Phys 111: Lecture 17

Ross Miller

University of Idaho

October 22, 2019

"The 17$^{\text{th}}$ Day: Pirate Ship"

Homework Wk #9 Due Thursday

Homework Wk #10 Due Exam Day T 10/29/19

Today's Topics

1. Torque
2. Mechanical Equilibrium
3. Center of Gravity

Motivation

We'll start the process to learn about:

1. How do forces affect rotation?
2. When is acceleration truly zero?
3. What's an efficient way to consider applications of torque?

Concept Example

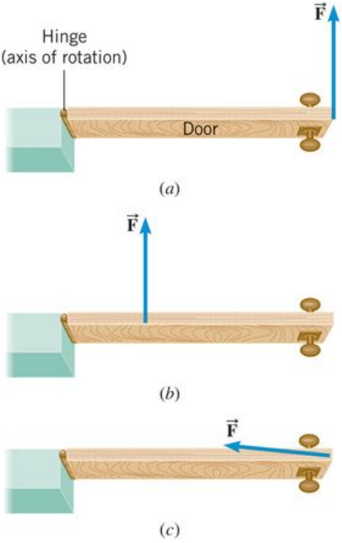

Figure: 17.1 Which force produces the largest torque?

Determining The Lever Arm (a)

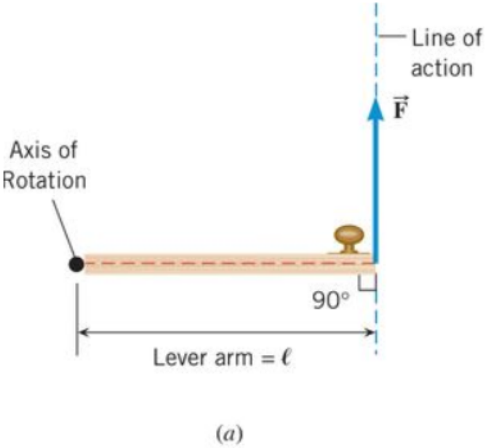

Figure: 17.2 Line of action and lever arm (a)

Determining The Lever Arm (b)

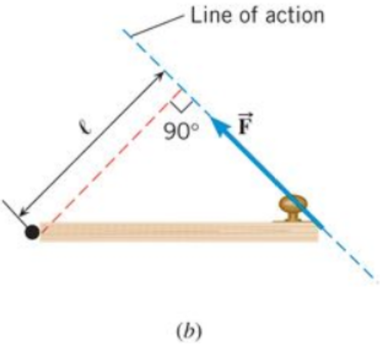

Figure: 17.3 Line of action and lever arm (b)

Determining The Lever Arm (c)

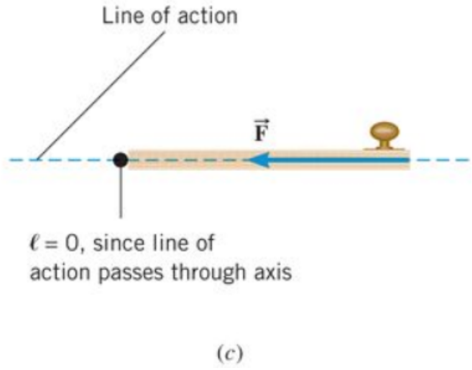

Figure: 17.4 Line of action and lever arm (c)

Reasoning Strategy #1

Calculating Torques

1. Draw a sketch of the object and clearly indicate the pivot point and the axis of rotation you are considering.
2. Decide which direction of rotation is positive and which is negative. **Do not change** your decision during the course of a calculation.
3. Draw a line of action for each force in separate sketches.
4. Draw a line from the pivot point that makes a right angle with the line of action. This line is your lever arm.
5. Use given information such as angles and lengths to calculate the lever arm length.
6. Calculate torques from the necessary information and add them up including $\pm$ signs.
7. *Double-check if the direction and magnitude of your answer seems reasonable or not.*

Exercise #1

C&J 9.3 You are installing a new spark plug in your car, and the manual specifies that it be tightened with a torque that has a magnitude of $45\ N \cdot m$. Using the data in the drawing, determine the magnitude F of the force that you must exert on the wrench.

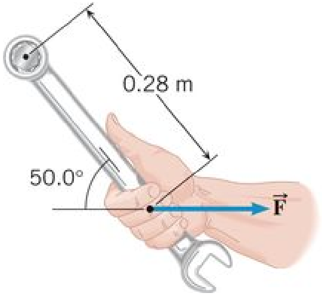

Figure: 17.5 Torque Calculation

Mechanical Equilibrium

If a rigid body is in **equilibrium**, **neither** its linear motion **nor** its rotational motion **changes**.

