Phys 111: Lecture 16

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“The 16th Day: Adventure of Zelda”

Homework Wk #8 Due Today

Tutor Schedule Update!

Today’s Topics

1. Rotational & Linear Motion
2. Rolling Motion
3. Preparing for Torque

Motivation

We'll start the process to learn about:

1. What if an object speeds up and moves in a circle?
2. How can we relate rotational and linear motion?
3. How does this relate to rolling motion?

Non-Uniform Circular Motion

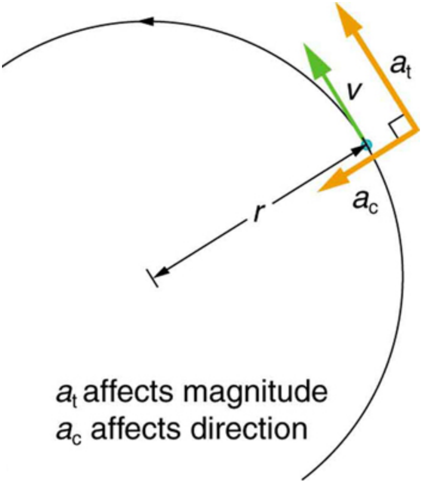

Figure: 16.1 $a_c = v^2/r$, $a_t = ?$

Constant Acceleration

Rotational Motion	Motion
$\omega = \omega_o + \alpha t$	$v = v_o + at$
$\theta = \frac{1}{2}\left(\omega + \omega_o\right)t$	$s = \frac{1}{2}\left(v + v_o\right)t$
$\theta = \omega_o t + \frac{1}{2}\alpha t^2$	$s = v_o t + \frac{1}{2}at^2$
$\omega^2 = \omega_o^2 + 2\alpha\theta$	$v^2 = v_o^2 + 2as$

Exercise #1

C&J 8.34 A fan blade is rotating with a constant angular acceleration of $+12.0\,rad/s^2$. At what point on the blade, as measured from the axis of rotation, does the magnitude of the tangential acceleration equal that of the acceleration due to gravity?

N-UCM Math

Even though v isn't constant, centripetal acceleration is still

$$a_c = \frac{v_t^2}{r}.$$

We know

$$v_t = r\omega \qquad \& \qquad a_t = \frac{\Delta v}{\Delta t} \qquad \& \qquad \alpha = \frac{\Delta \omega}{\Delta t}.$$

Hopefully it's not too surprising that

$$a_t = r\alpha$$

2D Rotational Acceleration Math

Polar Coordinates

$$\vec{\mathbf{a}} = \vec{\mathbf{a_c}} + \vec{\mathbf{a_t}}$$

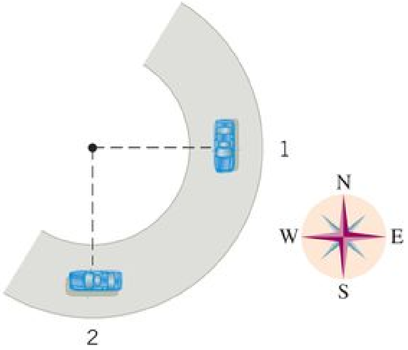

Figure: 16.2 Turning Cars

Working with Vectors

The Components of $\vec{\mathrm{a}}$

$$a_c = \frac{v_t^2}{r} = r\omega^2$$

$$a_t = r\alpha$$

$$a = \sqrt{a_c^2 + a_t^2}$$

for constant a

$$\alpha = \frac{\omega - \omega_o}{t}$$

Constant Acceleration

Rotational Motion	Motion
$\omega = \omega_o + \alpha t$	$v = v_o + at$
$\theta = \frac{1}{2}\big(\omega + \omega_o\big)t$	$s = \frac{1}{2}\big(v + v_o\big)t$
$\theta = \omega_o t + \frac{1}{2}\alpha t^2$	$s = v_o t + \frac{1}{2}at^2$
$\omega^2 = \omega_o^2 + 2\alpha\theta$	$v^2 = v_o^2 + 2as$

Exercise #2

C&J 8.42 A person lowers a bucket into a well by turning the hand crank. The crank handle moves with a constant tangential speed of $1.20\ m/s$ on its circular path. The rope holding the bucket unwinds without slipping on the barrel of the crank. Find the linear speed with which the bucket moves down the well.

