# Phys 111: Lecture 24

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# The 24$^{\text{th}}$ Day: “Keep 'Er Steady!”

**Homework Wk #12 Due Today**

**Today’s Topics**

1. Archimede’s Principle - Buoyancy
2. Fluid Flow
3. Equations for Fluid Flow

# Motivation

Why is the vent required by law?

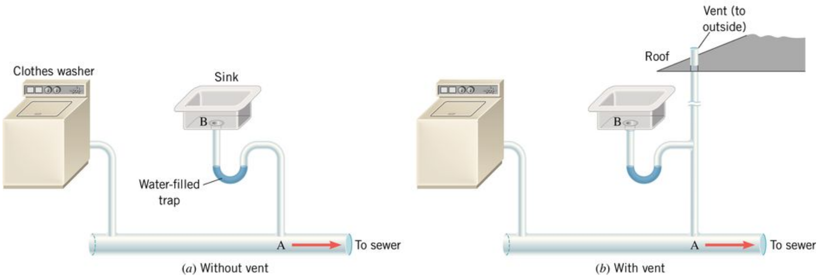


Figure: 24.1 Good vs bad plumbing

# Last Time

The **pressure** $P$ **(**$Pa$**)** exerted on a surface is indicative of how concentrated an applied force is:

$$P = \frac{F_{\perp}}{A}$$

The **mass density** $\rho$ **(**$kg/m^3$**)** of an object informs you how massive a chunk of material will be.

$$\rho = \frac{m}{V}$$

**Hydrostatic Pressure** applies to any non-accelerating fluid

$$P_2 = P_1 + \rho g h$$

$$\Delta P = P_2 - P_1$$

$$\Delta P = \rho g h$$

# Buoyancy

Recall that I didn't address **Buoyancy** last lecture.

Let's consider a different scenario, but keep in mind:

$$(P_2 - P_1)A = (\rho_{fl} V)g$$

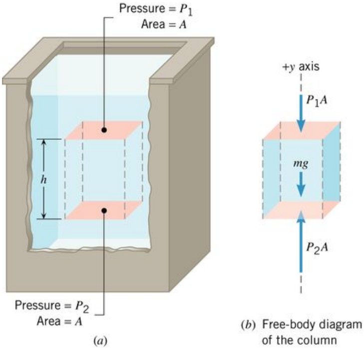


Figure: 24.2 Hydrostatic pressure differences

# Archimede's Principle

The upward buoyant force ($F_b$) from a fluid is the interaction force due to the pressure differences between the top and bottom of an object from the fluid it's in:

$$F_b = \text{Weight of fluid displaced}$$

$$F_b = (P_2 - P_1)A = (\rho_{fl} V)g$$

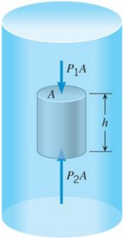


Figure: 24.3 Buoyancy, the result from displacing fluids

# Buoyant Force

Take careful consideration that the density used is the density of the fluid:

$$F_b = \rho A h g$$

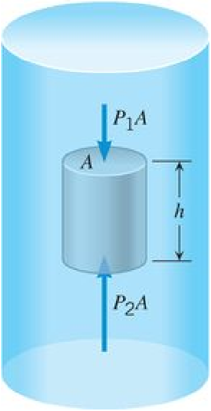


Figure: 24.3 Buoyancy, the result from displacing fluids

# Exercise #1

**C&J 11.40** The density of ice is $917\ kg/m^3$, and the density of seawater is $1025\ kg/m^3$. A swimming polar bear climbs onto a piece of floating ice that has a volume of $5.2\ m^3$. What is the weight of the heaviest bear that the ice can support without sinking completely beneath the water?

# Exercise #1 Translation

**C&J 10.43** Solve for the max weight that can be put on top of the ice such that the buoyant force isn't fully overcome by the weight of the ice and the bear as the ice is just fully submerged.

**Strategy:** Perform a force analysis of the ice to find the max weight of the bear. The result of the force analysis produces the following equation:

$$F_b - F_g = 0$$

where the total force due to gravity affecting the ice is from the bear and the ice.

$$\rho_w g V_i - F_{bear} - \rho_i g V_i = 0$$

# Exercise #1 Answer

From the force analysis we can solve for the weight of the bear $F_{bear}$:

$$F_{bear} = \rho_w g V_i - \rho_i g V_i$$

$$F_{bear} = (\rho_w - \rho_i) g V_i$$

Now let's put in our specific measurements

$$F_{max} = \left[(1025 - 917)\ kg/m^3\right]\left(10.0 m/s^2\right)\left(5.2\ m^3\right)$$

Then the max weight $F_{bear}$ is

$$F_{max} = 5616\ N$$

$$F_{max} \approx 1290\ lb$$

# Fluid Flow

There are three general types of fluid flow:

1. **Steady Flow:** The velocities for each individual point in the fluid are constant.
2. **Unsteady Flow:** The velocities for each individual point in the fluid vary.
3. **Turbulent Flow:** Extreme changes in velocity caused by objects and sharp bends.

