Name: Solutions

Homework Week # 10

Rotational Dynamics
Due Tues 10/29/19

Reading

C&J Physics:	**Tues – Ch. 9: 1-4**	**Thurs – Ch. 9: 4-7**
OS Coll Phys:	**Tues – Ch. 9: 1-6**	**Thurs – Ch. 10**

Problems

Problem 1. **9.1**

Problem 2. **9.2**

Problem 3. **9.7**

Problem 4. **9.12**

Problem 5. **9.18**

Problem 6. **9.32**

Problem 7. **9.33**

Problem 8. **9.33 (use the parallel-axis theorem)**

Problem 9. **9.49 (treat each mass as a point mass)**

Problem 10. **9.59 (hint:** $I_{tot}=I_1+I_2$ **)**

Prob. 1   9.1

Rolling w/o slipping means τ from static friction balances τ from car engine.

$$\tau_s = r f_s = \tau_{eng} = 295\ N\cdot m$$

for $r = 0.350m$.

$$r f_s = \tau$$

$$f_s = \frac{\tau}{r} = \frac{295\ N\cdot m}{0.350m}$$

$$\boxed{f_s = 843\ N}$$

Prob. 2 9.2

Two torques are accomplished with equal force, but different radii. What's the ratio of the two torques? $r_1 = 0.19m$, $r_2 = 0.25m$

Goal: $\frac{\tau_2}{\tau_1} = \frac{r_2 F_2}{r_1 F_1}$ when $F_1 = F_2$

$$\frac{\tau_2}{\tau_1} = \frac{r_2 \cancel{F}}{r_1 \cancel{F}}$$ (use given info)

$$\frac{\tau_2}{\tau_1} = \frac{r_2}{r_1}$$

$$= \left(\frac{0.25\,m}{0.19\,m}\right)$$ (plug in #'s)

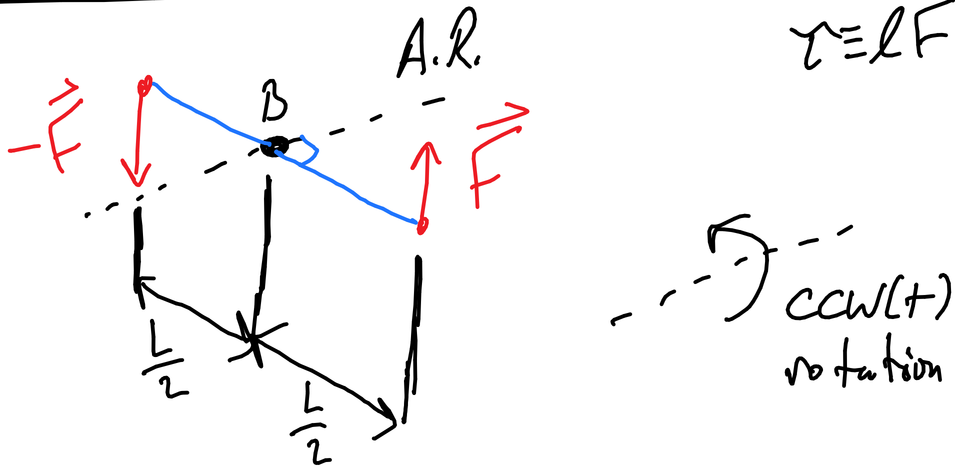
$$\boxed{\frac{\tau_2}{\tau_1} = 1.32}$$

Prob. 3 9.7

Find the torque for a given force "couple".

"couples" not on exam

$\tau \equiv \ell F$



Both forces on the shown couple produce CCW torques with magnitude $\tau = (\frac{L}{2})(F)$.

