Name: Example Scratch Work

Homework Week # 4

2D Motion & Forces
Due Thurs 9/19/19

Reading

Tuesday: **C&J 4.7-4.12**

Thursday: **C&J 5.1-5.7**

It is required that students provide a FBD and accompanying vector equations for the force problems.

Problems

Problem 1. **3.3.19**

Problem 2. **3.3.20**

Problem 3. **3.4.52**

Problem 4. **3.4.63**

Problem 5. **4.3.7**

Problem 6. **4.3.9**

Problem 7. **4.4.11**

Problem 8. **4.8.41**

Problem 9. **4.9.47**

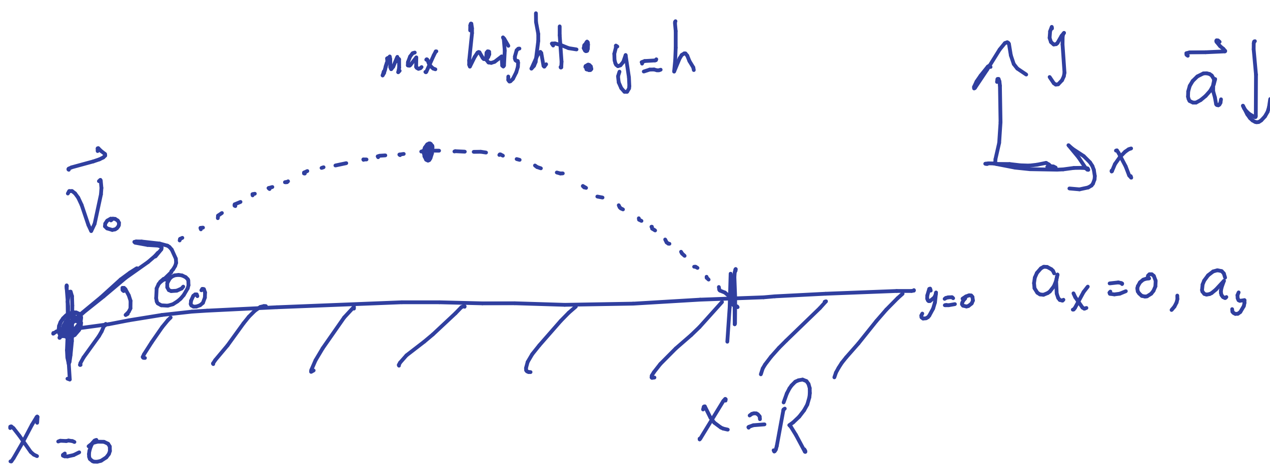
Problem 10. **4.9.51**

Prob. 1 3.3.19
1/S

ball w/ initial velocity $v_0 = 30.3 \frac{m}{s}$ @ angle θ s.t. max distance traveled when landing at same elevation.

Calculate (a) time in air $= t_{air}$

(b) At what distance does the ball land R_{max}

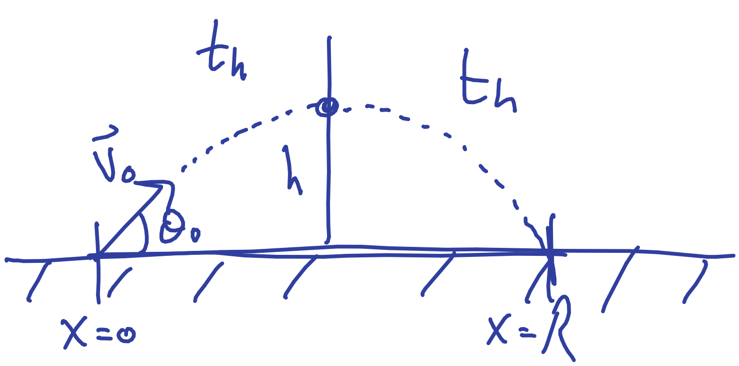


$a_x = 0, a_y = -g$

unless specified straight up, straight horizontal, always assume the launch velocity has an angle : $\vec{V}_0 = V_0, \theta \Rightarrow \vec{V}_0 = V_{0x}\hat{i} + V_{0y}\hat{j}$

Prob. 1 — 2/5

When an object lands at the same height it is launched at [& air resistance is negligible/ignorable], then we can say it takes the same amount of time for the object to reach max height when the velocity in the y direction $V_y = 0$ as it does to fall back down.



$$t_{air} = 2\,t_h$$

and the object travels a horizontal distance $R = \frac{V_0^2}{g}\sin(2\theta_0)$.

