Phys 111: Lecture 22

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“The 22nd Day: $(7)(3) + 1$”

Homework Wk #11 Due Today

Today’s Topics

1. Pendulum
2. Realistic Simple Harmonic Motion
3. Pressure

Motivation

Why are/were pendulums used in clocks?

Robert Mathena/Fundamental Photographs

Figure: 22.1 Clock utilizing a pendulum to tell time

A Different SHM

Where have we seen this type of graph before?

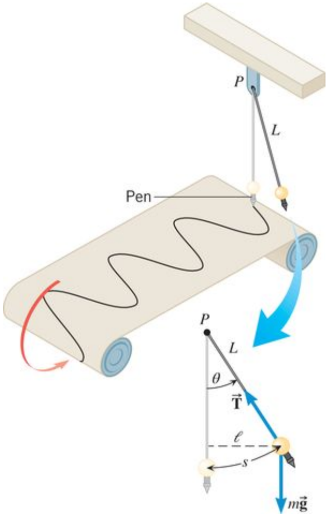

Figure: 22.2 Simple harmonic motion of a pendulum

Quantifying SHM of a Pendulum

How would we determine the frequency of a pendulum in general?

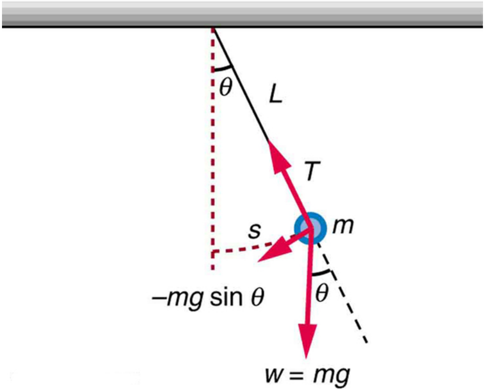

Figure: 22.3 Force [and torque] on a pendulum

Derivation of the Frequency of a Pendulum

Gravity produces a torque on the pendulum and similar to a spring force $F_{sp} = -kx$, the torque has the form:

$$\tau_g \approx -mgL\theta$$

when the angle displacement is small. [small $\theta \longrightarrow sin(\theta) \approx \theta$]

Utilizing $\sum \tau = I\alpha$ and $I = mL^2$ results in

$$(-mgL\theta) = (mL^2)\alpha$$

Which simplifies to

$$\alpha = -\left(\frac{g}{L}\right)\theta$$

Derivation Continued

Whenever an equation of the form

$$\alpha = -A\theta$$

occurs one can determine ω because the solution for such a 2nd order rate equation includes the relation

$$\omega^2 = A.$$

In our case we have

$$\omega^2 = \frac{g}{L}.$$

Which results in

$$\omega = \sqrt{\frac{g}{L}}.$$

Reading Graphs for Simple Harmonic Motion

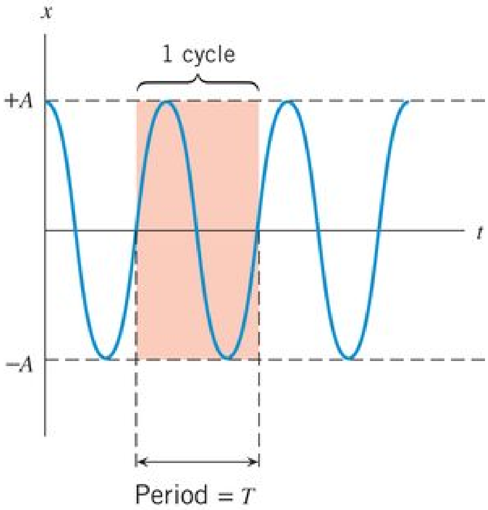

Figure: 22.3 Example graph for simple harmonic motion

Simple Harmonic Motion Universal Equations

The following equations can be used for modeling any simple harmonic motion that begins with max amplitude:

$$\theta = \omega t$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$x = Acos(\omega t), \qquad x_{max} = A$$

$$v_x = -A\omega sin(\omega t), \quad v_{max} = A\omega$$

$$a_x = -A\omega^2 cos(\omega t), \quad a_{max} = A\omega^2$$

Exercise #1

C&J 10.43 A simple pendulum is made from a $0.65\ m$ long string and a small ball attached to its free end. The ball is pulled to one side through a small angle and then released from rest. After the ball is released, how much time elapses before it attains its greatest speed?

Exercise #1 Translation

C&J 10.43 Utilize the relationships $v = -A\omega sin(\omega t)$ and $\omega = \sqrt{g/L}$ as well as the information $L = 0.65\ m$ to determine at what time a mass at the end of a pendulum will reach max speed?

Exercise #1 Answer

C&J 10.4 How much time elapses before a pendulum mass attains its greatest speed if the length of the pendulum is $0.65\ m$?

