Name: Key

Homework Week # 12

SHM Continued
Due Thurs 11/14/19

Reading

C&J Physics:	**Tues – Ch. 11: 1-6**	**Thurs – Ch. 11: 7-11**
OS Coll Phys:	**Tues – Ch. 11: 1-5**	**Thurs – Ch. 11: 5-9**

Problems

Problem 1. **10.21**

Problem 2. **10.22**

Problem 3. **10.25**

Problem 4. **10.26**

Problem 5. **10.27** +2

Problem 6. **10.37** +2

Problem 7. **10.46**

Problem ~~5~~. 8. **10.74**

Problem ~~6~~. 9. **FOC 10.17**

Problem ~~7~~. 10. **10.78 (a) only**

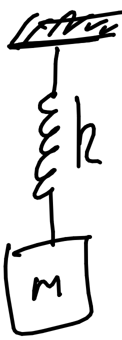
+6 for completion

Prob. 1 10.21

$X_{sp} = 0.018\,m$ for $F_{app} = m_1 g$

with $m_1 = 2.8\,kg$.

Which mass will have a natural / resonance frequency of $f_2 = 3.0\,Hz$ if attached to this spring?



$$k = \frac{m_1 g}{x_1} = 1556\,\frac{N}{m} \quad \& \quad \omega = 2\pi f = \sqrt{\frac{k}{m}}$$

square both sides

For m_2: $\dfrac{k}{m_2} = (2\pi f_2)^2$

(resolve for m) $\quad m_2 = \dfrac{k}{(2\pi f_2)^2}$

$$m_2 = \frac{\left(\frac{m_1 g}{x_1}\right)}{(2\pi f_2)^2} = \frac{\left(\frac{2.8 \cdot 10}{0.018}\right)\frac{N}{m}}{\left(2\pi(3.0)\frac{1}{s}\right)^2}$$

$$\boxed{m_2 = 4.38\,kg \approx 4.4\,kg}$$

Prob. 2 10.22 2/10

horizontal shm of a spring with

$V_{max} = 1.25 \frac{m}{s}$, $a_{max} = 6.89 \frac{m}{s^2}$.

What is time between V_{max} and a_{max} (Δt)?

For a spring:

$$\left.\begin{array}{l} V = -A\omega \sin(\omega t) \\ a = -A\omega^2 \cos(\omega t) \end{array}\right\} \frac{a_{max}}{V_{max}} = \frac{A\omega^2}{A\omega} = \omega$$

V_{max} when $\sin(\omega t) = 1 \rightarrow \omega t_1 = \frac{\pi}{2}$

a_{max} when $\cos(\omega t) = 1 \rightarrow \omega t_2 = \pi$

So $\omega t_2 - \omega t_1 = \omega(\Delta t) = \pi - \frac{\pi}{2}$

$$\downarrow$$

$$\left(\frac{a_{max}}{V_{max}}\right) \Delta t = \frac{\pi}{2}$$

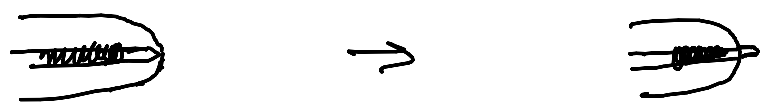
$$\boxed{\Delta t = \frac{\pi}{2} \cdot \frac{V_{max}}{a_{max}} = 0.28 s}$$

Prob. 3 10.25

Given $k = 250 \frac{N}{m}$, $x_0 = 5.0\,mm$, $x_f = 11.0\,mm$

Find W_{app} to the spring.

$x_0 = 0.0050\,m$ $x_f = 0.0110\,m$



$PE_0 = \frac{1}{2} k x_0^2$ $PE_f = \frac{1}{2} k x_f^2$

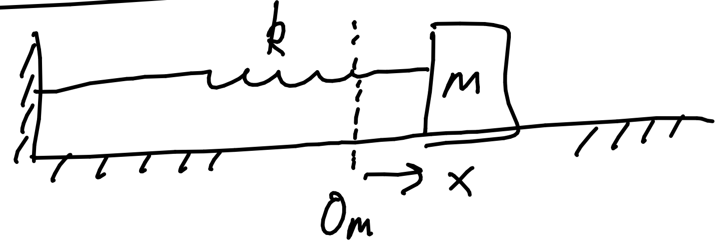
$W_{app} = +\Delta PE$ b/c sign matches as adding or subtracting energy from the spring.