Prob. 1 3/5 3/20

With the standard x-y coordinates and constant acceleration due to gravity [g] we can quantify the motion by components with the constant acceleration equations

$$V_s = V_{os} + a_s t$$

$$s = s_o + V_{os} t + \frac{1}{2} a_s t^2$$

$$V_s^2 = V_o^2 + 2a_s (s - s_o)$$

for both x & y for "given" values:

$$a_x = 0, \ a_y = -g$$

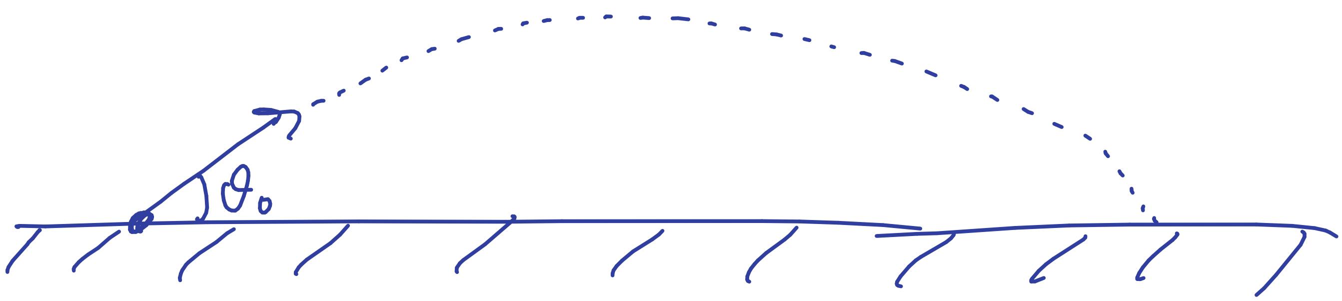
$$V_{ox} = V_o \cos\theta_o$$

$$V_{oy} = V_o \sin\theta_o$$

Prob. 1 $\xrightarrow{\quad}$ 4/5

4/20

"max distance" = R_{max} which occurs @ $\theta_0 = 45°$



$$R = \frac{V_0^2}{g}\sin(2\theta_0)$$

max @ $\sin(2\theta_0) = 1$

$2\theta_0 = 90°$

$\theta_0 = 45°$

From R w/ $\theta_0 = 45°$

$$R_{max} = \frac{(30.3\frac{m}{s})^2}{(10.0\frac{m}{s^2})} = 91.8\,m$$

(b) $\boxed{R_{max} = 91.8\,m}$

(a) $V_y = V_{oy} - gt$ $\qquad V_{oy} = V_0 \sin\theta_0 = (30.3\frac{m}{s})\sin 45° = 21.4\,\frac{m}{s}$

$V = 0$ @ $y = h$ b/c it stops moving up, pauses, and then moves back down $\rightarrow$ $V_{oy} - g t_h = 0 \rightarrow t_h = \frac{V_{oy}}{g}$

