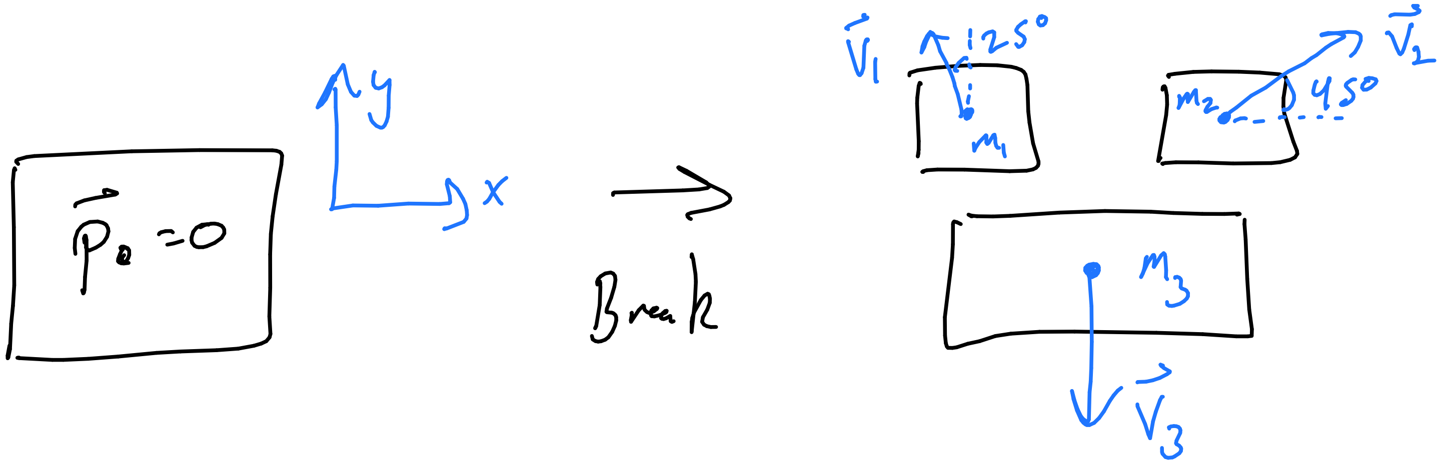
$$m_2 \vec{v}_2 = (m_1 + m_2)\, \vec{v}_0$$

$$\vec{v}_2 = \left(\frac{m_1 + m_2}{m_2}\right) \vec{v}_0$$

$$\boxed{\vec{v}_2 = +\,7.5\ \frac{m}{s}}$$

Prob. 7 7.25 1/3

Breaking Plate Problem from Lecture 14
Find 2 unknown masses with 2D conservation of momentum



The breaking occurs via internal forces so

$$\vec{P}_f = \vec{P}_0$$

$\vec{P}_f$ has 2 directions so we write

$$P_{fx} = P_{0x}$$

$$P_{fy} = P_{0y}$$

and both equal zero!

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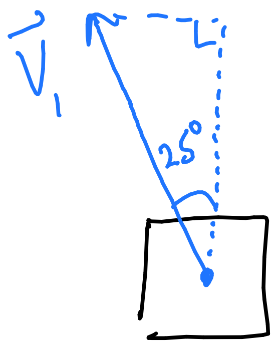
Prob. 7 2/3

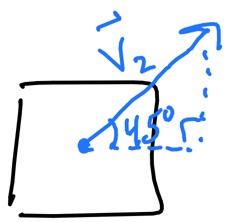
Using a standard x-y coordinate system we can analyze each piece:



$V_1 = 3.00 \frac{m}{s}$

$V_{1x} = V_1 \sin 25° = 1.27 \frac{m}{s}$

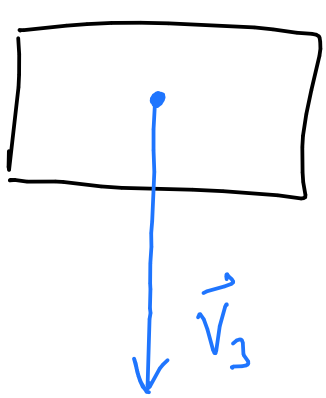
$V_{1y} = V_1 \cos 25° = 2.72 \frac{m}{s}$



$V_2 = 1.79 \frac{m}{s}$

$V_{2x} = V_2 \cos 45° = 1.27 \frac{m}{s}$

$V_{2y} = V_2 \sin 45° = 1.27 \frac{m}{s}$



$V_3 = 3.07 \frac{m}{s}$

$V_{3x} = 0$

$V_{3y} = 3.07 \frac{m}{s}$

Prob. 7 3/3

From the vector analysis and choice of $+x, +y$ we write our momenta equations as

$$p_{fx} = p_{2x} - p_{1x} = 0$$

$$p_{fy} = p_{1y} + p_{2y} - p_{3y} = 0$$

From p_x:

$$m_2 V_{2x} = m_1 V_{1x}$$

$$m_2 = \frac{V_{1x}}{V_{2x}} m_1$$

$$m_2 = \frac{1.27}{1.27} m_1 = m_1$$

From p_y:

$$m_1 V_{1y} + m_2 V_{2y} = m_3 V_{3y}$$

& subbing in $m_2 = m_1$

$$(V_{1y} + V_{2y}) m_1 = m_3 V_3$$

$$m_1 = \frac{m_3 V_3}{V_{1y} + V_{2y}}$$

$$\boxed{m_1 = m_2 = 1.00 \text{ kg}}$$

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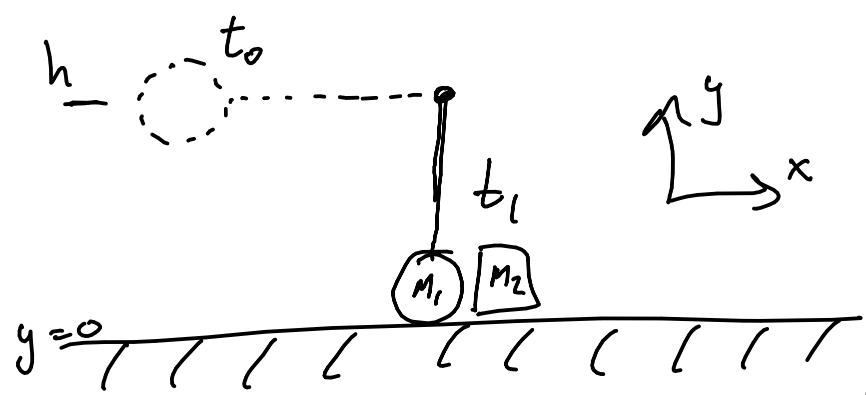
Prob 8 $\frac{1}{4}$ 7.38

Use conservation of energy and momentum to find the velocity of a ball before and after an elastic collision.



$h = 1.20\ m$

$m_1 = 1.60\ kg$

$m_2 = 2.40\ kg$

PE = 0

t_2 immediately after collision.

$$E_0 = E_1 \rightarrow PE_0 = KE_1$$

$$m_1 g h = \frac{1}{2} m_1 v_1^2$$

$$v_1^2 = 2gh$$

(a) Before collision $\boxed{\vec{V}_1 = 4.90\ \frac{m}{s}\ \hat{x}}$

Prob. 8 2/4 (b) Set up only

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(b) 1 After elastic collision

$$E_1 = E_2 \quad \& \quad \vec{p}_1 = \vec{p}_2$$

$$E_1 = E_2 \rightarrow KE_1 = KE_2$$

$$\frac{1}{2} m_1 V_1^2 = \frac{1}{2} m_1 V_{21}^2 + \frac{1}{2} m_2 V_{22}^2$$

$$\vec{p}_1 = \vec{p}_2 \rightarrow m_1 V_1 = m_1 V_{21} + m_2 V_{22}$$

stop

masses are known so relate speeds from momentum and plug into energy equation.

$$V_{22} = \frac{m_1 (V_1 - V_{21})}{m_2}$$

$$\frac{1}{2} m_1 V_1^2 = \frac{1}{2} m_1 V_{21}^2 + \frac{1}{2} m_2 \frac{m_1^2 (V_1 - V_{21})^2}{m_2^2}$$

(b)^2 $\frac{m_1}{2}$ can be multiplied/divided out

$$v_1^2 = v_{21}^2 + \frac{m_1}{m_2}(v_1 - v_{21})^2$$

Remember we want v_{21} and we know all of the other values.

$$v_1^2 = v_{21}^2 + \frac{m_1}{m_2}\left(v_1^2 - 2v_1 v_{21} + v_{21}^2\right)$$

If I write this as a quadratic equation for v_{21} as the variable I get

$$v_{21}^2 + \frac{m_1}{m_2}v_{21}^2 - \frac{2m_1}{m_2}v_1 v_{21} + \frac{m_1}{m_2}v_1^2 - v_1^2 = 0$$

$$\left(1 + \frac{m_1}{m_2}\right)v_{21}^2 - \frac{2m_1}{m_2}v_1 v_{21} + \left(\frac{m_1}{m_2} - 1\right)v_1^2 = 0$$

$$1.67\,v_{21}^2 - \left(6.53\,\frac{m}{s}\right)v_{21} - 8.00\,\frac{m^2}{s^2} = 0$$

Prob. 8 4/4

(b)^3

$$V_{21} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$V_{21} = \frac{-(-6.53\frac{m}{s}) \pm \sqrt{(-6.53\frac{m}{s})^2 - 4(1.67)(-8.00\frac{m^2}{s^2})}}{2(1.67)}$$

$$V_{21} = \frac{+6.53 \pm \sqrt{96.08}}{3.34}\ \frac{m}{s}$$

$$V_{21} = \frac{6.53 \pm 9.80}{3.34}\ \frac{m}{s}$$

$$V_{21} = 4.9\frac{m}{s} \text{ or } -0.97\frac{m}{s}$$

Then

$$V_{22} = \frac{m_1}{m_2}(V_1 - V_{21}) = 0 \text{ or } 3.91\frac{m}{s}$$

Sensible answer is $\boxed{V_{21} = -0.97\frac{m}{s},\ V_{22} = 3.91\frac{m}{s}}$

