Phys 111
R 10/24/19

Lect. 18 Problem Solving
(Scratch Work)

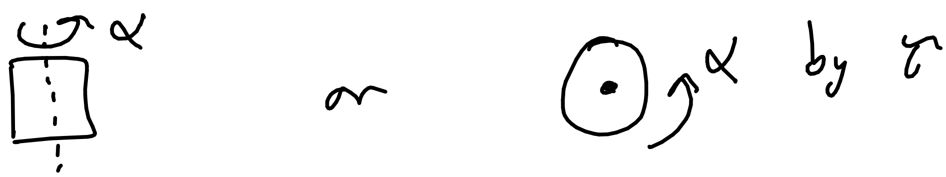
Look over the slides for more information about the problems and the problem-solving thought process.

Ex. 1 C&J 9.32

Find I given τ and α

using $\tau_{net} = I\alpha$.

$\tau = 15.0\ N\cdot m$ $\qquad$ $\alpha = 5.00\ \frac{rad}{s^2}$



Since $\tau = I\alpha$ then

$$I = \frac{\tau}{\alpha} \qquad \left(\text{also } \alpha = \frac{\tau}{I}\right)$$

$$I = \frac{(15.0)}{(5.00)} \frac{N\cdot m}{\frac{rad}{s^2}}$$

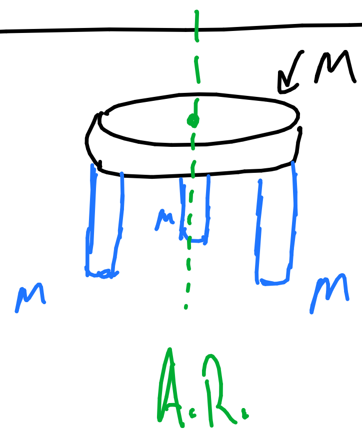
$$\boxed{I = 3.0\ kg\cdot m^2}$$

note

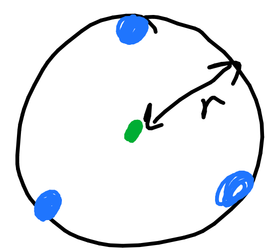
$N = \frac{kg\cdot m}{s^2}$

Ex. 2 C&J

Find the total I for an object with 4 distinct pieces of mass. $M = 1.4\,kg$, $m = 0.18\,kg$, $r = 0.18\,m$



Bottom View



• recall parallel-axis comment from class •

$$I = \sum I = \sum (\text{fraction})\, m r^2$$

$$I = I_{disk} + I_m + I_m + I_m$$

$$I = \left(\frac{1}{2} M r^2\right) + 3\left(m r^2\right)$$

$$\boxed{I = 0.040\ kg \cdot m^2}$$

Ex 3

How fast must a solid cylinder be spinning (ω) to store [RKE=] $1.2 \times 10^9 J$ of energy if it has a mass of $m = 15.0\ kg$ and a radius of $r = 0.40\ m$?

$$I_{disk} = I_{cyl} = \frac{1}{2} m r^2 \quad \& \quad RKE = \frac{1}{2} I \omega^2$$

Therefore

$$2\,RKE = I\omega^2$$

$$\omega^2 = \frac{2\,RKE}{I}$$

$$\omega = \sqrt{\frac{2\,RKE}{I}}$$

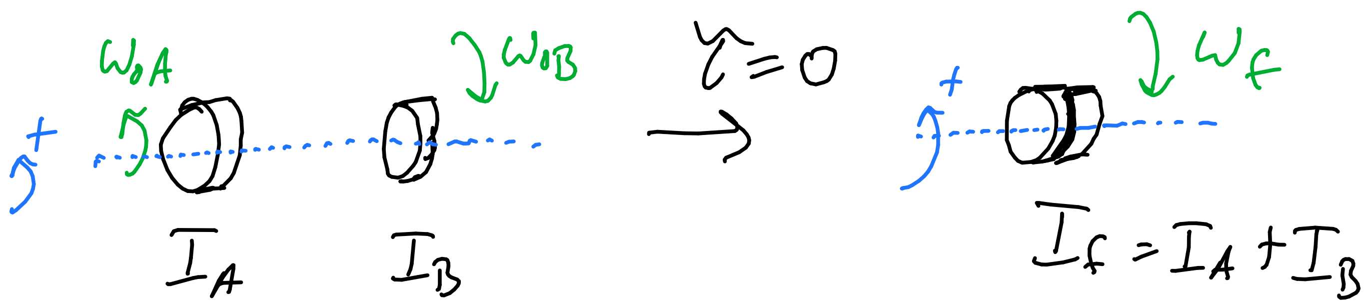
$$\omega = \sqrt{\frac{2(1.2 \times 10^9 J)}{(1.2\ kg \cdot m^2)}} \cong \boxed{4.5 \times 10^4 \frac{rad}{s}}$$

Ex 4 $\frac{1}{2}$ 9.59

Two disks are rotated and then connected together without adding any extra angular momentum. Solve for the unknown moment of inertia I_B.