Prob. 1

5/5

$5/20$

$$t_h = \frac{30.3\ \frac{m}{s}}{10.0\ \frac{m}{s^2}} = 3.03\,s$$

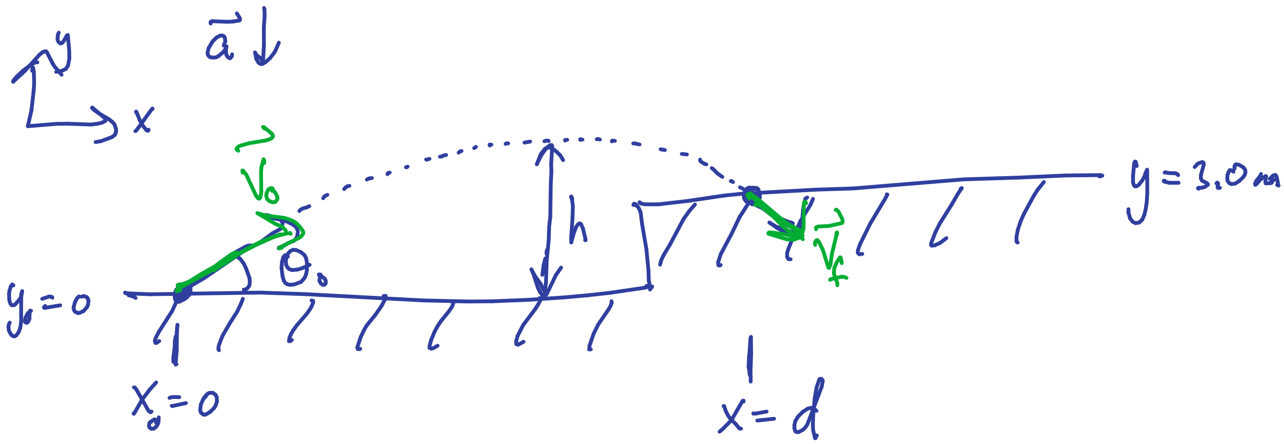
$$t_{air} = 2t_h = 2(3.03\,s)$$

$$\boxed{t_{air} = 6.06\,s}$$

Prob. 2 1/2 3.3.20

ball w/ initial velocity $v_0 = 14.0 \frac{m}{s}$, $\theta_0 = 40.0°$ rises to a max height and then falls back down landing at a height of 3.0 m above where it was launched

Find v_f, the speed of the ball right as it hits the ground.



Constant acceleration $\rightarrow$ Equations of Motion

$$a_x = 0, \quad a_y = -g \quad \rightarrow \quad \begin{cases} v_x = v_{0x} = v_0 \cos\theta_0 \\ x = x_0 + v_x t \\ \\ v_y = v_{0y} - g t \\ y = y_0 + v_{0y} t - \frac{g}{2} t^2 \end{cases}$$

Prob. 2 2/2

7/20

final velocity/speed @ same time as $y = 3.0\,m$.

Use $y = y_0 + V_0 \sin\theta_0\, t - \frac{g}{2} t^2$ solve for t, then plug t into $V_y = V_{0y} - gt$ & solve for V_f w/ $V_f = \sqrt{V_x^2 + V_y^2}$

$$3.0\,m = (14.0\,\tfrac{m}{s}) \sin 40.0^\circ\, t - \frac{10\,\frac{m}{s^2}}{2} t^2$$

$$\downarrow$$

$$(-5.0\,\tfrac{m}{s^2})t^2 + (9.00\,\tfrac{m}{s})t - 3.0\,m = 0$$

looks like $ax^2 + bx + c = 0$ so

$$t = \frac{(-9.00\,\frac{m}{s}) \pm \sqrt{(9.00\,\frac{m}{s})^2 - 4(-5.0\,\frac{m}{s^2})(-3.0\,m)}}{2(-5.0\,\frac{m}{s^2})}$$

$$t = \frac{(-9.00 \pm 11.87)\frac{m}{s}}{-10.0\,\frac{m}{s^2}} = 2.09\,s \text{ or } -0.287\,s$$