Math Version

$$\vec{\mathbf{a}} = 0 \qquad \& \qquad \alpha = 0$$

Formally we write that the **net force** and the **net torque** are both **zero**.

$$\sum \vec{\mathbf{F}} = 0 \qquad \& \qquad \sum \vec{\tau} = 0$$

Reasoning Strategy #2

Applying Conditions of Mechanical Equilibrium

1. Select the object to which the equations for equilibrium are to be applied.
2. Draw a free-body diagram that shows all the external forces acting on the object.
3. Choose a convenient set of x and y axes, and resolve all forces into components that lie along these axes.
4. Apply the equations that specify the balance of forces at equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$.

Reasoning Strategy #2 cont'd

4. Apply the equations that specify the balance of forces at equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$.
5. Select a convenient axis of rotation and identify the point where each external force acts on the object.
6. Calculate the torque produced by each force about the chosen axis. Set the sum of the torques equal to zero: $\sum \tau = 0$.
7. Solve the equations in Steps 4 and 5 for the desired unknown quantities.

Concept Check #2

C&J FOC 9.8 The drawing shows a top view of a square box lying on a friction-less floor. Three forces, which are drawn to scale, act on the box. Consider measuring angular acceleration with respect to an axis through the center of the box (perpendicular to the page).

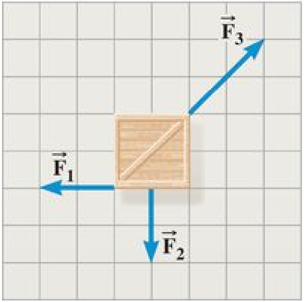

Figure: 17.6 Concept check for mechanical equilibrium

C&J FOC 9.8 Answer each question if the forces are drawn to scale and the axis of rotation is through the center of the box perpendicular to the page (out of the page).

a. Will the box have a linear acceleration?
b. Will the box have an angular acceleration?

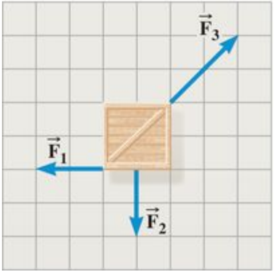

Figure: 17.6 Concept check for mechanical equilibrium

Reasoning Strategy #2 Full

Applying Conditions of Mechanical Equilibrium[1]

1. Select the object to which the equations for equilibrium are to be applied.
2. Draw a free-body diagram that shows all the external forces acting on the object.
3. Choose a convenient set of x and y axes, and resolve all forces into components that lie along these axes.
4. Apply the equations that specify the balance of forces at equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$.
5. Select a convenient axis of rotation and identify the point where each external force acts on the object.
6. Calculate the torque produced by each force about the chosen axis. Set the sum of the torques equal to zero: $\sum \tau = 0$.
7. Solve the equations in Steps 4 and 5 for the desired unknown quantities.

[1] to a rigid body

Two General Types of Rigid Bodies

1. If a body is **compact**, then the weight of the body won't cause a torque.
2. That is not true for an **extended** body or a body that is not compact.

The **Center of Gravity** of a rigid body is the point at which its weight can be considered to act when the torque due to the weight is being calculated.

Visualizing a Center of Gravity

The **Center of Gravity** of a rigid body is the point at which its weight can be considered to act when the torque due to the weight is being calculated.

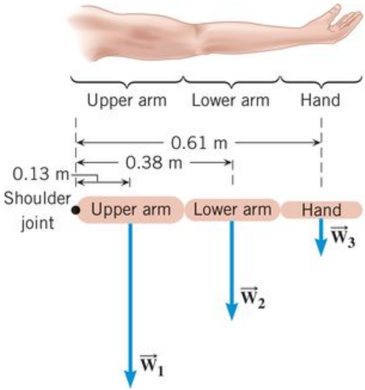

Figure: 17.7 Center of gravity of an adult arm

Calculating a Center of Gravity

(Total Weight)(Position cg) = (W1)(P1) + (W2)(P2) + (W3)(P3) ...

$$W_{tot}x_{cg} = W_1x_1 + W_2x_2 + W_3x_3 + ...$$

$$x_{cg} = \frac{W_1x_1 + W_2x_2 + W_3x_3 + ...}{W_{tot}}$$

$$x_{cg} = \frac{W_1x_1 + W_2x_2 + W_3x_3 + ...}{W_1 + W_2 + W_3 + ...}$$

$$x_{cg} = \frac{\sum(Wx)}{\sum W}$$

Calculating a Center of Gravity

Center of Gravity: $\mathbf{x_{cg}}$

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + W_3 x_3 + ...}{W_1 + W_2 + W_3 + ...}$$

$$x_{cg} = \frac{\sum (Wx)}{\sum W}$$

Exercise #3

Fig 17.7 Calculate the center of gravity of the arm if $W_1 = 20.0\ kg$, $W_2 = 12.0\ kg$, and $W_3 = 5.0\ kg$.

