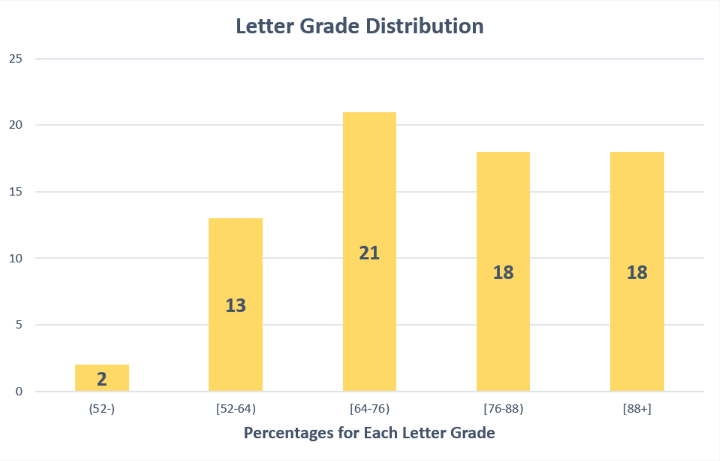
Study Habit Changes

My suggestions

1. Don't rush to do algebra. Think about the physics.
2. Trust the process.
3. Engage with each of the many ways to think about the relationships between quantities.
4. Don't struggle alone. Work with a small group and attend tutoring/recitation.
5. Think about what's similar and different between each problem assigned.

Grade Distribution



“The 21st Day: A Wandering Night”

Homework Wk #11 Due Thursday 11/07/19

Today’s Topics

1. Simple Harmonic Motion [Springs]
2. Reference Circles (trig)
3. Elastic Energy

Motivation

What are the purposes of the springs and the shocks in a car?

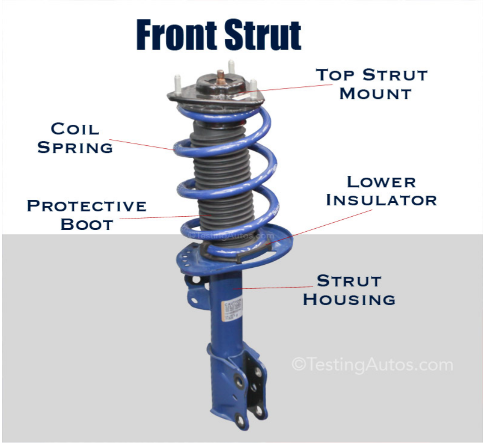


Figure: 21.1 Example Front Strut[1] for an automobile

[1] https://testingautos.com

Regime of Current Topic

We are focuing on elastic materials and simple oscillations.

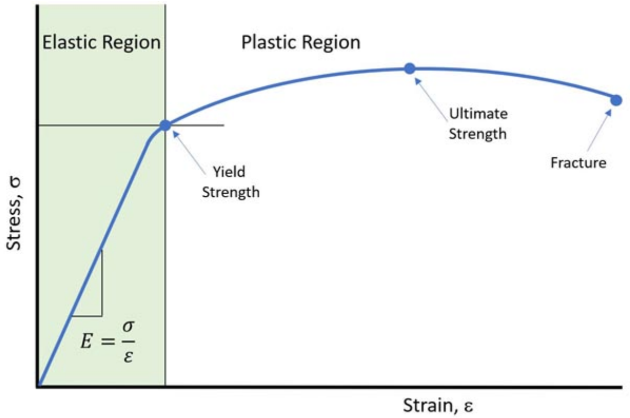


Figure: 21.2 Ideal stress-strain[2] curve for material tests

[2] https://www.simsolid.com/faq-items/material-properties/

Simple Harmonic Motion

Any object with a “restoring force” that is similar to an ideal spring without friction is called a **Simple Harmonic Oscillator.**

Any motion that can be quantified with the same mathematical tools as that of a simple, ideal spring is called **Simple Harmonic Motion.**

Quantifying a Force

What does the spring do in response to a change in length?