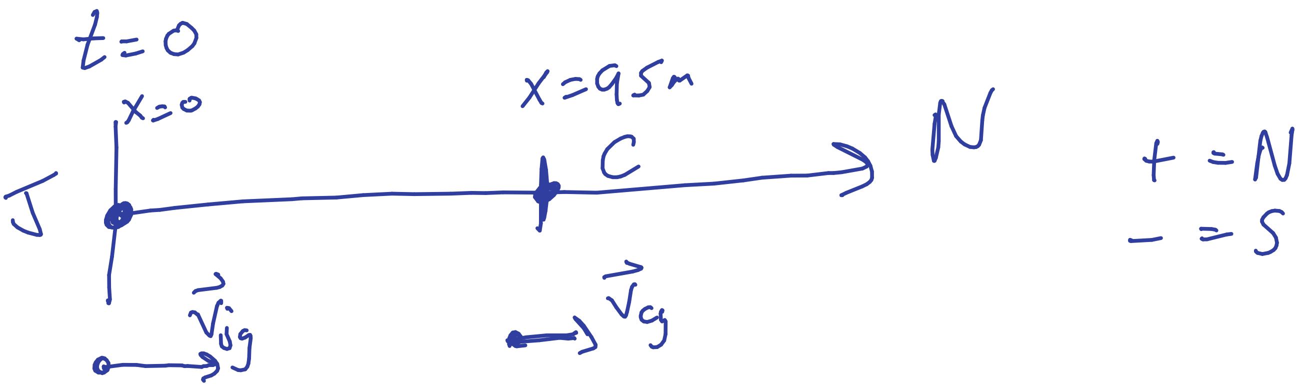
only $\underline{t = 2.09\,s}$ makes physical sense!

@ $t = 2.09\,s$ $\quad V_x = 10.72\,\frac{m}{s}$ $\quad V_y = -1.18\,\frac{m}{s}$

$$\boxed{V_f = 10.78\,\tfrac{m}{s}}$$

Prob. 3 1/1 3.4.52 8/20

Chad runs due north w/ speed $v_{cg} = 4.00 \frac{m}{s}$. John runs w/ speed $v_{jg} = 4.50 \frac{m}{s}$ and is initially 95 m behind Chad. How much time does it take for John to pass Chad?



$$\vec{v}_{jg} = \vec{v}_{jc} + \vec{v}_{cg} \rightarrow \vec{v}_{jc} = \vec{v}_{jg} - \vec{v}_{cg}$$

$$\vec{v}_{jc} = 4.5 \frac{m}{s} - 4.0 \frac{m}{s} = 0.5 \frac{m}{s}$$

meaning john runs $0.5 \frac{m}{s}$ faster than chad.

John has to run $\Delta x = 95\,m$ more than Chad to catch up so $v_{jc} = \frac{\Delta x}{\Delta t}$ can be used.

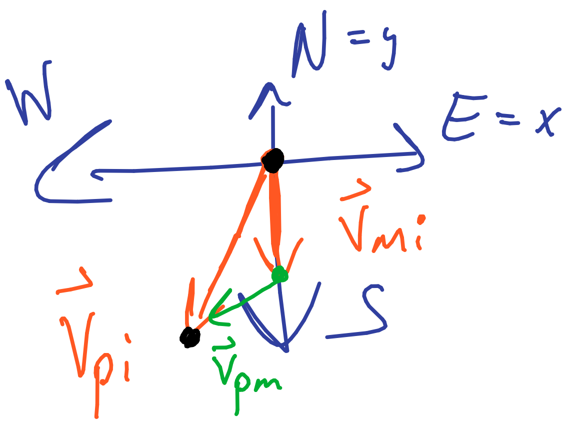
$$\Delta t = \frac{\Delta x}{v_{jc}} = \frac{95\,m}{0.5 \frac{m}{s}} \rightarrow \boxed{\Delta t = 190\,s}$$