$$W_{app} = PE_f - PE_0$$

$$W_{app} = \frac{1}{2} k \left(x_f^2 - x_0^2\right) = \boxed{+12.0\,mJ}$$

Prob. 4 10.26

Given x_0, x_f data for a spring with constant $k = 46.0\,\frac{N}{m}$, find $W_{sp} = -\Delta PE_{sp} = -\frac{1}{2}k(x_f^2 - x_0^2)$



(a) $x_0 = +1.00\,m$, $x_f = +3.00\,m$

$$\boxed{W_a = -184\,J}$$

(b) $x_0 = -3.00\,m$, $x_f = +1.00\,m$

$$\boxed{W_b = +184\,J}$$

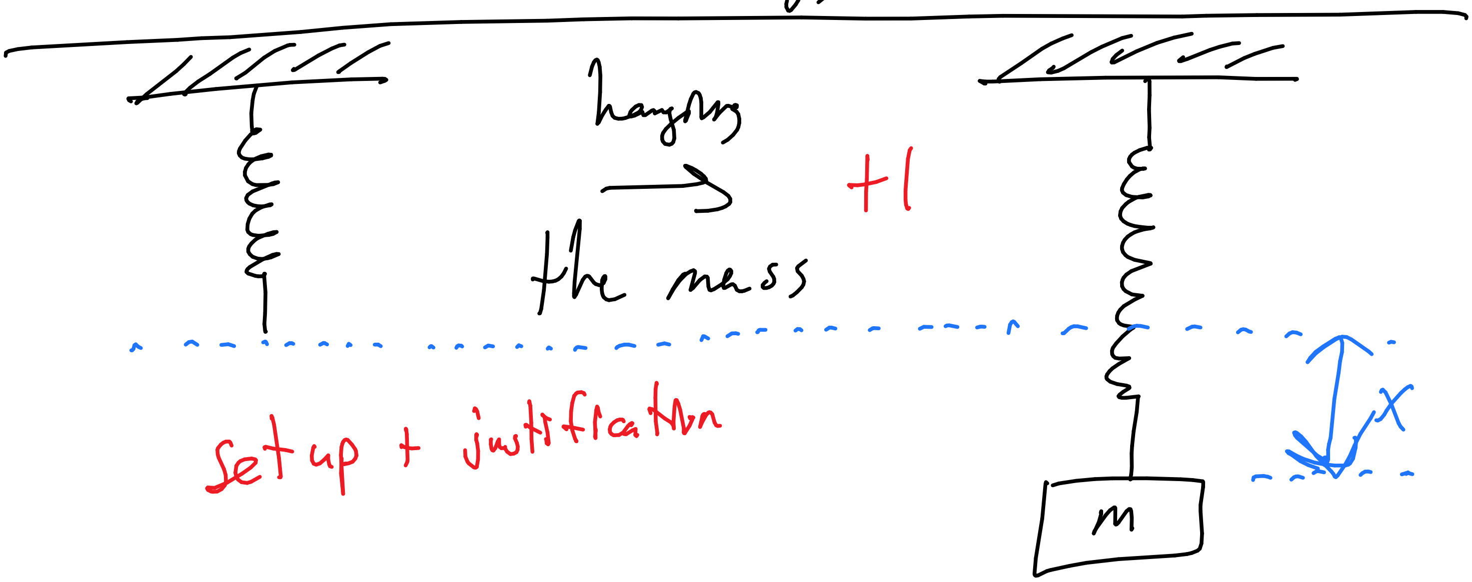
(c) $x_0 = -3.00\,m$, $x_f = +3.00\,m$ $\boxed{W_c = 0}$

5/10

Prob. 5 10.27

Given m, x for a hanging mass-spring system find (a) k (b) ω

$m = 0.450\ \text{kg}, \quad x = 0.150\,\text{m}$



$$F_{app} = mg \overset{+1}{=} |-kx| \;;\quad \omega_{sp} = \sqrt{\frac{k}{m}}$$

(a) $k = \dfrac{mg}{x} = \boxed{30\,\dfrac{N}{m}}$

(b) $\omega_{sp} = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{x}} = \boxed{8.2\,\dfrac{rad}{s}}$

Prob. 6 10.37

Given info about energy find the angular frequency if the object was attached so it would oscillate.