$$\tau = \tau_F + \tau_{-F}$$

$$\tau = 2\left(\frac{L}{2}\right)F$$

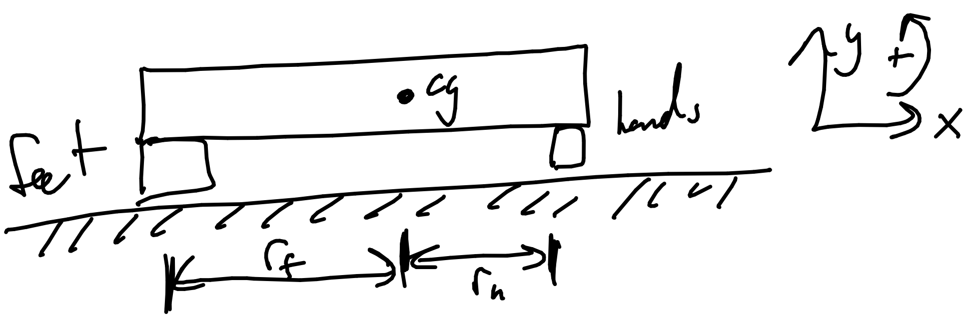
$$\boxed{\tau = LF}$$

$\tau = LF$ for application at A, B, or C because applying the same couple means doing whatever is necessary to get exactly the same torque.

Prob. 4 9.12
1/2

Knowing position of cg and that both hands exert equal force ($2N_h$) and same for feet ($2N_f$) find each normal force if $W = 584N$, $r_h = 0.410m$, and $r_f = 0.840m$.

Mechanical equilibrium: $\sum \vec{F} = 0$
$\sum \vec{\tau} = 0$



$$\sum F = 2N_h + 2N_f - W = 0$$

$$\sum \tau = \tau_h - \tau_f = 0$$

$\tau = rF$

$$\longrightarrow (2N_h) r_h - (2N_f) r_f = 0$$

means $$2N_f = \frac{r_h}{r_f}(2N_h)$$ (sub into forces)

Prob. 4 2/2

$$\Sigma F = 0 \rightarrow 2N_h + \frac{r_h}{r_f}(2N_h) = W$$

$$\left(1 + \frac{r_h}{r_f}\right)(2N_h) = W$$

$$2N_h = \frac{W}{1 + \frac{r_h}{r_f}}$$

recall: $2N_f = \frac{r_h}{r_f}(2N_h)$

$r_h = 0.410m$, $r_f = 0.840m$, $W = 584N$

Using #'s

$2N_h = 392\,N$ $\quad$ $2N_f = 192\,N$

So

$$\boxed{N_h = 196N \qquad N_f = 96N}$$

each hand ↑ $\quad$ each foot ↑

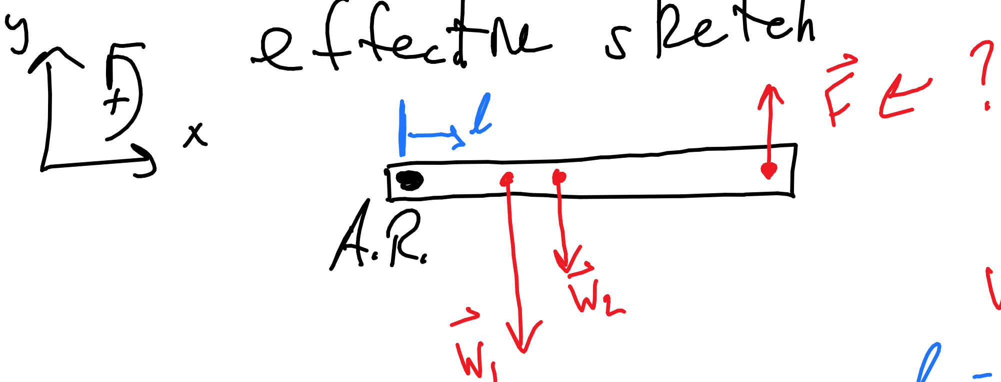
Prob. 5 1/2 9.18

7/15

For 2 designs use $\sum \vec{\tau} = 0$ to determine force applied by a person where 2 weights are equal, but one lever arm is changed.