Max speed occurs when

$$sin(\omega t) = 1$$

Which occurs at

$$\omega t = \frac{\pi}{2}.$$

Therefore

$$t = \frac{\pi}{2\omega}$$

and don't forget that

$$\omega = \sqrt{\frac{g}{L}}.$$

Exercise #1 Finished

C&J 10.4 How much time elapses before a pendulum mass attains its greatest speed if the length of the pendulum is $0.65\ m$?

Our useful version of this relationship is

$$t = \left(\frac{\pi}{2}\right)\sqrt{\frac{L}{g}}$$

and with our numbers we get

$$t = \left(1.571\right)\sqrt{0.065\ s^2}$$

$$t = 1.571 \times 0.255\ s$$

$$t = 0.40\ s$$

Exercise #2

C&J 10.45 The length of a simple pendulum is $0.79\ m$ and the mass of the particle (the bob) at the end of the cable is $0.24\ kg$. The pendulum is pulled away from its equilibrium position by an angle of 8.50° and released from rest. Assume that friction can be neglected and that the resulting oscillatory motion is simple harmonic motion.

a. What is the angular frequency of the motion?
b. Using the position of the bob at its lowest point as the reference level, determine the total mechanical energy of the pendulum as it swings back and forth.
c. What is the bob's speed as it passes through the lowest point of the swing?

Exercise #2 Physical Relationships

C&J 10.45 Simple pendulum with $L = 0.79\ m$, $m = 0.24\ kg$, and an initial angular displacement of $\theta_o = 8.5°$.

$$E_o = E_f$$

$$PE_o = PE_f + KE_f$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$PE_o = mgL\Big[1 - cos\theta_o\Big]$$

$$PE_{low} = 0$$

Exercise #2

C&J 10.45 Simple pendulum with $L = 0.79\,m$, $m = 0.24\,m$, and an initial angular displacement of $\theta_o = 8.5°$.

a. What is the angular frequency of the motion?

$$\omega = 3.56\,rad/s$$

b. Using the position of the bob at its lowest point as the reference level, determine the total mechanical energy of the pendulum as it swings back and forth.

$$E_{tot} = PE_o = 21\,mJ$$

c. What is the bob's speed as it passes through the lowest point of the swing?

$$v = 0.42 m/s$$

Damped Harmonic Motion

How do we quantify the process of harmonic motion diminishing?

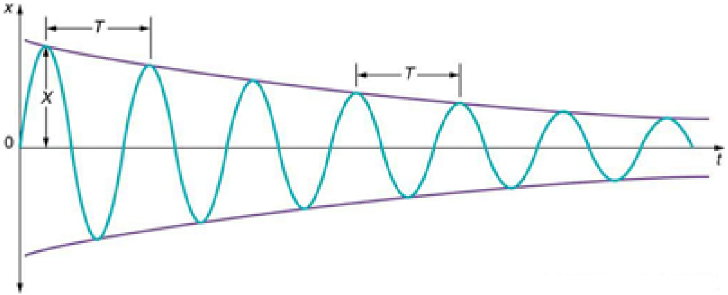

Figure: 22.4 Example graph of underdamped oscillations

The three types of damped harmonic motion are

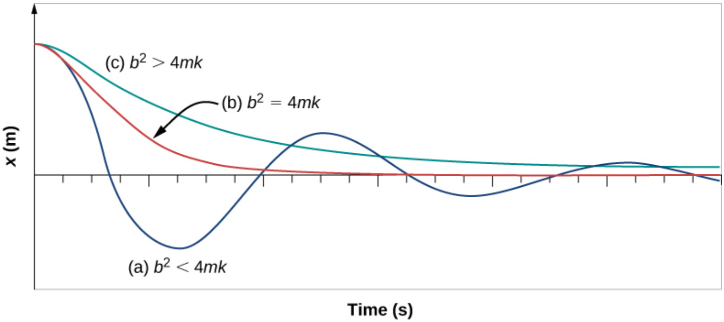

Figure: 22.5 (a) Under, (b) Critically, and (c) Over damped oscillations

Driven Harmonic Motion

What is special about forcing the ball at the frequency f_o?

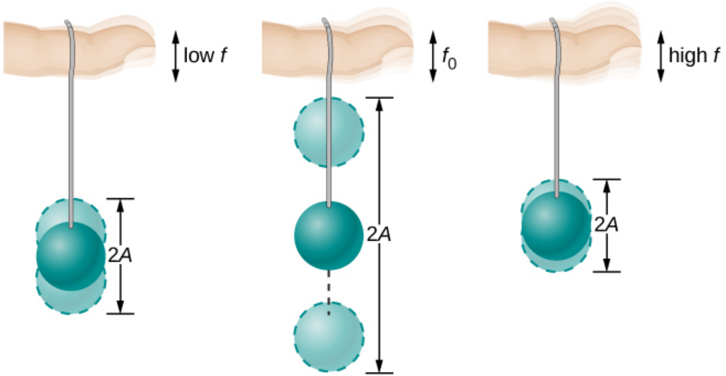

Figure: 22.6 Driven oscillations: Under, Resonant, and Over driven systems

Driven & Damped Harmonic Motion

In reality, a system can be driven and damped.

