Typical 2D Projectile Motion

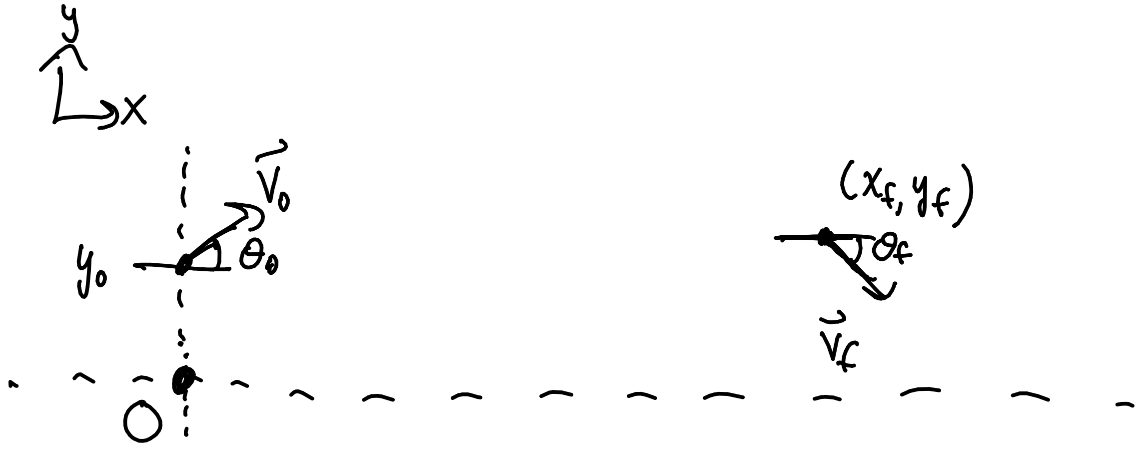
1. Object w/ initial position $\vec{r}_0 = x_0\hat{i} + y_0\hat{j}$

2. Object has an initial velocity
   $\vec{V}_0 = V_{0x}\hat{i} + V_{0y}\hat{j} = V_0, \theta_0$

3. Constant Acceleration (note: $g \equiv +9.81\frac{m}{s^2} \approx 10.0\frac{m}{s^2}$)
   $\vec{a} = a_x\hat{i} + a_y\hat{j}$; a_x, a_y constant

4. Goals range from solving for time of flight, final speeds, final positions, initial velocities, etc.

<u>Sketch</u> $\vec{a}$ specified; Often $\vec{a} = -g\hat{j}$



In general we use the equations

$$s_f = s_0 + v_{0s} t + \frac{1}{2} a_s t^2 \qquad v_s = v_{0s} + a_s t$$
$$v_s^2 = v_{0s}^2 + 2 a_s \Delta s$$

as needed for each direction. So in general we rewrite them for x and y knowing that <u>time</u> is the quantity that links all of the quantities/equations together.

This leads to an interesting result where the total horizontal distance or range R of an object that lands at the same height it is launched at can be calculated with:

$$R = \frac{v_0^2}{g} \sin(2\theta_0) \qquad \left[\text{when } \vec{a} = -g\hat{j}\right]$$

which has a maximum value when $\theta_0 = 45°$.

C&J 3.3.19

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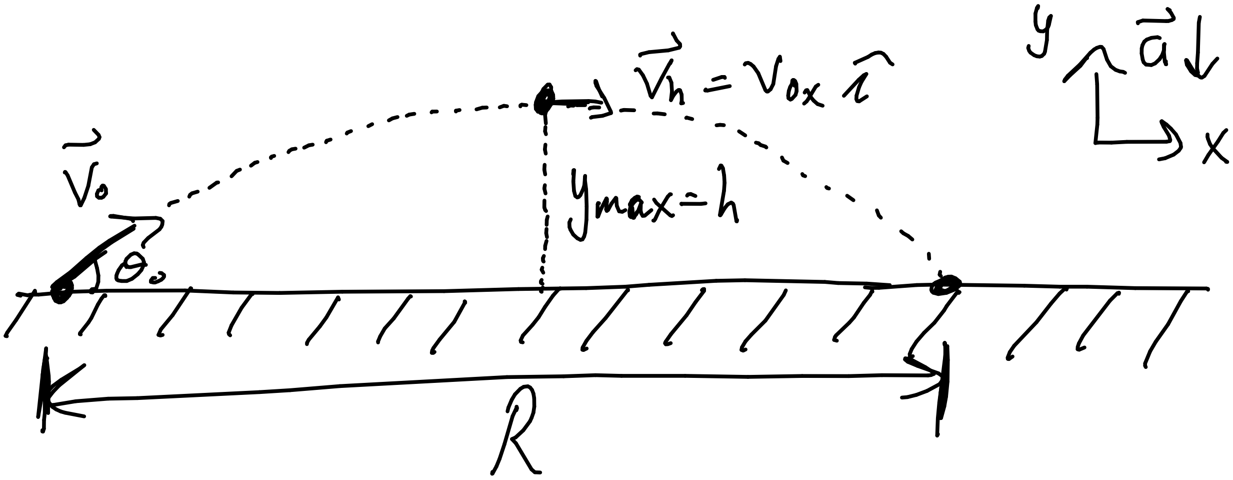
ball w/ $V_0 = 30.3\frac{m}{s}$ lands at the same height as launched and travels max x-distance before landing.

