Phys 111: Lecture 23

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"The 23rd Day: Under Pressure!"

Homework Wk #12 Due Thurs 11/14/19

Today's Topics

1. Pressure
2. Hydrostatic Fluids
3. Pascal's Principle

Motivation

Usually we don't cut our food with two spoons. Why?

Figure: 22.1 Plate setting with a steak knife[1]

[1] https://www.ehow.com/how_7703575_set-table-steak-knives.html

Pressure

The **pressure P** exerted on a surface is defined as the magnitude of the force acting perpendicular to a surface $(F_\perp)$ divided by the area A over which the force F acts:

$$P = \frac{F_\perp}{A}$$

SI Unit of Mass Density: **pascal, Pa**

$$1\ Pa = 1\ \frac{N}{m^2}$$

Pressure changes are often a more useful consideration than force directly.

Concept Image

$$P = \frac{F_\perp}{A}$$

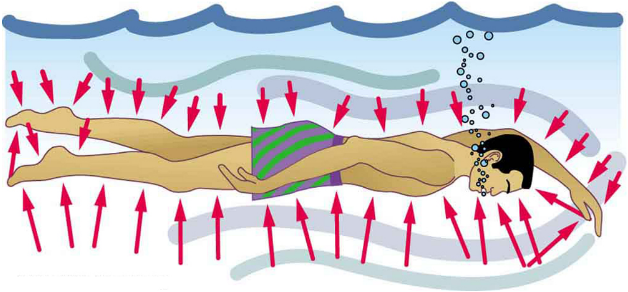

Figure: 23.2 Pressure exerted on all sides of a swimmer

Mass Density

The **mass density,** ρ**,** of a substance is the mass m of the substance divided by its volume V.

$$\rho = \frac{m}{V}$$

SI Unit of Mass Density: kg/m^3.

Alternatively:[2] g/cm^3

which is the same as g/mL

[2]Note that $1\ mL = 1\ cm^3$

Example Mass Densities

Some mass densities of real materials

Substance	$\rho(10^3\ \mathrm{kg/m^3}$ or g/mL)	Substance	$\rho(10^3\ \mathrm{kg/m^3}$ or g/mL)	Substance	$\rho(10^3\ \mathrm{kg/m^3}$ or g/mL)
Solids		**Liquids**		**Gases**	
Aluminum	2.7	Water (4ºC)	1.000	Air	1.29×10^{-3}
Brass	8.44	Blood	1.05	Carbon dioxide	1.98×10^{-3}
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	1.25×10^{-3}
Gold	19.32	Mercury	13.6	Hydrogen	0.090×10^{-3}
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	0.18×10^{-3}
Lead	11.3	Petrol	0.68	Methane	0.72×10^{-3}
Polystyrene	0.10	Glycerin	1.26	Nitrogen	1.25×10^{-3}
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	1.98×10^{-3}
Uranium	18.70			Oxygen	1.43×10^{-3}

Figure: 23.3 Partial Table 11.1 from Openstax College Physics[3]

[3] https://openstax.org/details/books/college-physics

Exercise #1

C&J 11.1 One of the concrete pillars that support a house is $2.2\ m$ tall and has a radius of $0.50\ m$. The density of concrete is about $2.2 \times 10^3\ kg/m^3$. Find the weight of this pillar in pounds $(1\ N = 0.2248\ lb)$.

This data suggests the concrete pillars are cylindrical. This is typical for pillars in construction.

Exercise #1 Translation

C&J 10.43 Utilize the relationship $\rho = m/V$ in the form

$$mass = density \times Volume$$

with the information $h = 2.2\ m$, $r = 0.5\ m$, and $\rho = 2.2 \times 10^3\ kg/m^3$ to find the weight of the cylindrical pillars?

For a cylinder $V = \pi r^2 h$.