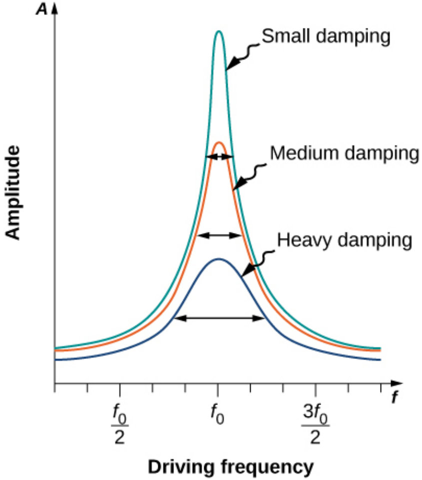

Figure: 22.7 Driven and damped oscillations

Exercise #3

C&J 10.15 When responding to sound, the human eardrum vibrates about its equilibrium position. Suppose an eardrum is vibrating with an amplitude of $6.3 \times 10^{-7}\ m$ and a maximum speed of $2.9 \times 10^{-3}\ m/s$.

a. What is the frequency (in Hz) of the eardrum's vibration?
b. What is the maximum acceleration of the eardrum?

Exercise #3 Answer

C&J 10.15 Consider SHM with an amplitude of $6.3 \times 10^{-7}\ m$ and a maximum speed of $2.9 \times 10^{-3}\ m/s$. The following answers were calculated using the relationships

$$v_{max} = A\omega,$$

$$a_{max} = A\omega^2,\ and$$

$$\omega = 2\pi f$$

a. What is the frequency (in Hz) of the eardrum's vibration?

$$f = 733\ Hz$$

b. What is the maximum acceleration of the eardrum?

$$a_{max} = 13.3\ m/s^2$$

Mini Break

Any questions before we continue with Pressure?

Before we move on, let's also demonstrate the discussed phenomena.

Regime of Simple Harmonic Motion

Stress, σ, is a specific type of pressure and Strain, ϵ, is a percent change in length.

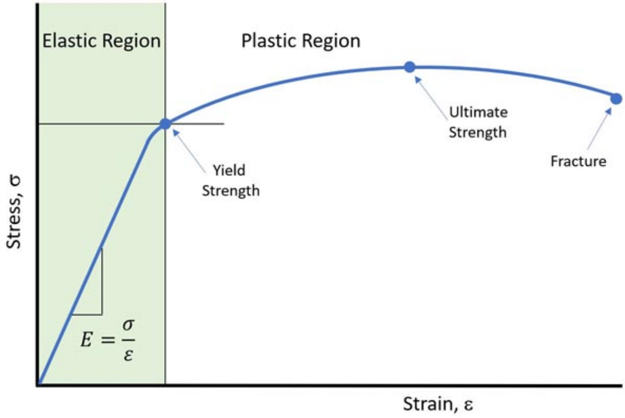

Figure: 22.8 Ideal stress-strain[2] curve for material tests

[2] https://www.simsolid.com/faq-items/material-properties/

Pressure

The **pressure P** exerted on a surface is defined as the magnitude of the force acting perpendicular to a surface ($F_\perp$) divided by the area A over which the force F acts:

$$P = \frac{F_\perp}{A}$$

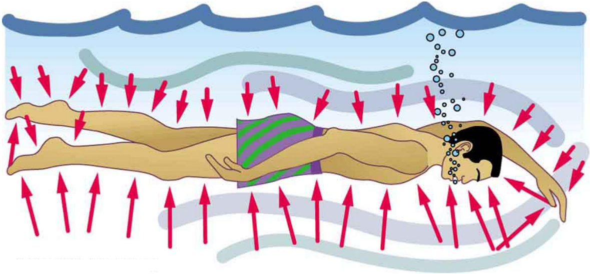

Figure: 22.9 Pressure exerted on all sides of a swimmer

Hydraulics Application/Motivation

Why can brake pedals in a car be so small?

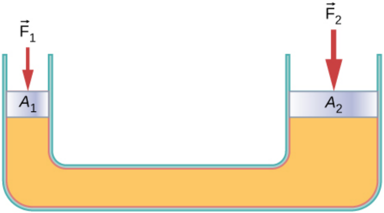

Figure: 22.10 Why does the car lift if a smaller area is used for the original force?