Note that solve for left design and when solving for right design, one lever arm is set to zero.

effective sketch



$\vec{F} \leftarrow ?$ $l_F = 1.30\,m$

$W_1 = 525\,N$

$W_2 = 60.0\,N$

$l_1 = 0.400\,m$

$l_2 = 0.600\,m$

$$\sum \tau = \tau_F - \tau_1 - \tau_2 = 0$$

$$\tau = lF$$

$$\tau_F = \tau_1 + \tau_2$$

Prob. 5 2/2

$$\frac{l_F F}{l_F} = \frac{l_1 W_1 + l_2 W_2}{l_F}$$

$l_F = 1.30\,m$

(a) $l_1 = 0.400\,m$

$l_2 = 0.600\,m$

$W_1 = 52.5\,N$

$W_2 = 60.0\,N$

$$F = \frac{l_1 W_1 + l_2 W_2}{l_F}$$

(b) same except $l_1 = 0$.

$$\boxed{\begin{array}{ll} \text{(a)} & F = 189\,N \\ \text{(b)} & F = 28.0\,N \end{array}}$$

Prob. 6 9.32

Given τ and α, find I

$\tau = 10.0\ N \cdot m$, $\alpha = 8.00\ \frac{rad}{s^2}$

$$\sum \tau = \tau_{net} = I\alpha$$

$$I = \frac{\tau}{\alpha}$$

$$I = \frac{10.0\ N \cdot m}{8.00\ \frac{rad}{s^2}}$$

$$\boxed{I = 1.25\ kg \cdot m^2}$$

Prob. 7 9.33

Calculate total I if disk and 3 compact masses at end of disk

$M_{disk} = 1.2\,kg$, $r_{disk} = 0.16\,m$ + 3 masses $m = 0.15\,kg$ at $r = r_{disk}$.

$$I_{tot} = I_{disk} + 3 I_m$$

$$I = \frac{1}{2} M_d r^2 + 3 m r^2$$

$$= \left(\frac{1}{2} M_d + 3m\right) r^2$$

$$I = \left(\frac{1}{2}(1.2) + 3(0.15)\right)(0.16)^2\ kg \cdot m^2$$

$$\boxed{I = 2.7 \times 10^{-2}\ kg \cdot m^2}$$

Prob. 8 9.34

Find α given τ and I.

$\tau = 1.8\ N\cdot m$, $I = 0.22\ kg\cdot m^2$

$$\sum \tau = I\alpha$$

$$\alpha = \frac{\tau}{I}$$

$$\alpha = \frac{1.8\ N\cdot m}{0.22\ kg\cdot m^2}$$

$$\boxed{\alpha = 8.2\ \frac{rad}{s^2}}$$

Prob. 9 9.49

1/2

Three compact/point masses rotate about the same axis with the same angular velocity $\omega = 5.00\,\frac{rad}{s}$, but different masses and radii.

$m_1 = 6.00\,kg$, $r_1 = 2.00\,m$

$m_2 = 4.00\,kg$, $r_2 = 1.50\,m$

$m_3 = 3.00\,kg$, $r_3 = 3.00\,m$

same ω, different m, r

(a) Calculate each tangential speed. $(V = r\omega)$

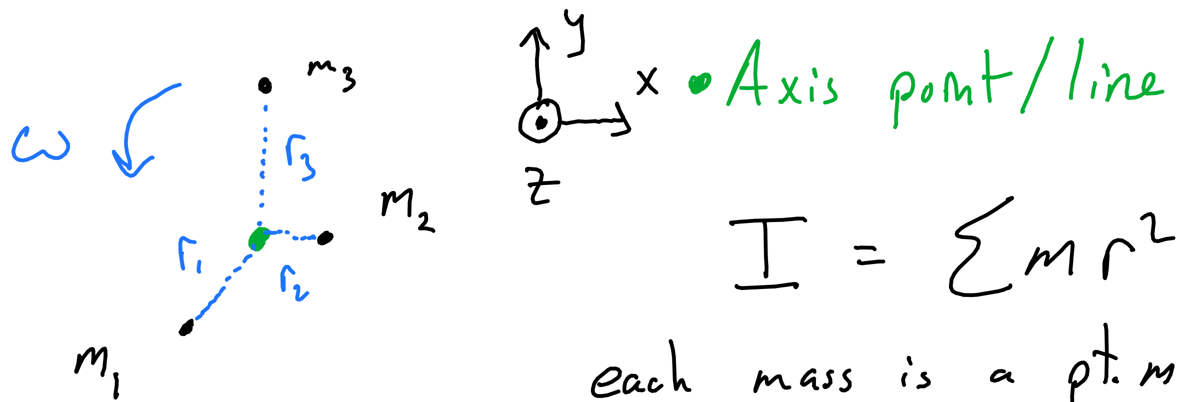
$$\boxed{V_1 = 10.0\,\frac{m}{s}, \quad V_2 = 7.5\,\frac{m}{s}, \quad V_3 = 15.0\,\frac{m}{s}}$$

