Phys 111: Lecture 18

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“The 18th Day: The Dread Isle”

Homework Wk #9 Due Today

Homework Wk #10 Due Exam Day T 10/29/19

Today’s Topics

1. Newton’s Second Law in Torque Form
2. Rotational Work & Energy
3. Angular Momentum

Motivation

Why does this happen when they have the same mass?

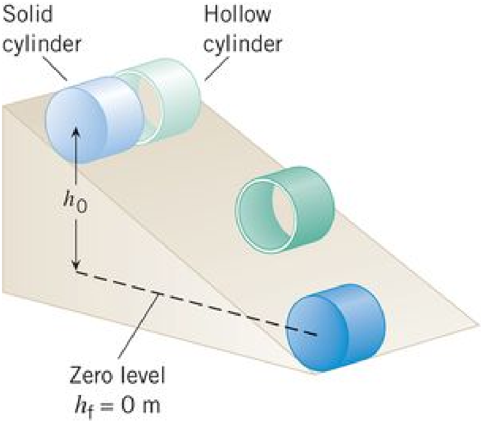

Figure: 18.1 Different rotational inertia with same mass

Comparing Force & Torque

Force causes linear accleration.

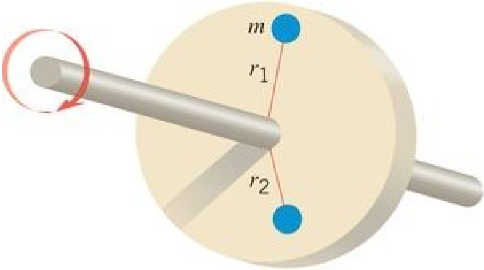

Figure: 18.2 analogues of Newton's second law

Torque causes rotational acceleration

Math for Force & Torque

If mass is linear inertia

$$\sum F = ma$$

is there any reason to not expect a **rotational inertia**?

$$\sum \tau = I\alpha$$

Utilizing Rotational Kinematics

Tangential and angular acceleration.

$$a_t = r\alpha$$

Then

$$F = ma$$

$$F = mr\alpha.$$

$$\tau = rF.$$

$$\tau = r(mr\alpha) = mr^2\alpha.$$

Therefore

$$I_{pt} = mr^2$$

Rotational Inertia

Rotational Inertia I is a measurement of how extended the mass of an object is from an axis of rotation or pivot point. Denoted I and called the Moment of Inertia.

Moment of Inertia for a point mass is

$$I_{pt} = mr^2$$

Any object can be thought of as a collection of point masses

$$I = \sum mr^2$$

Visualizing Rotational Inertia

$$I = \sum mr^2$$

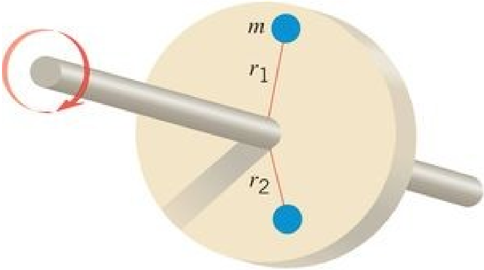

Figure: 18.2 reused

Exercise #1

C&J 9.32 A clay vase on a potter's wheel experiences an angular acceleration of $5.00\ rad/s^2$ due to the application of a $15.0N \cdot m$ net torque. Find the total moment of inertia of the vase and potter's wheel.

Translation Find a total moment of inertia given a net torque and a rotational acceleration.

$$\tau = I\alpha$$

Therefore

$$I = \frac{\tau}{\alpha}$$

$$I = 0.33\ kg \cdot m^2$$

Exercise #2

C&J 9.33 A solid circular disk has a mass of $M = 1.4\ kg$kg and a radius of $r = 0.18\ m$. Each of three identical thin rods has a mass of $m = 0.18\ kg$. The rods are attached perpendicularly to the plane of the disk at its outer edge to form a three-legged stool. Find the moment of inertia of the stool with respect to an axis that is perpendicular to the plane of the disk at its center.

Figure: 18.3 Multi-part object

Exercise #2

C&J 9.33 The object is a disk with three masses that are placed parallel to the axis through the center. $M = 1.4\ kg$, $m = 0.18\ kg$, $r = 0.18\ m$.

$$I = I_{disk} + 3I_{leg}$$

$$I = \frac{1}{2}Mr^2 + 3mr^2$$

Figure: 18.3 Multi-part object

Rotational Kinetic Energy

Rotational Kinetic Energy is the energy associated with an objects rotational velocity. Just one more type of energy.

$$KE = \frac{1}{2}mv^2$$

$$RKE = \frac{1}{2}I\omega^2$$

$$Mechanical\ Energy = KE + RKE + PE$$

Exercise #3

C&J 9.51 A flywheel is a solid disk that rotates about an axis that is perpendicular to the disk at its center. Rotating flywheels provide a means for storing energy in the form of rotational kinetic energy and are being considered as a possible alternative to batteries in electric cars. The gasoline burned in a $300\ mile$ trip in a typical midsize car produces about $1.210^9\ J$ of energy. How fast would a $15\ kg$ flywheel with a radius of $0.40\ m$ have to rotate to store this much energy? Give your answer in rev/min.