Prob. 4 3.4.63 — 1/2 9/20

velocity of Mario relative to ice $\vec{v}_{mi} = 7.0\frac{m}{s}$ S

velocity of puck passed to ice $\vec{v}_{pi} = 11.0\frac{m}{s}$ 22° W of S

Find magnitude and direction of the puck relative to Mario. dir. relative to S

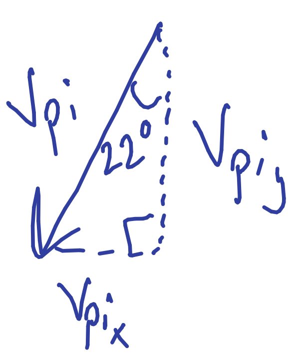


$$\vec{v}_{pi} = \vec{v}_{pm} + \vec{v}_{mi}$$

$$\downarrow$$

$$\vec{v}_{pm} = \vec{v}_{pi} - \vec{v}_{mi}$$

$$v_{mix} = 0, \; v_{miy} = -7.0\frac{m}{s}$$



$\rightarrow$

$$v_{pix} = -v_{pi}\sin 22° = -(11.0\tfrac{m}{s})\sin 22°$$

$$v_{pix} = -4.12\frac{m}{s}$$

$$v_{piy} = -v_{pi}\cos 22° = -(11.0\tfrac{m}{s})\cos 22°$$

$$v_{piy} = -10.2\frac{m}{s}$$

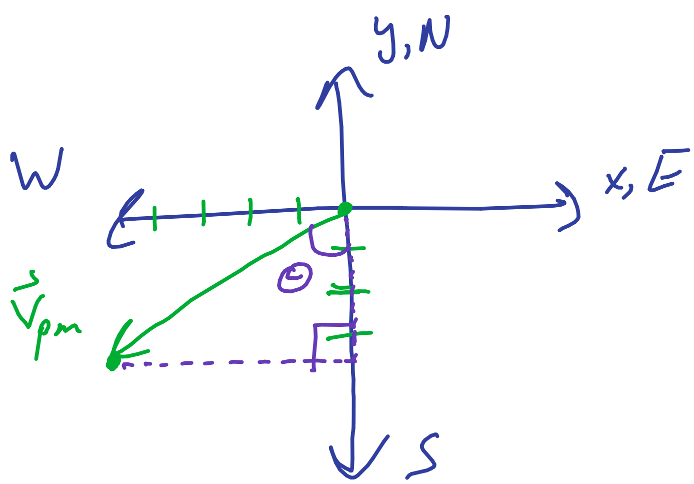
Prob. 4 2/2

10/20

$$\vec{V}_{pm} = \vec{V}_{pi} - \vec{V}_{mi}$$

$$= \left(-4.12\hat{i} - 10.2\hat{j}\right)\frac{m}{s} + \left(+7.0\hat{j}\right)\frac{m}{s}$$

$$\vec{V}_{pm} = \left(-4.12\hat{i} - 3.2\hat{j}\right)\frac{m}{s}$$



$$V_{pm} = \sqrt{V_{pmx}^2 + V_{pmy}^2}$$

$$\tan\theta = \frac{V_{pmx}}{V_{pmy}}$$

$$\theta = \tan^{-1}\left(\frac{V_{pmx}}{V_{pmy}}\right)$$

$V_{pmx} = -4.12\,\frac{m}{s}$ $\quad V_{pmy} = -3.2\,\frac{m}{s}$

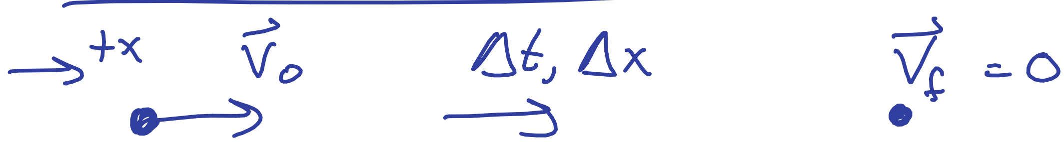
$$V_{pm} = \sqrt{(-4.12)^2 + (-3.2)^2}\;\frac{m}{s} = \sqrt{27.2}\;\frac{m}{s} = \boxed{5.2\,\frac{m}{s}}$$

$$\theta = \tan^{-1}\left(\frac{+4.12}{+3.2}\right) = \boxed{52.2^\circ}$$

Prob. 5, 4.3.7 11/20

1/2

How much force is required to stop a 1580 kg car with a speed of 15.0 $\frac{m}{s}$ in a distance of 50.0 m?