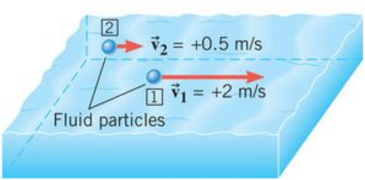


Figure: 24.5 Particle analysis of fluid flow

# White Water Rafting

The "~~Gem~~ Whitewater State." It could be both!


Frans Lemmens/Getty Images

Figure: 24.6 A fun, adventurous form of turbulence

# Visualization Tool

Most people studying fluid mechanics consider **streamlines**

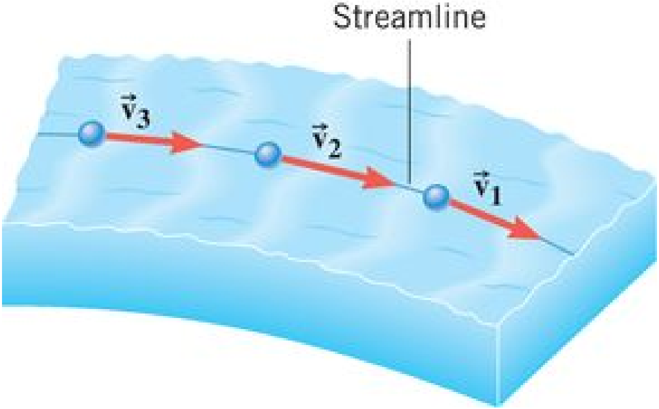


Figure: 24.7 Streamline visualations for modeling flow

# Movie Time

Fluid analysis is very difficult both experimentally and computationally. We've made many approximations. Meaningful, but still approximations.

1. **Turbulent Flow:** Simulation at

   `https://www.youtube.com/watch?v=_otrb-HJRtE`

2. **Unsteady Flow:** Standard Hose

   `https://www.youtube.com/watch?v=bQXlBsgApZk`

3. **Steady Flow:** Custom Hose

   `https://www.youtube.com/watch?v=MOBcdnRQ4jY`

# Fluid Flow Summary

There are three main qualities about fluid flow [that can be quantified].

1. **Fluid flow can be steady or unsteady.**

   Type of flow stems from many sources and causes very interesting phenomena.

2. **Fluid flow can be compressible or incompressible.**

   If a fluid is **compressible**, then the **density** of the fluid **will change** and that requires a more thorough analysis.

3. **Fluid flow can be viscous or non-viscous.**

   **Viscosity** is the measure of how readily a fluid flows. Honey has a high viscosity whereas water has a small viscosity.

An **ideal fluid** is a fluid that is incompressible and non-viscous.

# Continuity Equations

In many scenarios, scientists are practically obsessed with conservation of mass.

In physics, we call any equation for conservation of mass of a fluid flowing a **continuity equation**.

$$\text{Mass In} = \text{Mass Out}$$

which implies

$$\text{Mass Rate In} = \text{Mass Rate Out}$$

Let's reuse mass density to get

$$m = \rho V$$

# Continuity Equation

Again, it's easier to measure **volume flow rate** $Q$ by measuring the **cross-sectional area** $A$ of a pipe/trough and looking at the **velocity** $v$ of the fluid down the length of the channel:

$$Q = Av$$

$$\text{Mass Rate} = \text{Density * Volume Rate or}$$

$$\frac{\Delta m}{\Delta t} = \rho Q$$

Which is composed of

$$\frac{\Delta m}{\Delta t} = \rho A v$$

For a single entry-exit system we get the continuity equation:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

# One-Way Continuity Summarized

Generally for single entry-exit we have

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

and for an incompressible fluid $\rho_1 = \rho_2$ so we get

$$\frac{\rho_1 A_1 v_1}{\rho_1} = \frac{\rho_1 A_2 v_2}{\rho_1}$$

or

$$A_1 v_1 = A_2 v_2$$

# Exercise #2

**C&J 11.56** Use conservation of mass (continuity) to answer the following:

a. The volume flow rate in an artery supplying the brain is $3.6 \times 10^{-6}\ m^3/s$. If the radius of the artery is $5.2\ mm$, determine the average blood speed.

b. Find the average blood speed at a constriction in the artery if the constriction reduces the radius by a factor of $3$. Assume that the volume flow rate is the same as that in part 56(a).