Exercise #1 Answer

Volume V of a cylinder is

$$V = \pi(0.5)^2(2.2)\ m^3$$

$$V = 1.728\ m^3$$

so the mass is

$$m = (2.2 \times 10^3\ kg/m^3)(1.728\ m^3)$$

$$m = 3.80 \times 10^3\ kg$$

Then the weight ($F_g = mg$) is

$$F_g = 38.0kN$$

$$F_g \approx 8500\ lb$$

Exercise #2

C&J 11.12 A person who weighs $625\ N$ is riding a $98\ N$ mountain bike. Suppose that the entire weight of the rider and bike is supported equally by the two tires. If the pressure in each tire is $7.60 \times 10^5 Pa$, what is the area of contact between each tire and the ground?

Exercise #2 Answers

C&J 10.45 Each tire has the same area and the same force, pressure so we only need to solve once. Rewriting the pressure equation nets us the form:

$$A = \frac{F}{P}.$$

Assuming the ground is flat we can say $F_N = F_g$ and

$$A = \frac{\frac{1}{2}(723\ N)}{(7.60 \times 10^5\ Pa)}$$

Making sure we carefully complete the arithmetic we get

$$A = 48.0 \times 10^{-5}\ m^2$$

$$A = 0.48\ cm^2$$

Exercise #3

C&J 11.18 A cylinder is fitted with a piston, beneath which is a spring, as in the drawing. The cylinder is open to the air at the top. Friction is absent. The spring constant of the spring is $3600\ N/m$. The piston has a negligible mass and a radius of $0.024\ m$.

Figure: 23.4 Pressure-spring problem

Exercise #3 Questions

C&J 11.18 A cylinder is fitted with a piston, beneath which is a spring, as in the drawing. The cylinder is open to the air at the top. Friction is absent. The spring constant of the spring is $3600\ N/m$. The piston has a negligible mass and a radius of $0.024\ m$.

a. When the air beneath the piston is completely pumped out, how much does the atmospheric pressure cause the spring to compress?
b. How much work does the atmospheric pressure do in compressing the spring?

Exercise #3 (a) Only

C&J 11.18 (a) A cylinder of radius of $0.024\ m$ is fitted with a piston, beneath which is a spring of spring constant $3600\ N/m$ such that all of the simple approximations hold and the cylinder is open to the air at the top. Find compression when the air is sucked out of the bottom.

$F = PA$ and $F_{sp} = kx$ so we can calculate the compression of the spring to be

$$x = \frac{PA}{k}$$

$$x = \frac{\left[101.325 \times 10^5\ Pa\right]\left[\pi(0.024\ m)^2\right]}{\left[3600\ N/m\right]}$$

$$x = 0.51\ m$$

Pressure & Depth in a Static Fluid

How can we quanitfy the pressure from a static fluid?

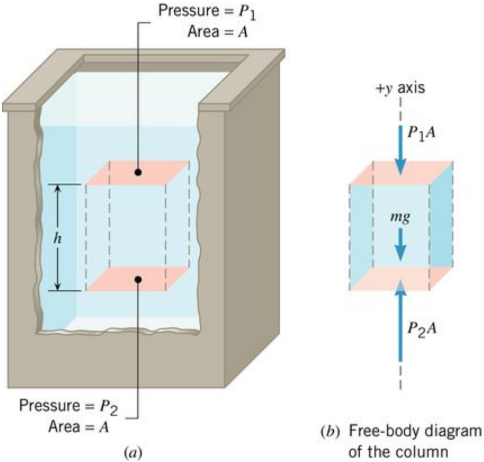

Figure: 23.5 Pressure as it relates to depth

Pressure Depth Math

Hydrostatics

If the water is staying still, then there is zero acceleration:

$$\sum F_y = P_2 A - P_1 A - mg = 0$$

Recall that there is a relation between mass, density, and volume:

$$P_2 A - P_1 A - (\rho V) g$$

$$P_2 A - P_1 A - \rho (A h) g$$

Which simplifies to

$$P_2 = P_1 + \rho g h$$

Concept Check

How does the pressure at A compare to the pressure at B?