$\vec{a}, \vec{F}$?

$$\vec{F} = m\vec{a} \Longrightarrow \vec{a} \rightarrow \Delta\vec{v}$$

force causes acceleration producing a change in velocity.

recall $v_x^2 = v_{0x}^2 - 2a(x - x_0)$

given $v_{0x} = 15.0 \frac{m}{s}$, $v_f = 0$, $x - x_0 = 50.0$ m

$$2a(x - x_0) = v_{0x}^2 - v_x^2$$

$$a = \frac{v_{0x}^2 - v_x^2}{2(x - x_0)}$$

Prob. 5 2/2

12/20

$$a_x = \frac{(15.0 \frac{m}{s})^2 - 0 \frac{m^2}{s^2}}{2\,(50.0 m)} = 2.25 \frac{m}{s^2}$$

$$\sum \vec{F} = m\vec{a} \longrightarrow F_x = m a_x$$

$$F_x = (1580\ kg)(2.25 \frac{m}{s^2})$$

$$F_x = 3555 \frac{kg \cdot m}{s^2}$$

$$\boxed{F_x = 3555\ N}$$

Prob 6 4.3.9 13/20

Two force $\vec{F}_A, \vec{F}_B$ applied to an 8.0 kg mass object. When both the forces add constructively the object accelerates $0.50 \frac{m}{s^2}$ to the right. When they add destructively the object accelerates $0.40\, m/s^2$ to the right. Find F_A, F_B.

Given $m = 8.0$ kg, $a_1 = 0.50 \frac{m}{s^2}$, $a_2 = 0.4 \frac{m}{s^2}$ &

Case 1

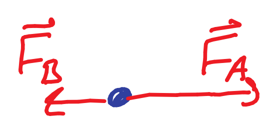
FBD

$$\sum \vec{F} = m\vec{a}_1$$

$$F_A + F_B = +ma_1$$

Case 2

FBD



$$\sum \vec{F} = m\vec{a}_2$$

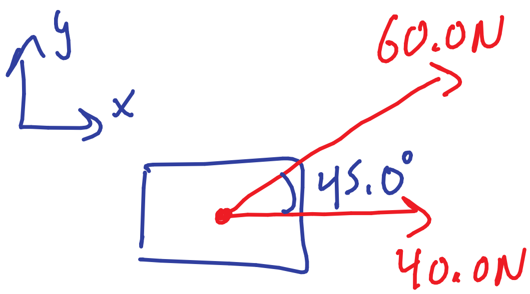
$$F_A - F_B = +ma_2$$

System of Linear Eqtn's

$$\begin{cases} F_A + F_B = 4.0\,N \\ F_A - F_B = 3.2\,N \end{cases} \rightarrow \boxed{\begin{aligned} F_A &= 3.6\,N \\ F_B &= 0.4\,N \end{aligned}}$$

Prob. 7 4.4.11

If two forces act on a block as shown, what is the magnitude and direction of the acceleration of the block?



$$\sum \vec{F} = m\vec{a}$$

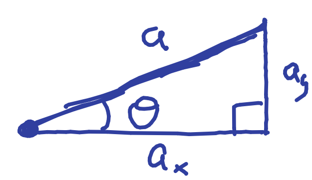
$$m = 3.00\,kg$$

$$\sum \vec{F} = \begin{cases} F_x = ma_x = (+40.0 + 60.0\cos 45°)\,N \\ F_y = ma_y = +60.0\sin 45°\ N \end{cases}$$

$$a_x = 27.5\,\frac{m}{s^2}$$

$$a_y = 14.1\,\frac{m}{s^2}$$

$\longrightarrow$



$$a = \sqrt{a_x^2 + a_y^2}$$

$$\tan\theta = \frac{a_y}{a_x}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

