Name: Solutions (Set-up + Answers)

Homework Week # 9

Rotational Motion
Due Thurs 10/24/19

Reading

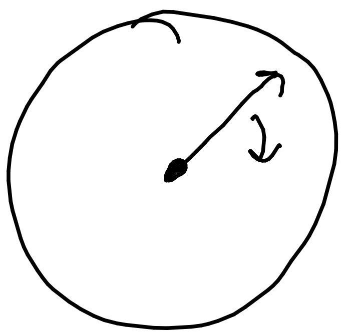
C&J Physics:	**Tues – Ch. 9: 1-4**	**Thurs – Ch. 9: 4-7**
OS Coll Phys:	**Tues – Ch. 9: 1-6**	**Thurs – Ch. 10**

Problems

Problem 1.	**FOC 8.3**
Problem 2.	**FOC 8.7**
Problem 3.	**8.2**
Problem 4.	**8.4**
Problem 5.	**8.6**
Problem 6.	**8.20**
Problem 7.	**8.21**
Problem 8.	**8.22**
Problem 9.	**8.34**
Problem 10.	**8.42**

Prob. 1 FOC 8.3 1/10

Determine t for an arc of $s = 24.0\,cm$ for a second hand of a circular clock that has a radius of $r = 6.00\,cm$.



for a second hand

$$V = \frac{2\pi r}{T}, \quad T = 60.0\,s$$

if v const.

$$V = \frac{s}{t} = \frac{2\pi r}{T}$$

$$\frac{t}{s} = \frac{T}{2\pi r}$$

$$t = \frac{s}{2\pi r} T$$

$$t = \frac{24.0\,cm}{2\pi(6.00\,cm)}(60.0\,s) = \frac{2.00}{\pi}(60.0)\,s$$

$$\boxed{t = 38.2\,s}$$

Prob. 2 FOC 8.7

$v_0 = 0, \omega_0 = 0$ for const. α. @ $t_1 = 7.0\,s$, $\omega_1 = 16\,\frac{rad}{s}$. Find ω_2 @ $t_2 = 14.0\,s$

Thinking: const. α & data given means ω increases by $16\,\frac{rad}{s}$ every $7.0\,s$

so $\boxed{\omega_2 = 32\,\frac{rad}{s}}$ b/c $7.0\,s$ more have passed @ $14.0\,s$.

Math: $\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{16}{7.0}\,\frac{rad}{s^2} = 2.3\,\frac{rad}{s^2}$

$\omega = \omega_0 + \alpha t$ $\qquad$ $\omega_2 = \alpha t_2$

$\omega_0 = 0$ $\longrightarrow$ $\boxed{\omega_2 = 32\,\frac{rad}{s^2}}$

Prob. 3 8.2 3/10

Angle data given for $\Delta t = 2.0\,s$ intervals. Calculate $\bar{\omega}$ for each pair.

$$\bar{\omega} = \frac{\theta_f - \theta_0}{\Delta t}$$

Use 4 times!

Note θ_f is labeled θ in the table.

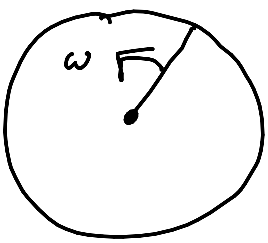
$\bar{\omega}_1 = +0.15 \frac{rad}{s}$ $\bar{\omega}_3 = -0.6 \frac{rad}{s}$

$\bar{\omega}_2 = -0.2 \frac{rad}{s}$ $\bar{\omega}_4 = +0.4 \frac{rad}{s}$

Prob. 4 8.4

Our sun rotates about a circle of $r = 2.2 \times 10^{20}$ m at a constant $\omega = 1.1 \times 10^{-15} \frac{\text{rad}}{s}$.

Calculate Δt for a full revolution in units of Earth years



Note $v = \frac{2\pi r}{T}$ means

$\omega = \frac{2\pi}{T}$

for a full revolution.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.1} \times 10^{+15} \text{ s}$$

conversion factor : $1 \text{ yr} = (365.25)(24)(3600) \text{ s}$

$$T = \left(\frac{2\pi}{1.1} \times 10^{15} \cancel{s}\right) \frac{1 \text{ yr}}{(365.25)(24)(3600) \cancel{s}}$$

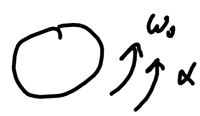
$$\boxed{T = 1.81 \times 10^{8} \text{ yr} \cong 181 \text{ million years}}$$

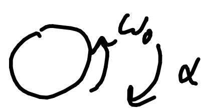
Prob. 5 8.6

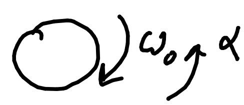
Given initial velocity and a constant acceleration, predict velocity after 2.0 s.