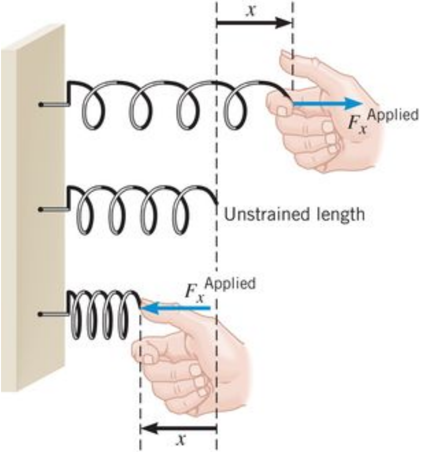


Figure: 21.3 Characterizing ideal springs

Quantifying Elastic Responses

The following apply to small displacements or deformations and have been concluded from experiments:

1. the applied force, F_x^{app}, is proportional to the displacement,

$$F_x^{app} \propto x.$$

2. The proportional constant, k, is called the spring constant,

$$F_x^{app} = kx.$$

3. k is a measure of how stiff a spring is.

Quantifying Ideal Springs

With what force does a spring return to it's original length if we need to apply a force in the $\pm x$ direction to stretch or compress the spring?

If we apply a force

$$F_x^{app} = kx$$

then[3] the spring will restore with a force

$$F_s = -kx$$

[3] Recall Newton's Third Law: $\mathbf{F_{12}} = -\mathbf{F_{21}}$

Exercise #1

C&J Example 10.1 When a tire pressure gauge is pressed against a tire valve, the air in the tire pushes against a plunger attached to a spring. Suppose the spring constant of the spring is $k = 160\ N/m$ and the bar indicator of the gauge extends $4.0\ cm$ when the gauge is pressed against the tire valve. What force does the air in the tire apply to the spring?

Exercise #1 Translation

C&J Example 10.1 What is the total applied force if a spring with constant $k = 160\ N/m$ is compressed $x = 4.0\ cm$?

Exercise #1 Answer

C&J Example 10.1 What is the total applied force if a spring with constant $k = 160\ N/m$ is compressed $x = 4.0\ cm$?

$$F_{app} = kx$$

$$x = (4.0\ \cancel{cm})\left(\frac{1.0\ m}{100\ \cancel{cm}}\right) = 0.040\ m$$

Therefore

$$F_{app} = (160\ N/m)(0.040\ m)$$

$$F_{app} = 6.4\ N$$

Exercise #2

C&J 10.04 A spring lies on a horizontal table, and the left end of the spring is attached to a wall. The other end is connected to a box. The box is pulled to the right, stretching the spring. Static friction exists between the box and the table, so when the spring is stretched only by a small amount and the box is released, the box does not move. The mass of the box is $0.80\ kg$, and the spring has a spring constant of $59\ N/m$. The coefficient of static friction between the box and the table on which it rests is 0.74. How far can the spring be stretched from its unstrained position without the box moving when it is released?

Exercise #2 Info Extraction

C&J 10.04 *A spring* lies on a horizontal table, and the *left end* of the spring is *attached to a wall. The other end is connected to a box.* **The box is pulled to the right, stretching the spring.** Static friction exists between the box and the table, so when the spring is stretched only by a small amount and the box is released, the box does not move. The **mass** of the box is $0.80\ kg$, and the spring has a **spring constant** of $59\ N/m$. The **coefficient of static friction** between the box and the table on which it rests is $\mu_s = 0.74$. **How far can the spring be stretched from its unstrained position without the box moving when it is released?**

Exercise #2 Math

C&J 10.04 How far can a string [attached to a block] be stretched before the force of the spring overcomes static friction on the block when the normal force is balancing the weight of a block. Given $m = 0.80\ kg$, $k = 59\ N/m$, and $\mu_s = 0.74$.

Recall quantifying static friction:

$$f_s \leq \mu_s F_N$$

The force analysis produces the equations

$$F_x = f_s - F_s = 0$$

$$F_y = F_N - mg = 0$$

with the condition $f_s = \mu_s F_N$.