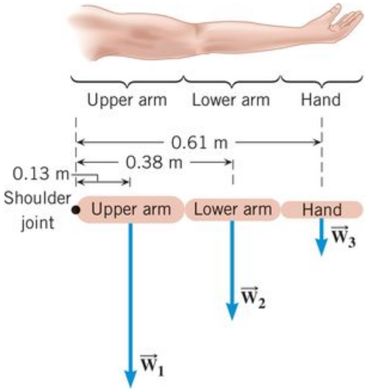

Figure: 17.7 Center of gravity of an adult arm

Exercise #4

C&J 9.22 A man holds a $178\ N$ ball in his hand, with the forearm horizontal. He can support the ball in this position because of the flexor muscle force $\vec{\mathbf{M}}$. The forearm weighs $22.0\ N$ and has a center of gravity as indicated. Find

a. the magnitude of $\vec{\mathbf{M}}$ and
b. the magnitude and direction of the force applied by the upper arm bone to the forearm at the elbow joint.

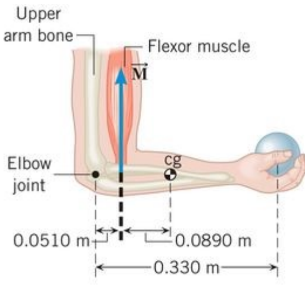

Figure: 17.8 Mechanical equilibrium and center of gravity example

Summary

Some Key Concepts

- Not all forces cause a torque.
  - Line of action
  - Axis of Rotation
- Consider the lever arm that maximizes applied torque (orientation).
- True Mechanical Equilibrium is two-fold.
  1. Linear Equilibrium
  2. Rotational Equilibrium
- Practice technical sketching when problem solving.
- Center of Gravity depends on distribution of weight from pivot
- Ross really meant don't "brain dump" after each exam.

The End

Thanks for your time and attention!

Any questions?

Answer Ex #1

C&J 9.3 You are installing a new spark plug in your car, and the manual specifies that it be tightened with a torque that has a magnitude of $45\ N \cdot m$.

The magnitude F of the force that you must exert on the wrench is about $210\ N$.

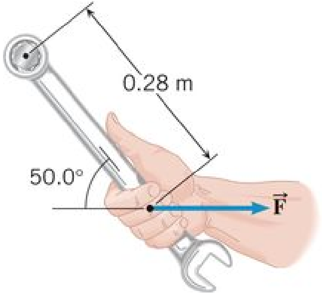

Figure: 17.5 Torque Calculation

C&J FOC 9.8 Answer each question if the forces are drawn to scale and the axis of rotation is through the center of the box perpendicular to the page (out of the page).

a. The box won't have a linear acceleration. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$
b. The box will have an angular acceleration? $\tau_{net} \neq 0$

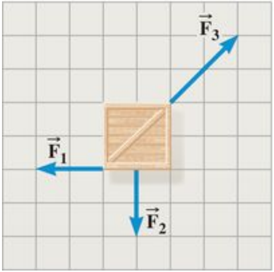

Figure: 17.6 Concept check for mechanical equilibrium

Fig 17.7 Calculate the center of gravity of the arm if $W_1 = 22.0\ kg$, $W_2 = 12.0\ kg$, and $W_3 = 5.0\ kg$. $x_{cg} = 10.47/39\ m$

The center of gravity is located $0.27\ m$ from the shoulder joint.
$x_{cg} = 0.27\ m$

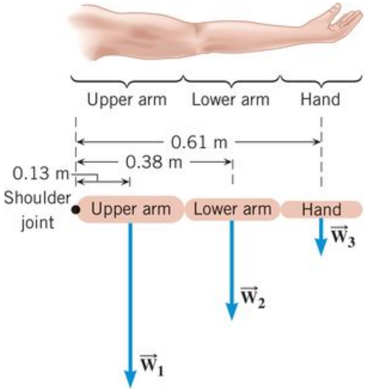

Figure: 17.7 Center of gravity of an adult arm

Answer Ex #4

C&J 9.22 A man holds a $178\ N$ ball in his hand, with the forearm horizontal. He can support the ball in this position because of the flexor muscle force $\vec{\mathbf{M}}$. The forearm weighs $22.0\ N$ and has a center of gravity as indicated.

a. The magnitude of $\vec{\mathbf{M}}$ is $1212\ N$. $M = 1212\ N$.
b. The force applied by the upper arm bone to the forearm at the elbow joint is $1012\ N$ down. $\vec{\mathbf{F}}_u = 1012\ N$, down.

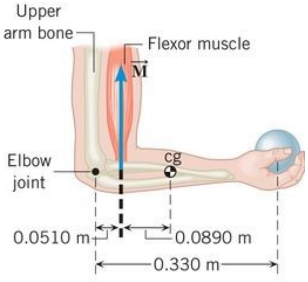

Figure: 17.8 Mechanical equilibrium and center of gravity example