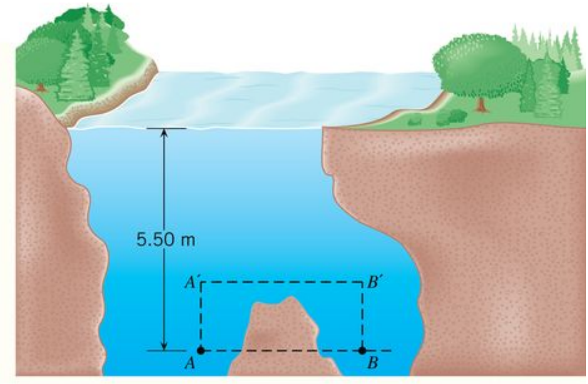

Figure: 23.6 Hydrostatic pressure example

Hydrostatic Summary

In a hydrostatic fluid the three largest factors are pressure at the surface, density of the fluid, and how much fluid is above.

$$P_2 = P_1 + \rho g h$$

Problem Solving Insight: The pressure at any point in a fluid depends on the vertical distance h of the point beneath the surface. However, for a given vertical distance, the pressure is the same, no matter where the point is located horizontally in the fluid.

Exercise #4

C&J 11.20 The Mariana trench is located in the floor of the Pacific Ocean at a depth of about $11,000\ m$ below the surface of the water. The density of seawater is $1,025\ kg/m^3$.

a. If an underwater vehicle were to explore such a depth, what force would the water exert on the vehicle's observation window (radius $= 0.10\ m$)?
b. For comparison, determine the weight of a jetliner whose mass is $1.2 \times 10^5\ kg$.

Exercise #4 Answer

C&J 10.15 Consider the pressure levels at a depth of $11,000\ m$ below the surface of the ocean. The density of seawater is $1,025\ kg/m^3$.

Using $P_h = P_{atm} + \rho g h$ and $P = F_{\perp}/A$ we can determine:

a. The force on a circular area with a radius of $0.10\ m$ would be

$$3.55 \times 10^6\ N$$

b. For comparison, the weight of a jetliner whose mass is $1.2 \times 10^5\ kg$ would be

$$1.2 \times 10^6\ N.$$

Gauge Pressure

Most, if not all, of our pressure gauges (barometers) read a difference from atmospheric pressure, not the absolute pressure:

$$P_{abs} = P_{atm} + P_g$$

$$P_{atm} = 101.325\ kPa = 14.7\ psi$$

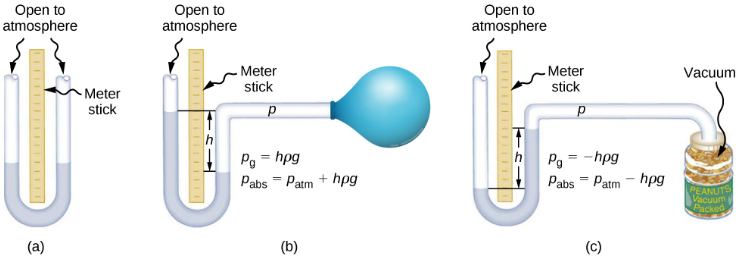

Figure: 23.7 Manometer diagrams

Pascal's Principle Demo

Why does the phenomena below occur even though there are very different volumes in each tube?

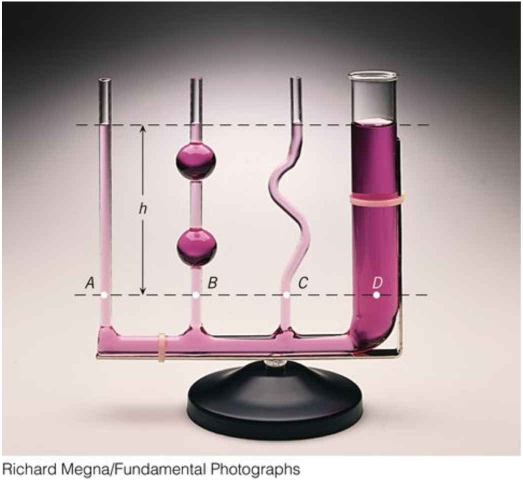

Figure: 23.8 Pascal's Principle demonstrated

Pascal's Principle

Pascal's Principle formally: Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and the enclosing walls.