Exercise #2 Math Continued

Force analysis produces the specific result

$$F_s = \mu_s mg$$

We're using the equation for an ideal spring so we can solve for x.

$$F_s = kx$$

$$kx = \mu_s mg$$

$$x = \frac{\mu_s mg}{k}$$

$$x \approx \frac{(0.74)(0.80\ kg)(10.0\ m/s^2)}{59\ N/m} \approx 10\ cm$$

Work Done By a Spring

If the force of a spring at any moment is

$$F_{sp} = kx$$

Then the average force in the spring is

$$F_{ave} = \frac{1}{2}k(x_f + x_o).$$

Since Work is $W = (F cos\theta)s$, then for a spring ($\theta = 0$)

$$W_{ave} = F_{ave}\Delta x$$

is quantified as

$$W_{sp} = \frac{1}{2}k(x_f + x_o)(x_f - x_o).$$

Energy Stored In a Spring

The work simplifies to

$$W_{sp} = \frac{1}{2}k(x_f + x_o)(x_f - x_o)$$

$$W_{sp} = \frac{1}{2}k(x_f^2 - x_o^2).$$

In a similar manner to gravity, such a spring force is conservative.

This means that one can store energy in a spring due to stretching or compressing it and can quantify it as

$$PE_{el} = \frac{1}{2}kx^2$$

Total Mechanical Energy

We can now say for any system the **total mechanical energy** can be quantified as the sum of the

- linear kinetic,
- rotational kinetic,
- gravitational potential, and the
- elastic[4] potential

energy of the system:

$$E_{tot} = KE + RKE + PE_g + PE_{el}$$

[4] Not all elastic objects are springs, but can behave the same way.

C&J 10.3.26 The drawing shows three situations in which a block is attached to a spring. The position labeled 0 m represents the unstrained position of the spring.. The block is moved from an initial position x_0 to a final position x_f, the magnitude of the displacement being denoted by the symbol s. Suppose the spring has a spring constant of $k = 46.0\ N/m$. Using the data provided in the drawing, determine the total work done by the restoring force of the spring for each situation.

Exercise #3 Image

C&J 10.26 Suppose the spring has a spring constant of $k = 46.0\ N/m$. Using the data provided in the drawing, determine the total work done by the restoring force of the spring for each situation.