$\omega_A = \omega_{0A} = +7.2$ rad/s $I_A = 3.4\ \text{kg}\cdot\text{m}^2$

$\omega_B = \omega_{0B} = -9.8$ rad/s $\omega_f = -2.4$ rad/s



Conservation: $L_0 = L_f$; $L = I\omega$

Thus, $L_{0A} + L_{0B} = L_f$

Specific to this problem

$$I_A \omega_A + I_B \omega_B = (I_A + I_B)\,\omega_f$$

Ex 4 2/2

$$I_A \omega_A + I_B \omega_B = (I_A + I_B)\omega_f$$

Now to rewrite/solve for I_B. (No zeros)

$$I_A \omega_A + I_B \omega_B = I_A \omega_f + I_B \omega_f$$

$$I_B \omega_B - I_B \omega_f = I_A \omega_f - I_A \omega_A$$

$$I_B (\omega_B - \omega_f) = I_A (\omega_f - \omega_A)$$

$$I_B = \frac{I_A (\omega_f - \omega_A)}{(\omega_B - \omega_f)}$$

Form ready for #'s.

$$I_B = \frac{(3.4\ kg \cdot m^2)(-7.2 - 2.4)\frac{rad}{s}}{(-9.8 + (+2.4))\frac{rad}{s}} = \frac{+(3.4)(9.6)}{+(7.4)}\ kg \cdot m^2$$

$$\boxed{I_B = 4.4\ kg \cdot m^2}$$

Ex 5 Ch.9 Example 15

Figure skater changes their moment of inertia with only internal torques. Is rotational energy conserved too.

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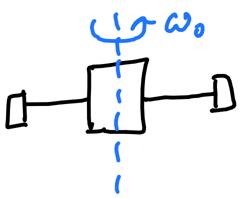
initial I_0, ω_0 final I, ω



$\tau = 0$

$I < I_0$

$$\tau = 0 \quad \rightarrow \quad L_0 = L_f$$

$I_0 \omega_0 = I\omega$ means if $I = \frac{1}{2} I_0$ then $\omega = 2\omega_0$.

Using $RKE_0 = \frac{1}{2} I_0 \omega_0^2$ and $RKE_f = \frac{1}{2} I \omega^2$

$$RKE_f = \frac{1}{2}\left(\frac{1}{2} I_0\right)(2\omega_0)^2$$

$$RKE_f = \frac{1}{2}\left(\frac{1}{2} I_0 \omega_0^2\right)(4)$$

$$RKE_f = 2\left(\frac{1}{2} I_0 \omega_0^2\right) > RKE_0$$

Think: The person had to use chemical/food potential energy to speed up.

Ex. 6 1/2

Three compact/point masses rotate about the same axis with the same angular velocity $\omega = 5.00 \frac{rad}{s}$, but different masses and radii.

$m_1 = 6.00\,kg$, $r_1 = 2.00\,m$

$m_2 = 4.00\,kg$, $r_2 = 1.50\,m$

$m_3 = 3.00\,kg$, $r_3 = 3.00\,m$

same ω, different m, r

(a) Calculate each tangential speed. ($v = r\omega$)

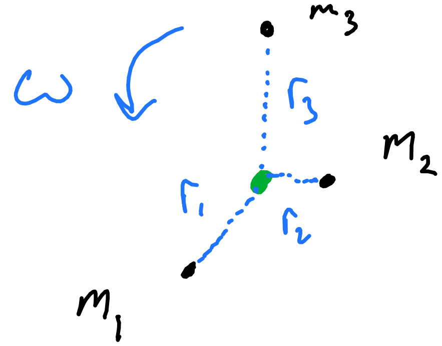
$$\boxed{V_1 = 10.0 \frac{m}{s}, \quad V_2 = 7.5 \frac{m}{s}, \quad V_3 = 15.0 \frac{m}{s}}$$

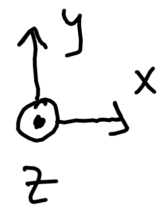
(b) Calculate $KE = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{1}{2} m_3 V_3^2$

$$\boxed{KE = 750\ J}$$

Ex. 6 2/2

(c) Calculate I for the whole system.





• Axis point/line

$$I = \sum m r^2$$

each mass is a pt. mass.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$I = (24.0 + 9.00 + 27.0)\ kg \cdot m^2$$

$$\boxed{I = 60.0\ kg \cdot m^2}$$

(d) Calculate $RKE = \frac{1}{2} I \omega^2$

$$RKE = \frac{1}{2}(60.0\ kg \cdot m^2)\left(5.00\ \frac{rad}{s}\right)^2$$

$$\boxed{RKE = 750\ J} = KE\ \checkmark$$