Prob. 9 1/5

e^- collides elastically with a stationary H with motion completely in 1D. Find KE_{Hf}/KE_{eo} knowing $m_H = 1837 m_e$

$m_e = m_1$

$m_H = m_2$

$\Delta \vec{J} = 0$, $W_{nc} = 0$

$\vec{p}_o = \vec{p}_f$ $\quad$ $E_o = E_f$

$\vec{p}_o = m_1 \vec{v}_0$

$E_o = \frac{1}{2} m_1 v_0^2$

$\vec{p}_f = m_1 \vec{v}_1 + m_2 \vec{v}_2$

$E_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

Solve $\vec{p}_o = \vec{p}_f$ for v_2 in terms of everything else then sub into $E_f = E_o$.

Required for set up

Prob. 9 2/5

Written for student knowledge. Not Required

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assumed e^- rebounds back since elastic collision.

$$m_2 v_2 - m_1 v_1 = m_1 v_0$$

$$m_2 v_2 = m_1 v_0 + m_1 v_1$$

$$v_2 = \frac{m_1}{m_2}(v_0 + v_1)$$

$$\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1 v_0^2 = m_1 v_1^2 + m_2 \left(\frac{m_1}{m_2}(v_0 + v_1)\right)^2$$

$$m_1 v_0^2 = m_1 v_1^2 + \frac{m_1^2}{m_2}(v_0^2 + 2v_0 v_1 + v_1^2)$$

$$m_1 m_2 v_0^2 = m_1 m_2 v_1^2 + m_1^2 (v_0^2 + 2v_0 v_1 + v_1^2)$$

Plug in $m_2 = 1837 m_1 = \alpha m_1$

$$\alpha m_1^2 v_0^2 = \alpha m_1^2 v_1^2 + m_1^2 (v_0^2 + 2v_0 v_1 + v_1^2)$$

Prob. 9 3/5

assume V_0 known so V_1 is the unknown.

Dividing m_1^2 out: $\alpha V_0^2 = \alpha V_1^2 + V_0^2 + 2V_0 V_1 + V_1^2$

$$(1+\alpha) V_1^2 + 2V_0 V_1 + (1-\alpha) V_0^2 = 0$$

$$V_1 = \frac{-2V_0 \pm \sqrt{(2V_0)^2 - 4(1+\alpha)(1-\alpha)V_0^2}}{1+\alpha}$$

$$V_1 = \frac{-2V_0 \pm \sqrt{4V_0^2 - 4V_0^2(1-\alpha^2)}}{1+\alpha}$$

$$V_1 = \frac{-2V_0 \pm 2V_0\sqrt{1-(1-\alpha^2)}}{1+\alpha}$$

Prob. 9 H/5

$$V_1 = \frac{-2v_0 \pm 2v_0\sqrt{\alpha^2}}{1+\alpha}$$

$$V_1 = \frac{-2v_0 \pm 2v_0\alpha}{1+\alpha}$$

$$V_1 = 2v_0\left(\frac{-1-\alpha}{1+\alpha}\right) \text{ or } 2v_0\left(\frac{-1+\alpha}{1+\alpha}\right)$$

$$V_1 = \cancel{-2v_0\left(\frac{1+\alpha}{1+\alpha}\right)} \text{ or } 2v_0\left(\frac{\alpha-1}{\alpha+1}\right)$$

* already assumed v_1 to be in negative dir. *

$$V_1 = \boxed{0.9989(2v_0)} \approx 2v_0$$

Note: $\dfrac{KE_{Hf}}{KE_{eo}} = \dfrac{\frac{1}{2}m_2 v_2^2}{\frac{1}{2}m_1 v_0^2} = \alpha\dfrac{v_2^2}{v_0^2}$

Prob. 9 5/5

$$V_2 = \frac{m_1}{m_2}(v_0 + v_1) \quad \text{from} \quad \vec{p}_0 = \vec{p}_f$$

$$\frac{KE_{Hf}}{KE_{e0}} = \alpha \frac{\left(\frac{1}{\alpha}(v_0 + v_1)\right)^2}{v_0^2} \quad ; \; v_1 = +1.9978 v_0$$

$$= \frac{\alpha}{\alpha^2} \frac{\left[(1 + 1.9978) v_0\right]^2}{v_0^2}$$

$$= \frac{1}{\alpha}(1 + 1.9978)^2$$

$$\frac{KE_{Hf}}{KE_{e0}} \cong \frac{9}{1837} = 0.0049$$

$$\boxed{\frac{KE_{Hf}}{KE_{e0}} = 4.9 \times 10^{-3}}$$

This means about 0.5% of the e^-'s initial kinetic energy is transferred to the H atom.