Phys 111: Lecture 13

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"The 13^{th} Day: Phantom Hourglass"

Homework Wk #7 Due Thursday 10/10/19

Exam #2 During Wk # 10 Tuesday 10/29/19

Today's Topics

1. Applications of Momentum
2. Momentum
3. 1D Collisions

Strong Points of Momentum

1. Discuss inertia and velocity simultaneously
2. Quantify interaction of collisions/scattering
3. Directly include time and direction
4. New Conservation Law!

Everyday Forensics

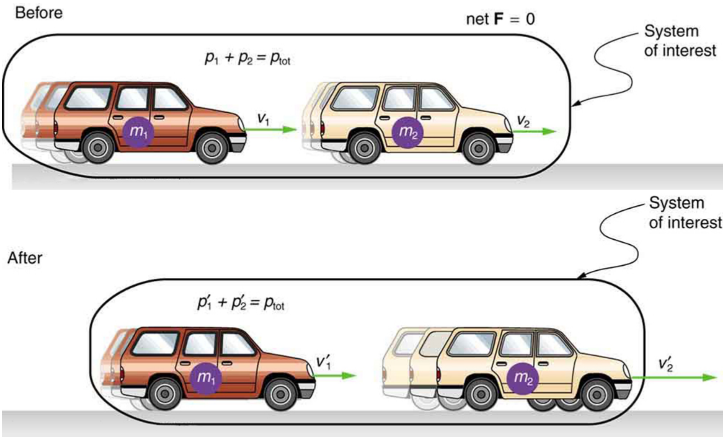

Figure: 13.1 Analysis of car collisions

Astronomical Transportation

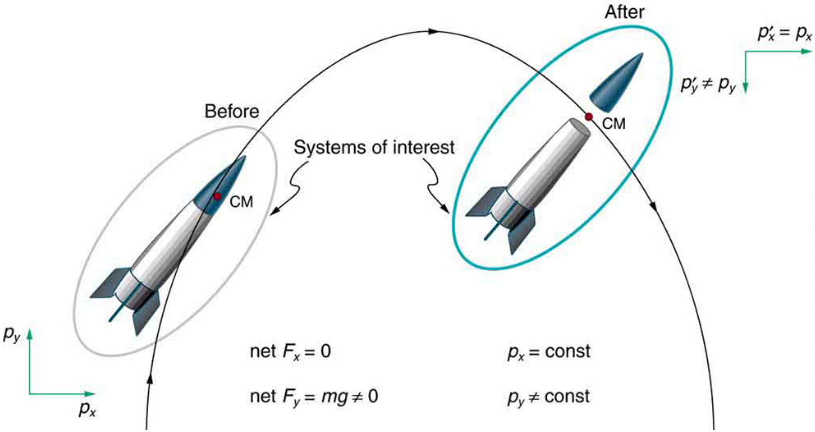

Figure: 13.2 Total rocket vs "payload"

Subatomic Forensics

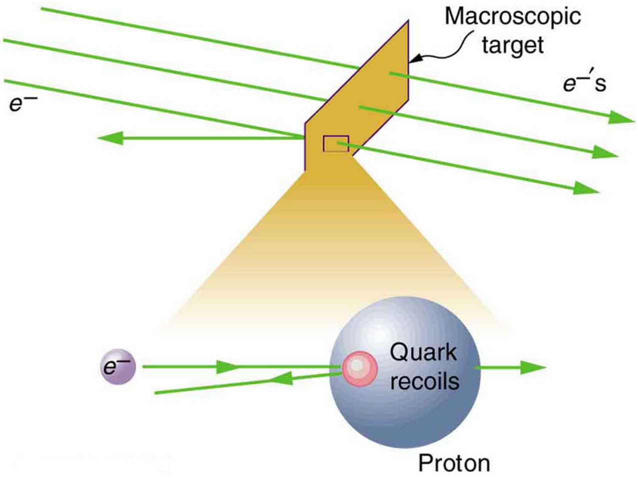

Figure: 13.3 Scattering and Backscattering

Planetary Science

Figure: 13.4 The Arizona Meteor Crater in Flagstaff, Arizona.