Rolling without Slipping

Slipping relates to a wheel spinning so fast it overcomes the friction that pushes it forward. Think about driving on sand, ice, and "burning out".

A wheel **rolls without slipping** perfectly if it the speed of the wheel rolling relative to the ground is equal to the speed of the center of mass of the wheel moving forward.

$$v = v_t = r\omega$$

$$a = a_t = r\alpha$$

Car Example Image

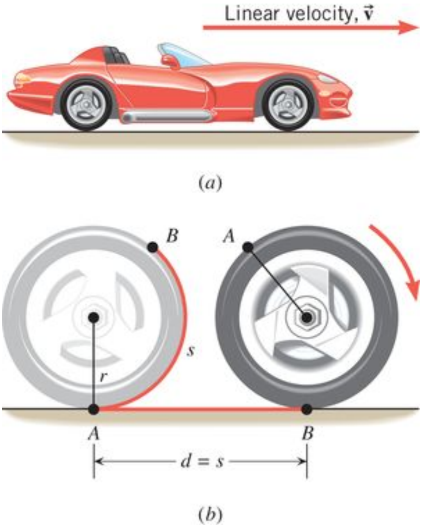

Figure: 16.3 Car wheel rolling without slipping

C&J 8.42 A person lowers a bucket into a well by turning the hand crank, as the drawing illustrates. The crank handle moves with a constant tangential speed of $1.20\ m/s$ on its circular path. The rope holding the bucket unwinds without slipping on the barrel of the crank. Find the linear speed with which the bucket moves down the well.

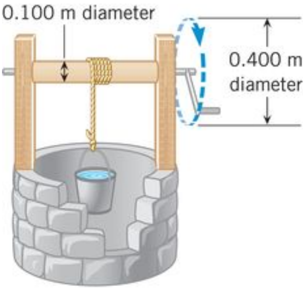

Figure: 16.2 Utilizing constant ω

Exercise #3

C&J 8.53 A motorcycle accelerates uniformly *from rest* and reaches a linear speed of $22.0\ m/s$ in a time of $9.00\ s$. The radius of each tire is $0.280\ m$. What is the magnitude of the angular acceleration of each tire?

Reasoning Strategy

Analyzing Rotational Motion

1. Make a drawing to represent the situation being studied, showing the direction of rotation.
2. Decide which direction of rotation is positive and which is negative. **Do not change** your decision during the course of a calculation.
3. Identify and write down known and unknown values and explicitly write down a note of what variables you are being asked to determine. Be careful about implied data such as reading "starts from rest".
4. Identify which physical relationships you are going to make use of and write down the relevant equations.
5. Simplify your equations by plugging int zeros and then solve.
6. *Double-check if the direction and magnitude of your answer seems reasonable or not.*

Exercise #4

C&J 8.54 An automobile tire has a radius of $r = 0.330\ m$, and its center moves forward with a linear speed of $v = 15.0\ m/s$.

a. Determine the angular speed of the wheel.
b. Relative to the axle, what is the tangential speed of a point located $0.175\ m$ from the axle?

The Cross Product

$$\vec{\mathbf{v}} = \vec{\omega} \times \vec{\mathbf{r}}$$

Also used as a way to write out the relationship between applied force (F) for a lever arm (l), and the corresponding torque (τ).

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

$$\tau = rF\sin(\theta)$$

$$\tau = Fl$$

Mag. of Torque = (Mag. of Force)(length of lever arm)

Right Hand Rule[2]

Grasp the axis of rotation with your right hand, so that your fingers circle the axis in the same sense as the rotation. Your extended thumb points along the axis in the direction of the angular velocity vector.

No part of the rotating object moves in the direction of the angular velocity vector.

[2] For quantifying the direction of an angular velocity

Direction of $\vec{\omega}$

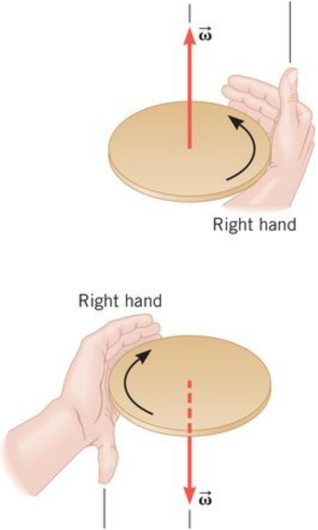

Figure: 16.4 Right hand rule for angular velocity

The End

Thanks for your time and attention!

Any questions?