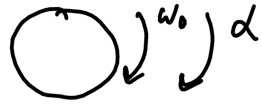
$$\alpha = \frac{\omega - \omega_0}{t} \iff \omega = \omega_0 + \alpha t$$

Use 4 sets of ω_0, α. (CCW +)*

$\omega_1 = 18 \frac{rad}{s}$ 

$\omega_2 = 6 \frac{rad}{s}$ 

$\omega_3 = -6 \frac{rad}{s}$ 

$\omega_4 = -18 \frac{rad}{s}$ 

* Please think about what type of rotation* each can be describing.

Prob. 6 8.20

$\omega_0 = +15 \frac{rad}{s}$, $\Delta\theta = \theta = +5.1\, rad$, $\omega = 0$

Calculate $\bar{\alpha}$, t

$\bar{\alpha} = \frac{\omega - \omega_0}{\Delta t}$ & we can assume

$\bar{\alpha} = \alpha$ since $\Delta t = t$ is small. (brief)

$\checkmark\omega^2 = \overset{\checkmark}{\omega_0}{}^2 + 2\overset{?}{\alpha}\,\overset{\checkmark}{\Delta\theta}$

$\bar{\alpha} = \frac{-\omega_0}{\Delta t}$

$t = \frac{-\omega_0}{\bar{\alpha}}$

$$\boxed{\begin{aligned}\bar{\alpha} &= -22.1 \frac{rad}{s^2} \\ t &= 0.68\, s\end{aligned}}$$

given ω_0 and $\Delta\theta$ both positive, $\bar{\alpha}$ should be negative and $t = 0.68s$ sounds brief.

Prob. 7 8.21 7/10

Calculate time (t) for a velocity change of 3.00 to 5.00 rev/s through a rotation of one-half revolution

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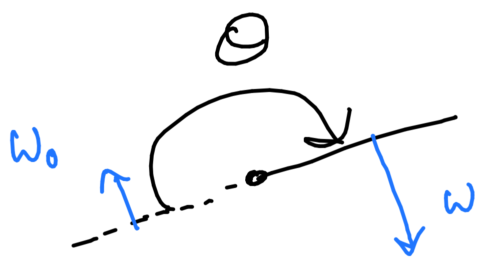

$$\theta = \frac{1}{2}(\omega_0 + \omega)t$$

$$t = \frac{2\theta}{\omega_0 + \omega}$$

$$t = \frac{2\left(\frac{1}{2}\text{ rev}\right)}{8.00\ \frac{\text{rev}}{\text{s}}}$$

No unit * conversion necessary!

$$\boxed{t = 0.13\text{ s}}$$

<u>Prob. 8</u> 8.22

$\omega_0 = 420\,\frac{rad}{s} \rightarrow \omega = 1420\,\frac{rad}{s}$ in $t = 5.00\,s$

Calculate $\Delta\theta$ and α for this.

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assume α const.

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$\boxed{\alpha = 200\,\frac{rad}{s^2} \qquad \Delta\theta = 4600\,rad}$$

$$4600\,rad \approx 732\,rev$$

Prob 9 8.34

For $\alpha = +12.0 \frac{rad}{s^2}$ find r @ which

$a_T = g$.

$a_T = r\alpha$

$g = 9.81 \frac{m}{s^2} \approx 10.0 \frac{m}{s^2}$

$r = \frac{a_T}{\alpha}$

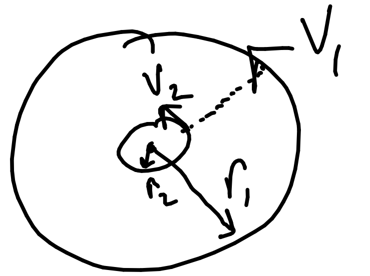
For $a_T = g$ $r = \frac{g}{\alpha}$

$\boxed{r \cong 0.83\,m}$

10/10

Prob. 10   8.42

move a wider, but connected circle @ const. V_1. Relate rotational quantities to solve for a linear speed if there is rolling w/o slipping.



$\rightarrow$

$\downarrow V_2$ b/c rolling w/o slipping

$$V_1 = r_1 \omega, \quad V_2 = r_2 \omega$$

$$\omega = \frac{V_1}{r_1}$$

$V_1 = 1.20 \, m/s$

$r_1 = 0.200 \, m$

$r_2 = 0.050 \, m$

$$V_2 = r_2 \omega$$

$$\boxed{V_2 = 0.30 \, \frac{m}{s}}$$