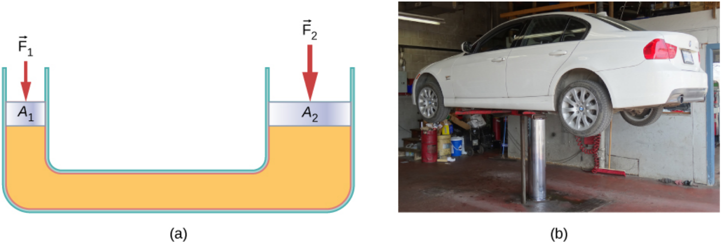

Figure: 22.9 Why does the car lift if a smaller area is used for the original force?

Exercise #5

C&J 11.34 A barber's chair with a person in it weighs 2100 N. The output plunger of a hydraulic system begins to lift the chair when the barber's foot applies a force of 55 N to the input piston. Neglect any height difference between the plunger and the piston. What is the ratio of the radius of the plunger to the radius of the piston?

Exercise #5 Answers

What is the ratio of the radius of the plunger (r_1) to the radius of the piston (r_2)? Pascal's Principle states

$$P_2 = P_1$$

$$\frac{F_2}{A_2} = \frac{F_1}{A_1}$$

So

$$\frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \frac{F_2}{F_1}$$

Taking the squareroot we get

$$\frac{r_2}{r_1} = \sqrt{\frac{2100\ N}{55\ N}}$$

$$\frac{r_2}{r_1} = 6.42$$

Exercise #6

C&J 11.38 The drawing shows a hydraulic chamber with a spring (spring constant = 1600 N/m) attached to the input piston and a rock of mass 40.0 kg resting on the output plunger. The piston and plunger are nearly at the same height, and each has a negligible mass. By how much is the spring compressed from its unstrained position?

Exercise #6 Image

C&J 11.38 The drawing shows a hydraulic chamber, $k = 1600\ N/m$,and the rock is of mass $40.0\ kg$ resting on the output plunger. By how much is the spring compressed from its unstrained position?

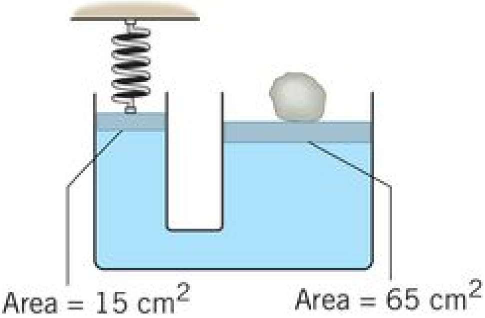

Figure: 23.10 A closed container with continuity of pressure

Exercise #6 Explanation

First we need to understand that this is a scenario where Pascal's Principle applies:

$$P_1 = P_2$$

$$F_1 = \frac{A_1}{A_2} F_2$$

$F_2 = mg$ is the weight of the rock.

Then we need to note that F_1 is going to be applied to the spring

$$F_1 = kx$$

So this force can be used to predict the compression of the spring:

$$x = \frac{F_1}{k}$$

Exercise #6 Answer

Combining all of our determinations we get the specific relationship

$$x = \frac{A_1 F_2}{A_2 k}.$$

Note that the units do in fact work out to units of length!!

$$x = \frac{(15\ cm^2)(400\ N)}{(65\ cm^2)(1600\ N/m)}$$

$$x = 5.77\ cm$$