(b) Calculate $KE = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{1}{2} m_3 V_3^2$

$$\boxed{KE = 750\ J}$$

Prob. 9 2/2

(c) Calculate I for the whole system.



$$I = \sum m r^2$$

each mass is a pt. mass.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$I = (24.0 + 9.00 + 27.0)\ kg \cdot m^2$$

$$\boxed{I = 60.0\ kg \cdot m^2}$$

(d) Calculate $RKE = \frac{1}{2} I \omega^2$

$$RKE = \frac{1}{2}(60.0\ kg \cdot m^2)\left(5.00\ \frac{rad}{s}\right)^2$$

$$\boxed{RKE = 750\ J} = KE \checkmark$$

Prob. 10 $\frac{1}{2}$ 9.59

Two disks are rotated and then connected together without adding any extra angular momentum. Solve for the unknown moment of inertia I_B.

$\omega_A = \omega_{0A} = +7.2$ rad/s $\qquad I_A = 3.4$ kg·m^2

$\omega_B = \omega_{0B} = -9.8$ rad/s $\qquad \omega_f = -2.4$ rad/s

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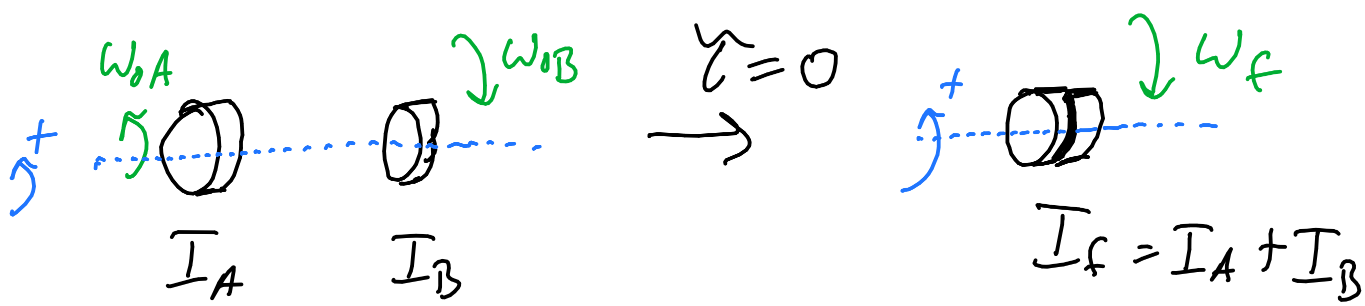

Conservation: $L_0 = L_f \quad ; L = I\omega$

Thus, $L_{0A} + L_{0B} = L_f$

Specific to this problem:

$$I_A \omega_A + I_B \omega_B = (I_A + I_B)\,\omega_f$$

Prob. 10 2/2

$$I_A \omega_A + I_B \omega_B = (I_A + I_B)\omega_f$$

Now to rewrite/solve for I_B. (No zeros)

$$I_A \omega_A + I_B \omega_B = I_A \omega_f + I_B \omega_f$$

$$I_B \omega_B - I_B \omega_f = I_A \omega_f - I_A \omega_A$$

$$I_B (\omega_B - \omega_f) = I_A (\omega_f - \omega_A)$$

$$I_B = \frac{I_A (\omega_f - \omega_A)}{(\omega_B - \omega_f)}$$

form ready for #'s.

$$I_B = \frac{(3.4\ kg \cdot m^2)(-7.2 - 2.4)\frac{rad}{s}}{(-9.8 + (+2.4))\frac{rad}{s}} = \frac{+(3.4)(9.6)}{+(7.4)} kg \cdot m^2$$

$$\boxed{I_B = 4.4\ kg \cdot m^2}$$