# Exercise #2 Start

**C&J 11.56** Let's revisit the definition of volume flow rate $Q$ for part (a) and then relate the two radii to get a new average speed:

$$Q = Av_{ave} = (\pi r^2)v_{ave}$$

$$v = \frac{Q}{\pi r^2}$$

# Exercise #2 Answers

**C&J 10.56** The strategy is to calculate one velocity and then use the differences in radii to calculate the second velocity very quickly.

Since $r_2 = r_1/3$, then $A_2 = A_1/9$. Using

$$v_{ave} = \frac{Q}{A}$$

a. $v_1 = 4.24\ cm/s$

b. $v_2 = 9v_1 = 38.2\ cm/s$

# Exercise #3

**C&J 11.57** A room has a volume of $120\ m^3$. An air-conditioning system is to replace the air in this room every twenty [20] minutes, using ducts that have a square cross section. Assuming that air can be treated as an incompressible fluid, find the length of a side of the square if the air speed within the ducts is

a. $3.0\ m/s$ and
b. $5.0\ m/s$.

# Exercise #3 Start

**C&J 11.57** The strategy for this problem is the start with the same relationship as the previous problem, but solve for area and then side length since the area of a square is the side length squared:

We need to convert $Q$ to $m^3/s$: 120 cubic meters every 20 minutes

$$Q = 1.0 \, \frac{m^3}{s}$$

Then start with

$$Q = Av$$

# Exercise #3 Answers

From

$$Q = Av$$

$$A = \frac{Q}{v}$$

Since we're dealing with a square then $A = l^2$

So we just need to take the square root of the fraction we're considering

$$l = \sqrt{\frac{Q}{v}}$$

a. $v = 3.0\ m/s$ then $l = 0.58\ m$
b. $v = 5.0\ m/s$ then then $l = 0.45\ m$

# Bernoulli's Principle

How does pressure relate to fluid speed?

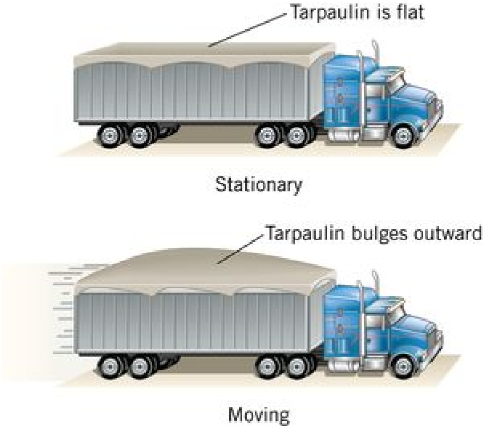


Figure: 24.8 Application matching Bernoulli Effect

# Before We Continue

**Ideal Fluids**

1. Incompressible
2. Non-viscous

Repeat after me:

"Bernoulli's Principle only applies to steady flow ideal fluids!"

# Straight to the "Punch Line""

Bernoulli's equation is a result from applying the Law of Conservation of Energy and knowledge of how Work is done on a fluid .

$$E_o + Wnc = E_f$$

and

$$W_{nc} = (\Delta P)V$$

produce

**Bernoulli's Equation:**

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

# Explanation of Derivation of Bernoulli's Equation

Non-Conservative Work = Change in Mechanical Energy

$$W_{nc} = \Delta(KE + PE)$$

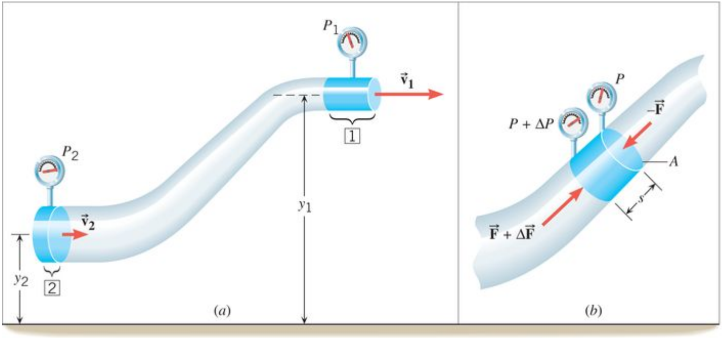


Figure: 24.9 Pressure drop in a pipe

$$W_{fluid} = P(\Delta V) \; or \; (\Delta P)V$$

# Bernoulli's Equation Derivation

For us we have constant volume in the pipe so we use

$$W_{nc} = (\Delta P)V = (P_2 - P_1)V$$

Let's use the definition of $W_{nc}$

$$W_{nc} = (KE_f + PE_f) - (KE_o + PE_o)$$

To get

$$(P_2 - P_1)V = (\frac{1}{2}mv_2^2 + mgy_2) - (\frac{1}{2}mv_1^2 + mgy_1)$$

# Bernoulli's Equation

$$(P_2 - P_1)V = (\frac{1}{2}mv_2^2 + mgy_2) - (\frac{1}{2}mv_1^2 + mgy_1)$$

Let's divide by volume $V$ and use the definition of density $\rho$

$$P_2 - P_1 = (\frac{1}{2}\rho v_2^2 + \rho g y_2) - (\frac{1}{2}\rho v_1^2 + \rho g y_1)$$