Consider the following

1. How should you catch an egg or a hard ball?
2. Should you land with stiff legs or bend your legs as you land?
3. Why are seat belts and safety seats required by law?
4. How do forensics experts interpret the measurements from a crime scene?

Momentum of an Object

Momentum = Total Motion

Linear momentum is measured by the velocity and the inertia of the object. Denoted $\vec{\mathbf{p}}$ and p for magnitude.

$$\text{linear momentum} = \text{inertia x motion}$$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

$$[p] = kg \cdot m/s$$

Impulse

Impulse = Change in Linear Momentum.

An impulse is the net change in momentum of an object from force being applied for however much time of interaction. Denoted $\vec{\mathbf{J}}$.

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_o$$

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$$

$$\vec{\mathbf{J}} = \vec{\mathbf{F}}_{ave}\Delta t$$

Impulse cont'd

Impulse = Change in Momentum.

$$\vec{\mathbf{F}}_{ave}\Delta t = \Delta\vec{\mathbf{p}}$$

$$\vec{\mathbf{F}}_{ave} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$$

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$$

Exercise #1

C&J 7.8 When jumping straight down, you can be seriously injured if you land stiff-legged. One way to avoid injury is to bend your knees upon landing to reduce the force of the impact. A $75.0\ kg$ man just before contact with the ground has a speed of $6.4\ m/s$.

a. In a stiff-legged landing he comes to a halt in $2.0\ ms$. Find the average net force that acts on him during this time.
b. When he bends his knees, he comes to a halt in $0.10\ s$. Find the average net force now.

Exercise #1 Answers

C&J 7.8 $\Delta t_a = 2.0\ ms$, $\Delta t_b = 100\ ms$, $v_o = 6.4\ m/s$, $v_f = 0\ m/s$, $m = 75.0\ kg$.

Find F_{ave} using

$$F_{ave} = \left| \frac{\Delta \vec{\mathbf{p}}}{\Delta t} \right|$$

a. In the stiff-legged landing, $F_{ave} = 240,000\ N$
b. When he bends his knees, $F_{ave} = 4,800\ N$

C&J 7.11 A student ($m = 63\ kg$) falls freely from rest and strikes the ground. During the collision with the ground, he comes to rest in a time of $0.040\ s$. The average force exerted on him by the ground is $+18000\ N$, where the upward direction is taken to be the positive direction. From what height did the student fall? Assume that the only force acting on him during the collision is that due to the ground.

C&J 7.17 $m = 63\ kg$, $t = 0.040\ s$, $F_{ave} = +18000\ N$. Find h by using conservation of momentum and energy together.

$$KE_f = PE_o \qquad \vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}}$$

$$h = \frac{v^2}{2g} \qquad v = \frac{Ft}{m}$$

$$h = \frac{1}{2g}\left(\frac{Ft}{m}\right)^2$$

$$h = 6.53\ m$$

Applying the Principle of Conservation of Linear Momentum

1. Decide which objects are included in the system.
2. Relative to the system that you have chosen, identify the internal forces and the external forces.
3. Verify that the system is isolated. In other words, verify that the sum of the external forces applied to the system is zero. Only if this sum is zero can the conservation principle be applied. If the sum of the average external forces is not zero, consider a different system for analysis.
4. Set the total final momentum of the isolated system equal to the total initial momentum. Remember that linear momentum is a vector. If necessary, apply the conservation principle separately to the various vector components.