Exercise #3

C&J 9.51 What ω is necessary to store $RKE = 1.210^9\ J$ with a $15\ kg$ flywheel with a radius of $0.40\ m$. Give your answer in rev/min.

Need to know

$$I_{disk} = \frac{1}{2}mr^2$$

$$RKE = \frac{1}{2}I\omega^2$$

$$\omega^2 = \frac{2(RKE)}{I}$$

$$\omega = \sqrt{\frac{2(RKE)}{I}}$$

Angular Momentum

Surprise, surprise! There's an **angular momentum.**

Think about impulse

$$F\Delta t = \Delta p.$$

What should a torque cause?

$$\tau\Delta t = ??$$

It should be the change in angular momentum!

Angular Momentum Defined

Angular Momentum L is total rotational motion of a system and it is measured by how fast something is rotating and how hard it is to change that rotation.

Math Version

$$L = I\omega$$

Exercise #4

C&J 9.59 Two disks are rotating about the same axis. Disk A has a moment of inertia of $I_A = 3.4\, kg \cdot m^2$ and an angular velocity of $\omega_A = +7.2\, rad/s$. Disk B is rotating with an angular velocity of $\omega_B = -9.8\, rad/s$. The two disks are then linked together without the aid of any external torques, so that they rotate as a single unit with an angular velocity of $\omega_f = -2.4\, rad/s$. The axis of rotation for this unit is the same as that for the separate disks. What is the moment of inertia of disk B?

Exercise #4

C&J 9.59 Use conservation of angular momentum to determine the moment of inertia for a disk if you know information about angular speeds and the moment of inertia of the other disk.

Exercise #5

C&J Example 9.15 Assume the skater in Figure 18.4(a) has a moment of inertia I_0 and is rotating with an angular velocity ω_0. When she brings her arms in close as in Figure 18.4(b) assume her rotational velocity doubles. Use conservation of angular momentum to determine whether or not her rotational kinetic energy is conserved.

Exercise #5

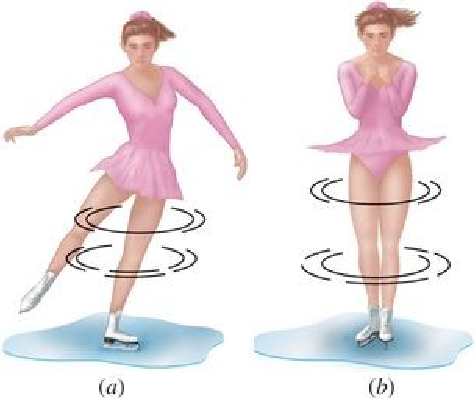

Figure: 18.4 Figure skater example

Exercise # 5

C&J Example 9.15 Use conservation of angular momentum to determine whether or not her rotational kinetic energy is conserved.

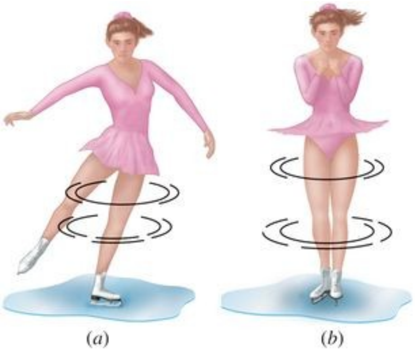

Figure: 18.4 Figure skater example

Exercise # 6

C&J 9.49 Three objects lie in the x, y plane. Each rotates about the z axis with an angular speed of $\omega = 5.00\ rad/s$. The mass m of each object and its perpendicular distance r from the z axis are as follows:

1. $m_1 = 6.00\ kg$ and $r_1 = 2.00\ m$
2. $m_2 = 4.00\ kg$ and $r_2 = 1.50\ m$
3. $m_3 = 3.00\ kg$ and $r_3 = 3.00 m$.

Exercise # 6

C&J 9.49 Three objects rotate about the z axis with an angular speed of $\omega = 5.00\ rad/s$. We're given mass and radii for each.

a. Find the tangential speed of each object.
b. Determine the total kinetic energy of this system using the expression

$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$$

c. Obtain the moment of inertia of the system.
d. Find the rotational kinetic energy of the system using the relation

$$RKE = \frac{1}{2}I\omega^2$$

to verify that the answer is the same as the answer to (b).

Exercise # 6

C&J 9.49 Three objects rotate about the z axis with an angular speed of $\omega = 5.00\, rad/s$. We're given mass and radii for each.

a. Find v of each object.
b. Determine the total kinetic energy of this system.
c. Obtain I of the system.
d. Find the total rotational kinetic energy of the system.