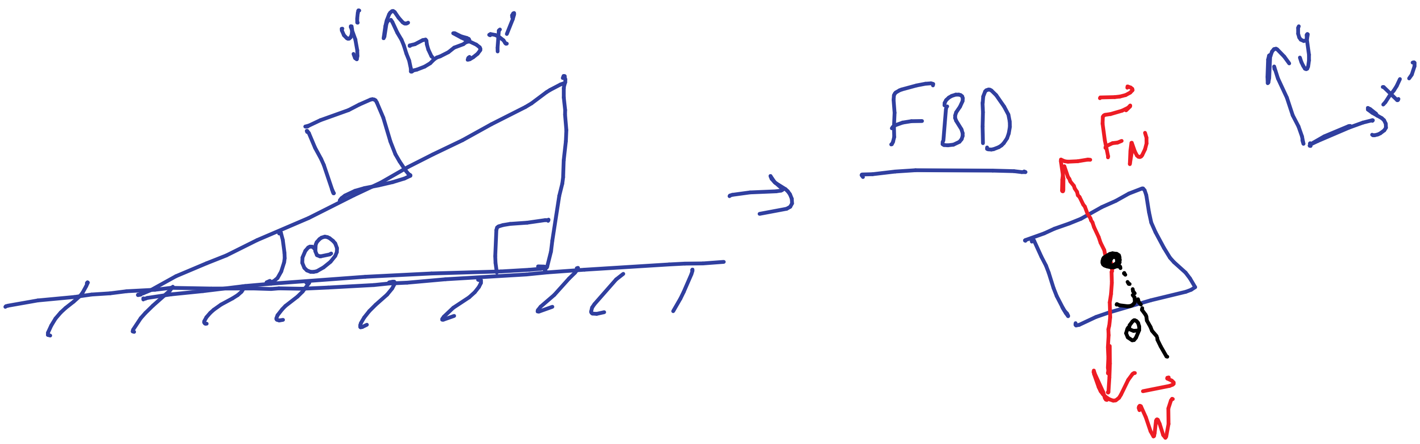
$\longrightarrow$

$$\boxed{\begin{aligned} a &= 30.9\ m/s^2 \\ \theta &= 27.24° \end{aligned}}$$

Prob. 8 4.8.41 15/20

Determine the ratio of the normal force to the weight of a car on inclines for 2 different angles.

$\frac{F_N}{W}$ @ (a) $\theta = 15°$ & (b) $\theta = 35°$



In both cases $\Sigma F_{y'} = F_N - W\cos\theta = 0$

b/c the car doesn't sink into the road.

$$F_N = W\cos\theta$$

$$\frac{F_N}{W} = \cos\theta \longrightarrow$$

$$\boxed{\begin{array}{l} (a)\ \frac{F_N}{W} = 0.96 \\ (b)\ \frac{F_N}{W} = 0.82 \end{array}}$$

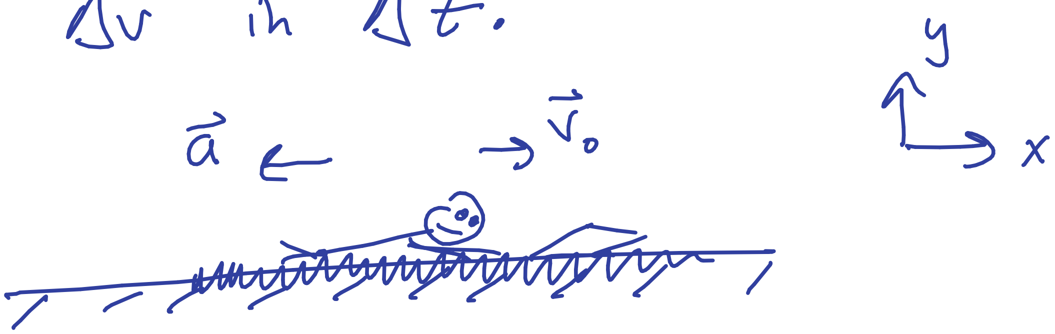
Prob. 9 $\frac{1}{2}$ 4.9.47 16/20

A 81 kg person slides with a coefficient of kinetic friction $\mu_k = 0.49$.

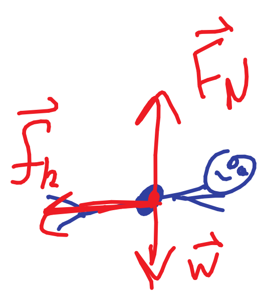
(a) Calculate $f_k = \mu_k N$

(b) If $v_f = 0$ at $t = 1.6s$, what was v_0?

Solve for $\vec{f}_k$ which produces an acceleration that causes Δv in Δt.