If energy can be conserved, then why not momentum?

$$E_o + W_{nc} = E_f$$

$$W = (F cos\theta)s$$

$$W_{net} = \Delta KE = KE_f - KE_o$$

Anticipating new laws with the same physics.

$$\vec{\mathbf{p}}_o + \vec{\mathbf{J}}_{ext} = \vec{\mathbf{p}}_f$$

Conservation of Momentum

Energy conserved when all forces are conservative!

$$W_{nc} = E_f - E_o = 0$$

$$E_f = E_o$$

Momentum is conserved when all external forces balance!

$$\vec{\mathbf{J}}_{ext} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_o = 0$$

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_o$$

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_o \quad \rightarrow \quad \left\{ \begin{matrix} p_{ox} = p_{fx} \\ \\ p_{oy} = p_{fy} \end{matrix} \right\}$$

Two Related Conservation Laws

Energy

$$E_o + W_{nc} = E_f$$

$$W = (F cos\theta)\Delta s$$

Momentum

$$\vec{\mathbf{p}}_o + \vec{\mathbf{J}}_{ext} = \vec{\mathbf{p}}_f$$

$$\vec{\mathbf{J}} = \vec{\mathbf{F}}\Delta t$$

Relating Momentum & Kinetic Energy

$$Claim:\ p^2 = 2mKE$$

$$\left|\vec{\mathbf{p}}\right| = p = mv$$

$$KE = \frac{1}{2}mv^2$$

$$v = \frac{p}{m}$$

$$KE = \frac{1}{2}m\left(\frac{p}{m}\right)^2$$

$$Answer:\ KE = \frac{p^2}{2m}$$

Exercise #3

C&J 7.17 A $2.3\ kg$ cart is rolling across a frictionless, horizontal track toward a $1.5\ kg$ cart that is held initially at rest. The carts are loaded with strong magnets that cause them to attract one another. Thus, the speed of each cart increases. At a certain instant before the carts collide, the first cart's velocity is $+4.5\ m/s$, and the second cart's velocity is $-1.9\ m/s$.

a. What is the total momentum of the system of the two carts at this instant?
b. What was the velocity of the first cart when the second cart was still at rest?

Exercise #3 Answer

C&J 6.2.13 $m_1 = 2.3\ kg$, $m_2 = 1.5\ kg$, $v_{2o} = 0\ m/s$, $v_{1f} = 4.5\ m/s$, $v_{2f} = -1.9\ m/s$.

$$\vec{\mathbf{p}}_o = \vec{\mathbf{p}}_f$$

$$\vec{\mathbf{p}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$$

a. $\vec{\mathbf{p}}_f = +7.5\ kg \cdot m/s$
b. $\vec{\mathbf{v}}_{1o} = +3.26\ m/s$

Problem Solving with Energy & Momentum

1. Determine the *system of interest* and *identify knowns, unknowns, and goals*. A sketch will help.
2. Verify if the system is isolated and set up momentum equations.
3. Verify if the system of interest has any non-conservative forces acting on it and set up energy equations.
4. Before solving, eliminate terms wherever possible to simplify the algebra. Ex. Set initial or final height to be zero.
5. Solve momentum and energy equations algebraicly so you have unknowns solved for in terms of knowns.
6. Plug in specific quantities to finish calculations.
7. *Check the answer to see if it is reasonable.* Check for $\pm$ signs and if speeds or heights are too large or too small.

Collisions and Explosions are often classified according to whether the total kinetic energy changes during the collision:

1. **Elastic collision:** One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision.
2. **Inelastic collision:** One in which the total kinetic energy of the system is not the same before and after the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.

For 1D Collisions

1. Verify if the collision is elastic or inelastic.
2. Verify if the objects colliding form an isolated system.
3. Apply Energy & Momentum Problem Solving Steps appropriately.

Exercise #4

C&J 7.21 A two-stage rocket moves in space at a constant velocity of $4900\ m/s$. The two stages are then separated by a small explosive charge placed between them. Immediately after the explosion the velocity of the $1200\ kg$ upper stage is $5700\ m/s$ in the same direction as before the explosion. What is the velocity (magnitude and direction) of the $2400\ kg$ lower stage after the explosion?