$x_0 = -0.0620\,m$ $\quad x_f = 0$

$v_0 = 0$ $\quad v_f = 1.50\,\frac{m}{s}$

Recall: $\omega_{sp} = \sqrt{\frac{k}{m}}$ +1

Cons. of Energy: $E_0 + \cancel{W_{nc}}^{\,0} = E_f$

expands to $PE_0 + KE_0 = PE_f + KE_f$,

but KE_0 and PE_f are both zero.

$$PE_{sp_0} = KE_f \quad +1$$

$$\frac{1}{2} k x_0^2 = \frac{1}{2} m v_f^2 \rightarrow \frac{k}{m} = \frac{v_f^2}{x_0^2}$$

$$\boxed{\omega = \left|\frac{V_f}{x_0}\right| = 24.2\,\frac{rad}{s}}$$

Prob. 7 10.46 7/10

A simple pendulum hung to measure the height of an old tower. Period measured to be

$T = 9.2\,s$.

- height of tower $\simeq$ length of pend. l
- For simple pendulums $\omega = 2\pi f = \sqrt{\frac{g}{l}}$
- Always true that $f = \frac{1}{T}$.

Therefore, $$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

$$\sqrt{\frac{l}{g}} = \frac{T}{2\pi}$$

$$\frac{l}{g} = \left(\frac{T}{2\pi}\right)^2$$

$$\boxed{l = \frac{gT^2}{4\pi^2} \simeq 21.4\,m}$$

8/10

Prob. 8 10.74

A speaker diaphragm operates with shm for 2.5s at an angular frequency of $\omega = 7.54 \times 10^4 \frac{rad}{s}$.
How many cycles does the diaphragm complete in the time of operation?

• $\omega = 2\pi f$ always true! •

Note: • $T = \frac{1}{f}$ means time T for 1 cycle if operating at frequency f.

$T = \frac{1}{f}$ leads to $t = \frac{\#\text{ cycles}}{f} = \frac{N}{f}$

So, $N = tf$ (using $f = \frac{\omega}{2\pi}$)

$N = t\left(\frac{\omega}{2\pi}\right)$

$\boxed{N = 3.00 \times 10^4}$ or 30,000 cycles

Prob. 9 FOC 10.17

If the resonant frequency f_0 for a spring of constant k paired with a mass m, which pair would have a resonant frequency of $f_1 = 2f_0$?

Think: $k = k_0 = 100\ \frac{N}{m}$

$m = m_0 = 1.0\ kg$

Strategy

$$\omega_{sp} = 2\pi f_{sp} = \sqrt{\frac{k}{m}}$$. If f doubles, ω doubles!

$$\omega_0 = \sqrt{\frac{k}{m}} \rightarrow \omega_1 = 2\omega_0$$

$$\omega_1 = 2\sqrt{\frac{k}{m}} = \sqrt{2^2}\sqrt{\frac{k}{m}}$$

$$\omega_1 = \sqrt{4\left(\frac{k}{m}\right)}$$

$$\text{Double } f/\omega \longleftrightarrow \text{Quadruple } \left(\frac{k}{m}\right)$$

The only k_1, m_1 pair that produces $\frac{k_1}{m_1} = 4\left(\frac{k}{m}\right)$ is $8k$ & $2m$: $\boxed{\text{(a) Pair A}}$

Prob. 10 10.78(a) 10/10

$A_\perp = 4.0 \times 10^{-4} m^2$ and $F_{max} = 6.8 \times 10^{4} N$ for a femur bone. (a) Calculate σ_{max}

σ_{max} = Max Stress which means max compression pressure the bone can withstand:

$$\sigma \equiv \frac{F}{A_\perp} \quad \text{means} \quad \sigma_{max} = \frac{F_{max}}{A_\perp}$$

$$\sigma_{max} = \frac{6.8 \times 10^4 N}{4.0 \times 10^{-4} m^2}$$

$$= \frac{6.8}{4.0} \times \frac{10^4}{10^{-4}} \frac{N}{m^2}$$

$$\sigma_{max} = 1.7 \times 10^8 Pa$$

$$\boxed{\sigma_{max} = 170 \text{ MPa} \simeq 24.7 \times 10^3 \text{ psi}}$$