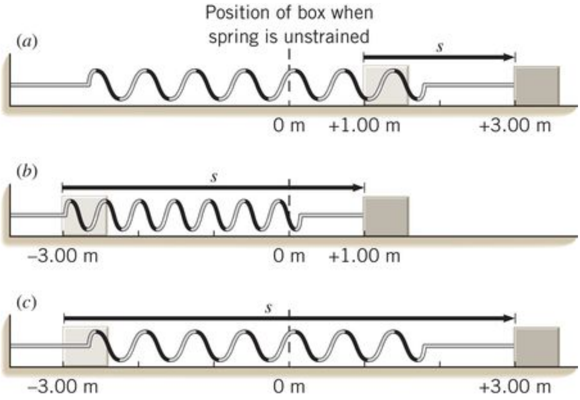


Figure: 21.4 Data for Exercise #3

Exercise #3 Math

For all three start with

$$W_{sp} = \frac{1}{2}k(x_f^2 - x_o^2)$$

a. $W = -184\ J$
b. $W = -184\ J$
c. $W = 0\ J$

Graphing Motion of a Spring

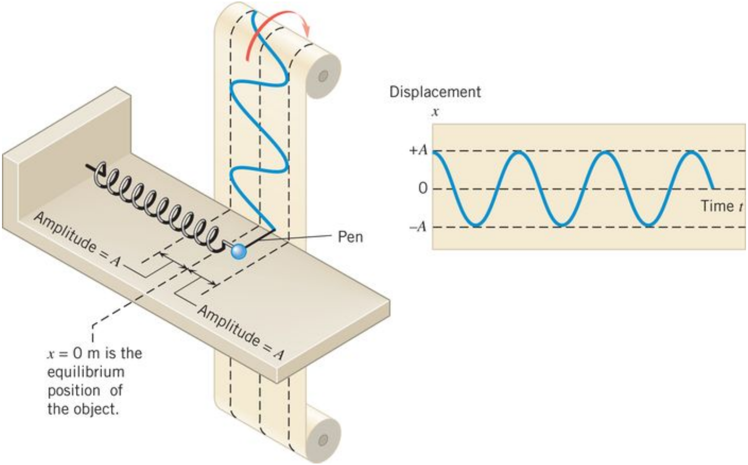


Figure: 21.5 Graphing the position of a spring

Reference Circle Motivation

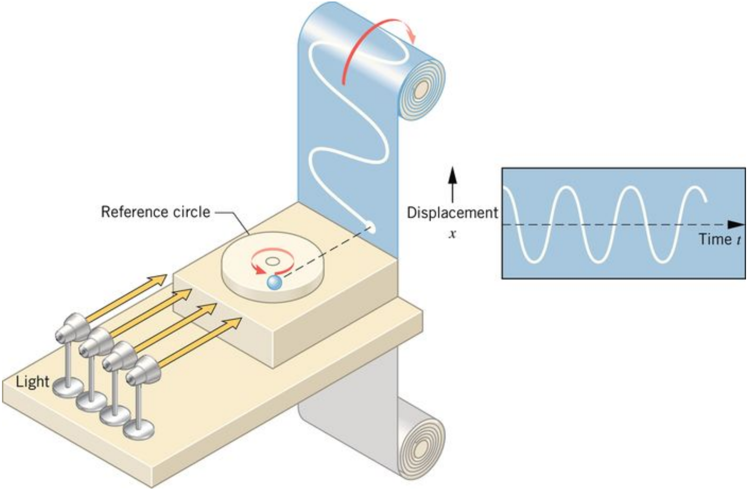


Figure: 21.6 Graphing the x position of uniform circular motion

Reference Circle

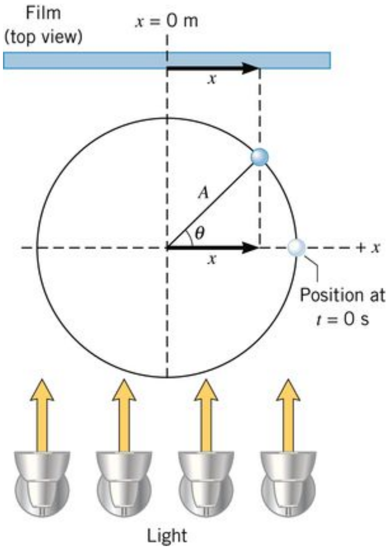


Figure: 21.7 Quantifying the x position of uniform circular motion

Reading Graphs for Simple Harmonic Motion

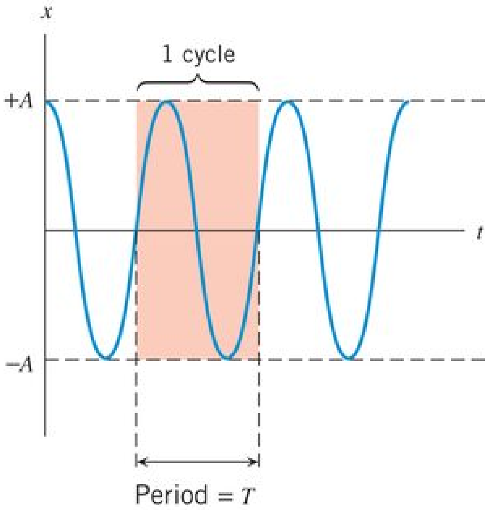


Figure: 21.8 Example graph for simple harmonic motion

Kinematics for Oscillations

The following are equations that can be used for modeling the behavior we've discussed today:

$$\theta = \omega t$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$x = Acos(\omega t), \quad x_{max} = A$$

$$v_x = -A\omega sin(\omega t), \quad v_{max} = A\omega$$

$$a_x = -A\omega^2 cos(\omega t), \quad a_{max} = A\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

Exercise #4

C&J CYU 10.2 Which object: I, II, or III, has the greatest maximum velocity? $v_{max} = A\omega$

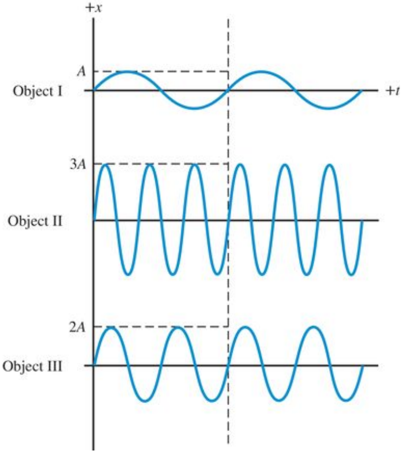


Figure: 21.9 Check your understanding of the graphs