FBD of person



$\Sigma \vec{F} = m\vec{a} \rightarrow$

$$F_N - W = 0$$

$$-f_k = -ma$$

Prob. 9 2/2 17/20

$F_N - W = 0$ $\rightarrow$ $f_k = \mu_k N = \mu_k W$

$F_N = W$ $f_k = \mu_k mg$

$W = mg$

$\mu_k = 0.49, \quad m = 81\ \text{kg}, \quad g \cong 10.0\ \frac{m}{s^2}$

$$f_k = (0.49)(81\ \text{kg})\left(10.0\ \frac{m}{s^2}\right)$$

(a) $\boxed{f_k = 40.0\ N}$

(b) $f_k = ma \rightarrow a = \mu_k g = 4.9\ \frac{m}{s^2}$

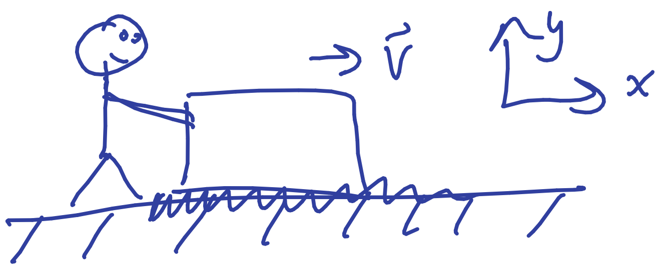
$$a \equiv \frac{\Delta v}{\Delta t} = \frac{V_f - V_0}{t}$$

$\cancel{V_f}^{\,0} - V_0 = at \rightarrow V_0 = -\left(-4.9\frac{m}{s^2}\right)(1.6\,s)$

$$\boxed{V_0 = +7.8\ \frac{m}{s}}$$

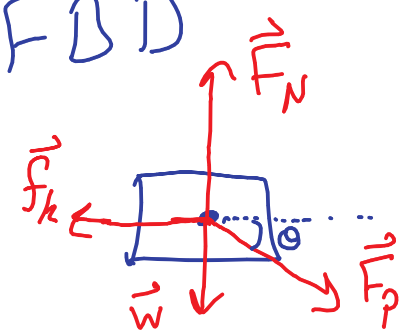
Prob. 10 1/3 4.9.51 18/20

A person pushes a box with a constant speed at an angle θ from horizontal on a floor that has a coeffecient of kinetic friction of 0.41. Find the smallest angle that would result in the box to stop moving.



$\vec{v}$ constant means $\vec{a} = 0$

FBD



$$\Sigma \vec{F} = 0$$

$$\Sigma F_x = F_p \cos\theta - f_k = 0$$

$$\Sigma F_y = F_N - W - F_p \sin\theta = 0$$

Note $f_k = \mu_k F_N$!

Prob. 10 2/3

Let's solve for F_p in terms of everything else and then think about what portion of the math would match/produce somet

From $F_x = 0 \rightarrow F_p \cos\theta = f_k = \mu_k F_N$

From $F_y = 0 \longrightarrow F_N = F_p \sin\theta + W$

Together we get $F_p \cos\theta = \mu_k (F_p \sin\theta + W)$

$$F_p \cos\theta - F_p \mu_k \sin\theta = \mu_k W \quad ; \mu_k = 0.41$$

$$F_p (\cos\theta - 0.41 \sin\theta) = 0.41 W$$

Prob. 10 3/3

20/20

$$F_p = \frac{0.41\,W}{\sin\theta - 0.41\cos\theta}$$

$F_p \to \infty$ as $\sin\theta - 0.41\cos\theta \to 0$

so let's check $\sin\theta = 0.41\cos\theta$

or $\tan\theta = 0.41$.

$$\theta = \tan^{-1}(0.41)$$

$\theta = 22.3° \pm 90° \to -68°, +112°$ ← pushing a different direction

$$\boxed{\theta = -68° \text{ CCW} = 68° \text{ CW}}$$

Remember that the push is currently balanced by friction and the normal force so the friction and normal force become very large at this angle and people can't push hard enough to move the box at a constant speed.