(a) Calc. t in air. (b) Calc. R_{max}



$\underline{x:}$ $a_x = 0$

$V_x = V_{0x}$

$X = X_0 + V_{0x} t$

$\underline{y:}$ $a_y = -g \approx -10.0\frac{m}{s^2}$

$V_y = V_{0y} + a_y t$

$y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2$

$V_y^2 = V_{0y}^2 - 2g\Delta y$

$V_{0x} = V_0 \cos\theta_0$

$V_{0y} = V_0 \sin\theta_0$

Notice that when $\vec{a} = a_y\hat{\jmath}$, or $a_x = 0$, the x equations are a lot simpler than in general like for y.

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Need to Know / Figure Out to Solve

(a) If $y_f = y_0$, then $t_{air} = 2t_{maxy}$. t_{maxy} @ $v_y = 0$

or $v_{0y} - gt = 0 \Rightarrow t_{maxy} = \frac{v_{0y}}{g} = \frac{v_0 \sin\theta_0}{g}$

(b) max x-distance or max Range = R_{max} occurs at $\theta_0 = 45°$ b/c $\sin(2\theta_0) = \max = 1$ @ $2\theta_0 = 90° \rightarrow \theta_0 = 45°$ for $R = \frac{v_0^2 \sin(2\theta_0)}{g}$.

(a) $t = 2t_h = \frac{2 v_0 \sin\theta_0}{g}$

(b) $R = \frac{v_0^2}{g} \sin(2\theta_0)$

$v_0 = 30.3 \frac{m}{s}$

$g \cong 10.0 \, m/s^2$

$\theta_0 = 45°$

$$t = \frac{2(30.3)\left(\frac{\sqrt{2}}{2}\right)}{(10.0)} s = \boxed{4.29 s}$$

$$R = \frac{(30.3)^2}{(10.0)}(1) \, m = \boxed{91.8 \, m}$$

C&J 3.3.15 (Lect. 04 Ex. 7)

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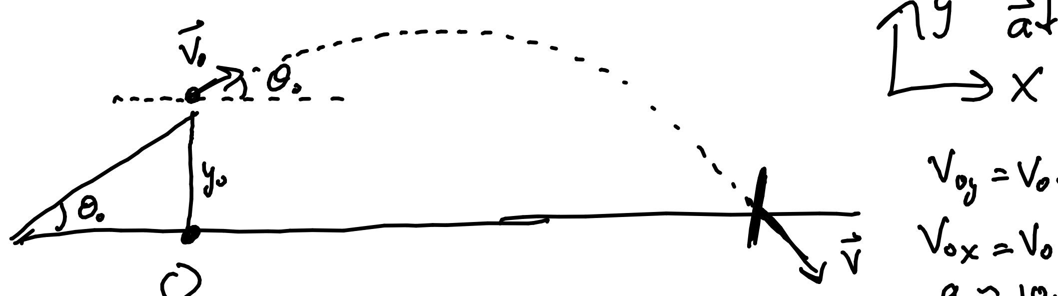
proj. mot. ($\vec{a} = -g\hat{j}$): $\vec{V}_0 = 6.6 \frac{m}{s}$, $58.0°$ from horizontal

$y_0 = 1.2\,m$, $x_0 = 0$. Calculate

(a) $y_{max} = h$ (b) x @ $y = h$

(c) total air time (d) v when skater lands



$V_{0y} = V_0 \sin\theta_0$

$V_{0x} = V_0 \cos\theta_0$

$g \approx 10.0\,m/s^2$

(a)' y_{max} when $V_y = 0$ so I can use either

$$\begin{cases} V_y = V_{0y} - g\,t = 0 \\ y_f = y_0 + V_{0y}t - \frac{g}{2}t = h \end{cases} \quad \text{or} \quad V_y^2 = V_{0y}^2 - 2g(h - y_0) = 0.$$

$V_{0y}^2 - 2g(h - y_0) = 0$ is better for (a), but keep in mind $t_h = \frac{V_{0y}}{g} = 0.467s$ is need for other parts.

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(a)[2] $V_{0y}^2 - 2g(h - y_0) = 0 \rightarrow h = \frac{V_{0y}^2}{2g} + y_0$

$\boxed{h = 2.77\,m}$

(b) $X_h = V_{0x} t_h = \frac{V_{0x} V_{0y}}{g} = \frac{V_0^2}{g} \cos\theta_0 \sin\theta_0$

$\boxed{X_h = 1.63\,m}$

(c) $t_{air} = 2t_h = \boxed{0.93\,s}$

(d) $V = \sqrt{V_x^2 + V_y^2}$

$V_x = V_{0x}$

$V_y = V_{0y} - g\,t_{air}$

$V_x = 3.50\,m/s, \quad V_y = -3.74\,m/s$

$\boxed{V = 5.1\,\frac{m}{s}}$