Now let's move all of the initial (1) and final values (2) to separate sides respectively

**Bernoulli's Equation:**

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

# Motivation Answered

$$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

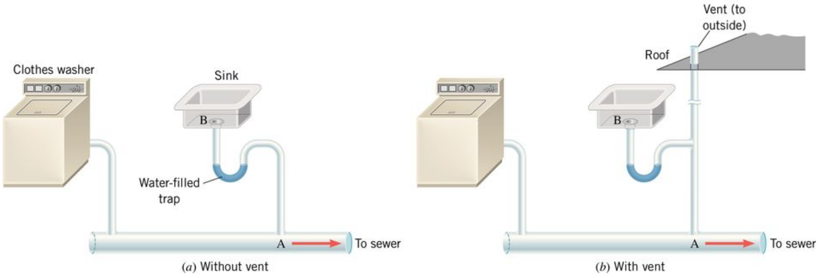


Figure: 24.1 Good vs bad plumbing

"The vent ensures that the pressure at A remains the same as that at B (atmospheric pressure), even when water from the clothes washer is rushing through the pipe. Thus, the purpose of the vent is to prevent the trap from being emptied, not to provide an escape route for sewer gas." - C&J pg. 311

# Exercise #4

**C&J 11.64** Water flowing out of a horizontal pipe emerges through a nozzle. The radius of the pipe is $1.9\ cm$, and the radius of the nozzle is $0.48\ cm$. The speed of the water in the pipe is $0.62\ m/s$. Treat the water as an ideal fluid, and determine the absolute pressure of the water in the pipe.

# Exercise #4 Start

We need to use the continuity equation

$$\rho A_1 v_1 = \rho A_2 v_2$$

as well as Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y$$

to solve for the absolute pressure $P_1 = P_p$ in the pipe.

We're given the quantities:

$$v_1 = 0.62\ m,\ r_1 = 1.9\ cm,\ r_2 = 0.48\ cm,$$

and it's implied that the pressure at the end of the nozzle is atmospherice pressure

$$P_2 = P_{atm} = 101.325\ kPa.$$

# Exercise #4 Answer

From continuity we get the relationship for the fluid speed coming out of the nozzle

$$v_n = \left(\frac{r_p}{r_n}\right)^2 v_p.$$

Using that result and the fact that the second pressure is atmospheric we get

$$P_p = P_{atm} + \frac{1}{2}\rho(v_n^2 - v_p^2)$$

$$P_p = 105.9\ kPa$$

# Exercise #5

**C&J 11.65** The blood speed in a normal segment of a horizontal artery is $0.11\ m/s$. An abnormal segment of the artery is narrowed down by an arteriosclerotic plaque to one-fourth the normal cross-sectional area. What is the difference in blood pressures between the normal and constricted segments of the artery?

# Exercise #5 Start

Similar physics as previous question, but this time we can't assume anything about what each absolute pressure is so we can only solve for the differences in pressure utilizing the same equations. Approximate by treating blood as an ideal fluid.

Let's use the relationships

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y$$

with the information $v_1 = 0.11\ m/s$, $A_2 = A_1/4$, and for whole blood at $37\ ^{\circ}C$, $\rho_{blood} = 1060\ kg/m^3$.

# Exercise #5 Answers

From continuity we get that the speed of the fluid after the arterial narrowing is 4 times the speed before

$$v_2 = 4v_1$$

From Bernoulli's equation we can approxmiate the pressure difference as

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$\Delta P = \frac{1}{2}\rho(1 - 16)v_1^2$$

Now we can use $\rho_b = 1060 kg/m^3$

$$\Delta P = -\frac{15}{2}(1060\ kg/m^3)(0.11\ m/s)^2$$

$$\Delta P \approx -96\ Pa = -0.96\ mbar$$

# Summary of New Topics

**For the simplest motions of flowing fluids:**

- Buoyancy Force: $F_b = \rho g V$
- Continuity of Mass Flow: $(\rho A v)_{in} = (\rho A v)_{out}$
- Ideal fluids are incompressible and non-viscous
- Real fluids are compressible and viscous, but some are close to ideal
- Work done by fluids: $W_{nc} = P(\Delta V)\ or\ V(\Delta P)$
- Bernoulli's Equation: $P + \frac{1}{2}\rho v^2 + \rho g y = constant$