Exercise #4 Answers

C&J 7.21 Find $\vec{\mathbf{v}}_{lf}$ if $m_u = 1200\ kg$, $m_l = 2400\ kg$, $\vec{\mathbf{v}}_o = +4900\ m/s$, $\vec{\mathbf{v}}_{uf} = +5700\ m/s$.

$$\vec{\mathbf{p}}_o = \vec{\mathbf{p}}_f$$

$$(m_u + m_l)\vec{\mathbf{v}}_o = m_u\vec{\mathbf{v}}_{uf} + m_l\vec{\mathbf{v}}_{lf}$$

$$\vec{\mathbf{v}}_{lf} = \frac{(m_u + m_l)\vec{\mathbf{v}}_o - m_u\vec{\mathbf{v}}_{uf}}{m_l}$$

$$\vec{\mathbf{v}}_{lf} = +4500\ m/s$$

Exercise #5

C&J 7.29 A $2.50 \times 10^{-3}\ kg$ bullet, traveling at a speed of $425\ m/s$, strikes the wooden block of a ballistic pendulum. The block has a mass of $0.215\ kg$.

a. Find the speed of the bulletblock combination immediately after the collision.
b. How high does the combination rise above its initial position?

Exercise #5 Answers

C&J 6.4.35 $m_1 = 2.50 \times 10^{-3}\ kg$, $v_{1o} = 425\ m/s$, $m_2 = 0.215\ kg$ on him, and $v_{2o} = 0\ m/s$. $m_t = m_1 + m_2$

$$\vec{\mathbf{p}}_o = \vec{\mathbf{p}}_1$$

$$E_1 = E_2 \quad \rightarrow \quad KE_1 = PE_2$$

$$\frac{p_1^2}{2m_t} = m_t g h$$

$$h = \frac{p^2}{2gm^2}$$

a. $p_1 = 1.06\ kg \cdot m/s$
b. $h = 0.260\ m$

The End

Thanks for your time and attention!
Any questions?

C&J 7.16 In a science fiction novel two enemies, Bonzo and Ender, are fighting in outer space. From stationary positions they push against each other. Bonzo flies off with a velocity of $+1.5\ m/s$, while Ender recoils with a velocity of $-2.5\ m/s$.

a. Without doing any calculations, decide which person has the greater mass. Give your reasoning.
b. Determine the ratio m_B/m_E of the masses of these two enemies.

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a. Without doing any calculations, decide which person has the greater mass. Give your reasoning.

   Bonzo has a larger mass because lower speed from same applied force or acceleration.

b. Determine the ratio m_B/m_E of the masses of these two enemies.

   $\mathbf{1.67}$ because equal to reciprocal of the ratio of speeds from conservation of momentum with $\vec{\mathbf{p}}_o = 0 = \vec{\mathbf{p}}_f$.

C&J 7.20 An astronaut in his space suit and with a propulsion unit (empty of its gas propellant) strapped to his back has a mass of 146 kg. The astronaut begins a space walk at rest, with a completely filled propulsion unit. During the space walk, the unit ejects some gas with a velocity of $+32$ m/s. As a result, the astronaut recoils with a velocity of 0.39 m/s. After the gas is ejected, the mass of the astronaut (now wearing a partially empty propulsion unit) is 165 kg. What percentage of the gas was ejected from the completely filled propulsion unit?

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Approximately **9.5%** of the gas was ejected during the space walk.

C&J 7.28 After sliding down a snow-covered hill on an inner tube, Ashley is coasting across a level snowfield at a constant velocity of $+2.7\ m/s$. Miranda runs after her at a velocity of $+4.5\ m/s$ and hops on the inner tube. How fast do the two slide across the snow together on the inner tube? Ashley's mass is $71\ kg$ and Miranda's is $58\ kg$. Ignore the mass of the inner tube and any friction between the inner tube and the snow.

Additional Practice #3 Answer

C&J 7.28 After sliding down a snow-covered hill on an inner tube, Ashley is coasting across a level snowfield at a constant velocity of $+2.7\ m/s$. Miranda runs after her at a velocity of $+4.5\ m/s$ and hops on the inner tube. How fast do the two slide across the snow together on the inner tube? Ashley's mass is $71\ kg$ and Miranda's is $58\ kg$. Ignore the mass of the inner tube and any friction between the inner tube and the snow.

The speed of the two after Miranda jumps on is approximately $\mathbf{3.5\ m/s}$.